recall that

$$p_\mu = (w_{\vec{p}}, \vec{p})$$

with

$$w_{\vec{p}} = \sqrt{|\vec{p}|^2 + m^2} > 0$$

the <u>relativistic energy</u> of a particle of 3-momentum $\vec{p}$ and (rest) mass m.

also

$$p_\mu p^\mu = p^2 = w_{\vec{p}}^2 - \vec{p}^2 = m^2$$

so this looks / is all consistent

Let me now discuss only one term in the 2nd line of (1.14). e.g.

$$\int d^3x \, d^3\tilde{p} \, d^3\tilde{q} \, \left[w_{\vec{p}} \, w_{\vec{q}} \, a(p) \, a^\dagger(q) \, e^{-i(p-q)x} \right]$$

here

$$d^3\tilde{p} = \frac{d^3p}{(2\pi)^3}\,\frac{1}{2\omega_{\vec{p}}} \qquad \text{etc.}$$

we will also need

$$\int d^3x \; e^{-i(\vec{p}-\vec{q})\vec{x}} = (2\pi)^3\,\delta^{(3)}(\vec{p}-\vec{q})$$

we first write out the scalar product in the exponential

$$\int d^3x\, d^3\tilde{p}\, \frac{d^3q}{(2\pi)^3}\,\frac{1}{2\omega_{\vec{q}}} \Big[\omega_{\vec{p}}\,\omega_{\vec{q}}\, a(p)\, a^\dagger(q) \cdot e^{-i[(\omega_{\vec{p}}-\omega_{\vec{q}})\cdot t - (\vec{p}-\vec{q})\cdot\vec{x}]} \Big]$$

now we integrate over d^3x

$$\int d^3\tilde{p}\, \frac{d^3q}{(2\pi)^3}\,\frac{1}{2\omega_{\vec{q}}} \Big[\omega_{\vec{p}}\,\omega_{\vec{q}}\, a(p)\, a^\dagger(q)\, e^{-i(\omega_{\vec{p}}-\omega_{\vec{q}})\cdot t}\, (2\pi)^3\,\delta^{(3)}(\vec{q}-\vec{p}) \Big]$$

now we integrate over d^3q using $\delta^{(3)}(\vec{q}-\vec{p})$
which sets

$$\vec{q} = \vec{p}$$

we get

$$\int d^3\tilde{p}\ \frac{1}{2w_{\vec{p}}}\left[w_{\vec{p}}\, w_{\vec{p}}\ a(p)\, a^{\dagger}(p)\ e^{-i(w_{\vec{p}}-w_{\vec{p}})\cdot t}\right]$$

note that I identified

$$a^{\dagger}(q) = a^{\dagger}(p)$$

this is correct since

$$\vec{q} = \vec{p}$$

implies

$$w_{\vec{q}} = w_{\vec{p}}$$

and thus

$$q_\mu = (w_{\vec{q}}, \vec{q}) = (w_{\vec{p}}, \vec{p}) = p_\mu$$

the latter expression can be simplified to

$$\frac{1}{2}\int d^3\tilde{p} \; w_{\tilde{p}} \; a(p)\, a^{+}(p)$$

which appears in the last line in (1.14) and also in (1.15)

The other terms of (1.15) are done in a similar fashion

hope this helps!