

# Parton shower from QCD (II): An implementation based on the dipole formalism

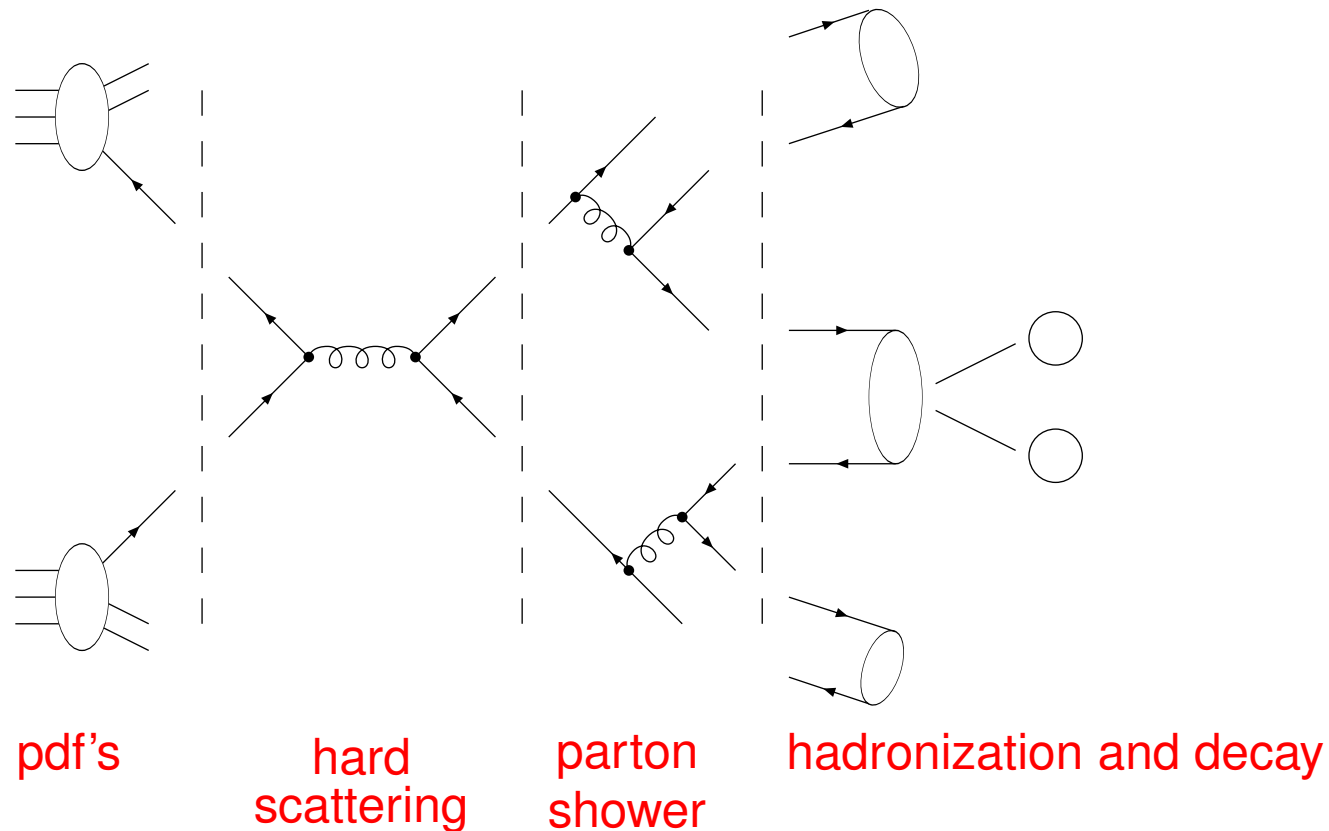
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- Introduction:** **Event generators and perturbative calculations**
- I.:** **Parton showers**
- II.:** **Showers from dipoles**
- III.:** **Numerical results**

# Event generators



**Underlying event:**

Interactions of the proton remnants.

**Multiple interactions:**

more than one pair of partons undergo hard scattering

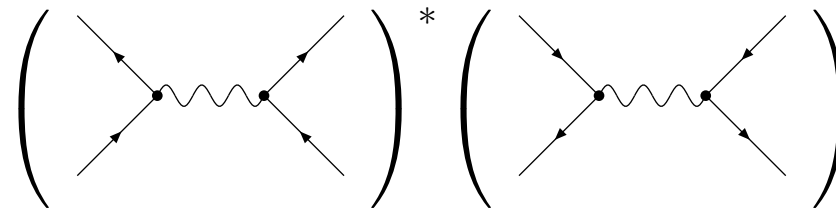
**Pile-up events:**

more than one hadron-hadron scattering within a bunch crossing

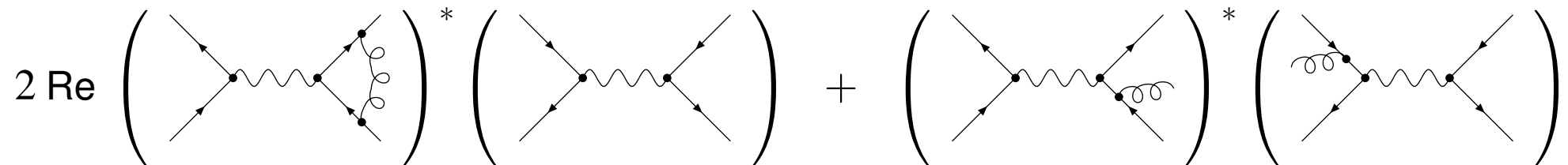
# Exact perturbative calculations

Leading order (LO) and next-to-leading order (NLO):

At leading order only **Born amplitudes** contribute:



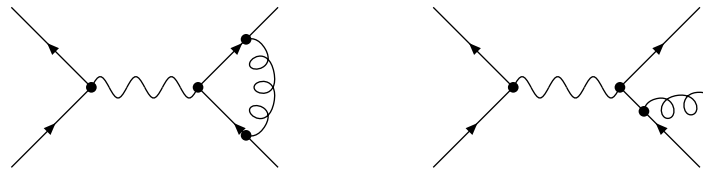
At next-to-leading order: **One-loop amplitudes** and Born amplitudes with an additional parton.



# Infrared divergences and the Kinoshita-Lee-Nauenberg theorem

In addition to ultraviolet divergences, **loop integrals** can **have infrared divergences**.

For each IR divergence there is a **corresponding divergence with the opposite sign** in the real emission amplitude, when particles becomes **soft** or **collinear** (e.g. unresolved).



The **Kinoshita-Lee-Nauenberg** theorem: Any observable, summed over all states degenerate according to some resolution criteria, will be finite.

# General methods at NLO

Fully differential **NLO Monte Carlo programs** need a general method to handle the cancelation of infrared divergencies.

- **Phase space slicing**

- $e^+e^-$ : W. Giele and N. Glover, (1992)
- **initial hadrons**: W. Giele, N. Glover and D.A. Kosower, (1993)
- **massive partons, fragmentation**: S. Keller and E. Laenen, (1999)

- **Subtraction method**

- **residue approach**: S. Frixione, Z. Kunzst and A. Signer, (1995)
- **dipole formalism**: S. Catani and M. Seymour, (1996)
- **massive partons**: L. Phaf and S.W. (2001), S. Catani, S. Dittmaier, M. Seymour and Z. Trócsányi, (2002)

## The dipole formalism

The dipole formalism is based on the subtraction method. The NLO cross section is rewritten as

$$\begin{aligned}\sigma^{NLO} &= \int_{n+1} d\sigma^R + \int_n d\sigma^V \\ &= \int_{n+1} (d\sigma^R - d\sigma^A) + \int_n \left( d\sigma^V + \int_1 d\sigma^A \right)\end{aligned}$$

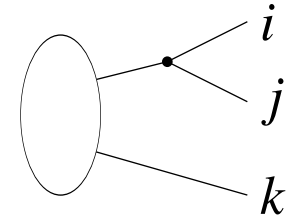
The approximation  $d\sigma^A$  has to fulfill the following requirements:

- $d\sigma^A$  must be a proper approximation of  $d\sigma^R$  such as to have the **same pointwise singular behaviour in  $D$  dimensions** as  $d\sigma^R$  itself. Thus,  $d\sigma^A$  acts as a local counterterm for  $d\sigma^R$  and one can safely perform the limit  $\varepsilon \rightarrow 0$ .
- **Analytic integrability in  $D$  dimensions** over the one-parton subspace leading to soft and collinear divergences.

## The subtraction terms

The approximation term  $d\sigma^A$  is given as a sum over dipoles:

$$d\sigma^A = \sum_{\text{pairs } i,j} \sum_{k \neq i,j} \mathcal{D}_{ij,k}.$$



Each dipole contribution has the following form:

$$\mathcal{D}_{ij,k} = -\frac{1}{2p_i \cdot p_j} \mathcal{A}_n^{(0)*} (p_1, \dots, \tilde{p}_{(ij)}, \dots, \tilde{p}_k, \dots) \frac{\mathbf{T}_k \cdot \mathbf{T}_{ij}}{\mathbf{T}_{ij}^2} V_{ij,k} \mathcal{A}_n^{(0)} (p_1, \dots, \tilde{p}_{(ij)}, \dots, \tilde{p}_k, \dots).$$

- Colour correlations through  $\mathbf{T}_k \cdot \mathbf{T}_{ij}$ .
- Spin correlations through  $V_{ij,k}$ .

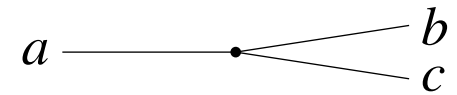
The dipoles have the correct soft and collinear limit.

## The physical origin of the correlations

- In the **soft limit**, amplitudes **factorize completely in spin space**, but **colour correlations** remain.
- In the **collinear limit**, amplitudes **factorize completely in colour space**, but **spin correlations** remain.  
Complete factorization after average over azimuthal angle.

# Basics of shower algorithm

Starting point: **Collinear factorization**.



**Probability** for particle  $a$  to split into particles  $b$  and  $c$ :

$$d\mathcal{P}_a = \sum_{b,c} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z) dt dz, \quad t = \ln \left( \frac{Q^2}{\Lambda^2} \right)$$

**Splitting kernels:**

$$P_{q \rightarrow qg}(z) = C_F \frac{1+z^2}{1-z},$$

$$P_{g \rightarrow gg}(z) = C_A \frac{(1-z(1-z))^2}{z(1-z)},$$

$$P_{g \rightarrow q\bar{q}}(z) = T_R (z^2 + (1-z)^2).$$

$P_{q \rightarrow qg}$  has a **soft singularity** for  $z \rightarrow 1$ ,  $P_{g \rightarrow gg}$  has a **soft singularity** for  $z \rightarrow 1$  and  $z \rightarrow 0$ .

# The Sudakov factor

Probability that a branching occurs during a small range of  $t$ :

$$dI(t) = dt \int_{z_-(t)}^{z_+(t)} dz \sum_{b,c} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z),$$

**Sudakov factor:** Probability that no branching occurs between  $t_0$  and  $t_1$ :

$$\Delta(t_1, t_0) = \exp \left( - \int_{t_0}^{t_1} dt \int_{z_-(t)}^{z_+(t)} dz \sum_{b,c} \frac{\alpha_s}{2\pi} P_{a \rightarrow bc}(z) \right)$$

## A typical shower algorithm

- Choose the **next scale**  $t$  according to the Sudakov factor.
- Choose the **momentum fraction**  $z$  according to  $P_{a \rightarrow bc}(z)$ .
- Choose the **azimuthal angle** uniform or according to spin-dependent splitting functions.
- **Insert the new particle.**
- If  $t > t_{min}$  goto first step, otherwise stop.

## Angular ordering

Amplitude for the emission of a soft gluon from a  $q\bar{q}$ -antenna:

$$d\sigma_g = d\sigma_0 \frac{\alpha_s C_F}{\pi} \frac{dk^0}{k^0} \frac{d\phi}{2\pi} d\cos\theta \frac{1 - \cos\theta_{q\bar{q}}}{(1 - \cos\theta_{qg})(1 - \cos\theta_{g\bar{q}})}$$

$$\frac{1 - \cos\theta_{q\bar{q}}}{(1 - \cos\theta_{qg})(1 - \cos\theta_{g\bar{q}})} = W_q + W_{\bar{q}}, \quad W_q = \frac{1}{2} \left[ \frac{\cos\theta_{g\bar{q}} - \cos\theta_{q\bar{q}}}{(1 - \cos\theta_{qg})(1 - \cos\theta_{g\bar{q}})} + \frac{1}{(1 - \cos\theta_{qg})} \right],$$

$$W_{\bar{q}} = \frac{1}{2} \left[ \frac{\cos\theta_{qg} - \cos\theta_{q\bar{q}}}{(1 - \cos\theta_{qg})(1 - \cos\theta_{g\bar{q}})} + \frac{1}{(1 - \cos\theta_{g\bar{q}})} \right].$$

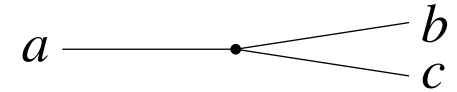
$$\int \frac{d\phi}{2\pi} W_q = \begin{cases} \frac{1}{1 - \cos\theta_{qg}}, & \text{if } \theta_{qg} < \theta_{q\bar{q}}, \\ 0 & \text{otherwise} \end{cases}$$

Angular ordering: No emission if  $\theta_{qg} > \theta_{q\bar{q}}$  !

# Momentum conservation

1. Momentum conservation:

$$p_a = p_b + p_c$$



2. Momenta are on-shell, for massless particles:

$$p_a^2 = p_b^2 = p_c^2 = 0.$$

3. Momenta are real.

For  $1 \rightarrow 2$  splittings it is not possible to satisfy all three requirements.

# Recent developments

- Rewriting and improvement of [Pythia](#), [Herwig](#) and [Ariadne](#), [Sherpa](#) as a new event generator  
Sjöstrand, Skands; Gieseke, Stephens, Webber; Lönnblad, Krauss, Kuhn, Schälicke, Soff;
- [Uncertainties](#) of parton showers  
Gieseke; Stephens, van Hameren; Bauer, Tackmann
- Parton showers in the framework of [SCET](#)  
Bauer, Schwartz, Tackmann, Thaler
- [Matching](#) of parton showers [with fixed-order tree level matrix elements](#)  
Catani, Krauss, Kuhn, Webber; Mangano, Moretti, Pittau; Mrenna and Richardson;
- [Matching](#) of parton showers [with NLO](#): → next page

# Recent developments

- Matching of parton showers with NLO

Frixione, Gieseke, Laenen, Latunde-Dada, Motylinski, Nason, Oleari, Ridolfi, Webber; Krämer, Mrenna, Soper; Odaka, Kurihara; Giele, Kosower, Skands;

- Parton shower based on the dipole formalism

- \* Foundation: Nagy, Soper, '05, '06

- \* For electron-positron annihilation very similar to Ariadne.

- \* Implementation: Schumann, Krauss '07 and Dinsdale, Ternick, SW. '07

## Parton shower based on the dipole formalism

$2 \rightarrow 3$  splittings: An emitter-spectator pair radiates off an additional particle.  
Can satisfy momentum conservation and on-shell conditions.

Splitting kernels of the Sudakov factors are given by the dipole splitting functions.  
Correct behaviour in the collinear and the soft limit.

No conceptual distinction between initial- and final-state shower.

Natural choice to combine with NLO

## Technical details

- 4 cases for emitter-spectator-pair: final-final, final-initial, initial-final, initial-initial.
- Only the singular terms of the dipole splitting functions are unique.
- Freedom to choose the finite terms.
- For a parton shower we would like to have a probabilistic interpretation:  
The splitting functions have to be non-negative everywhere.
  - Adjust finite terms
  - Rearrange terms between  $\mathcal{D}_{ij,k}$  and  $\mathcal{D}_{kj,i}$

## Technical details

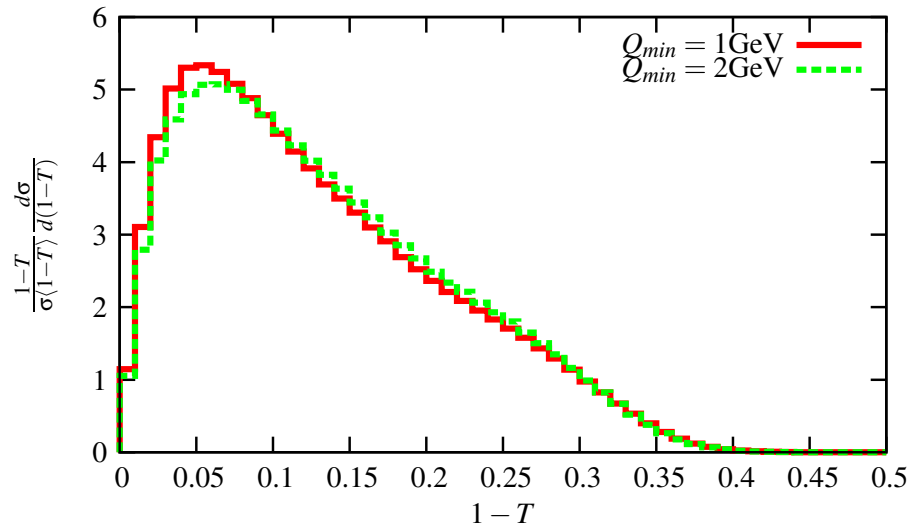
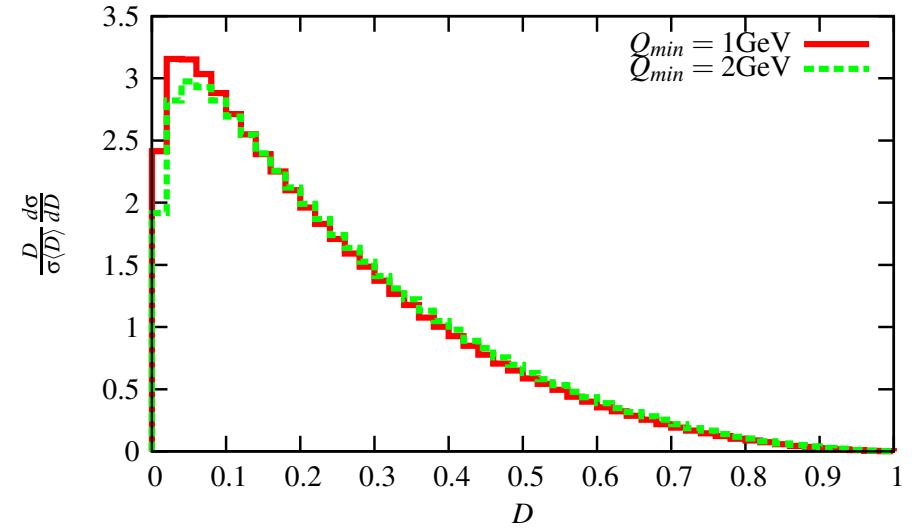
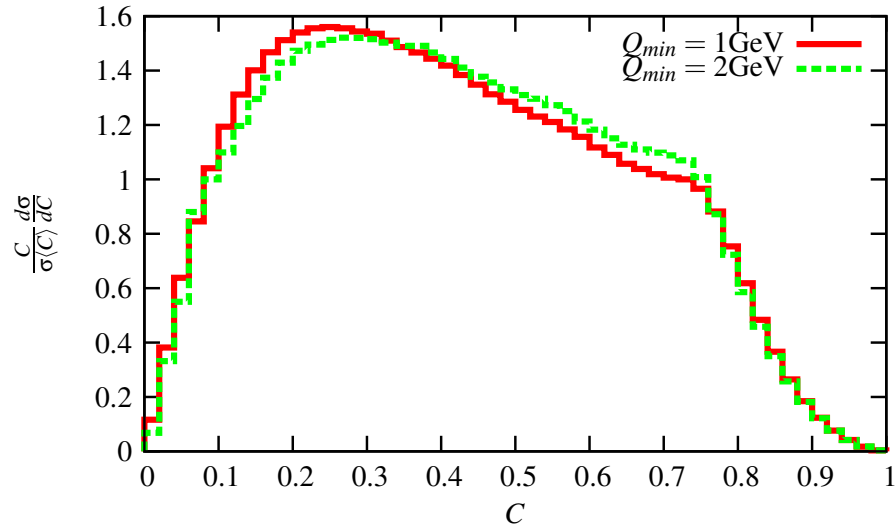
Sudakov factor:

$$\Delta_{ij,k}(t_1, t_2) = \exp \left( - \int_{t_2}^{t_1} dt C_{\tilde{i}, \tilde{k}} \int d\phi_{unres} \delta(t - T_{\tilde{i}, \tilde{k}}) \mathcal{P}_{ij,k} \right), \quad t = \ln \frac{-k_{\perp}^2}{Q^2}.$$

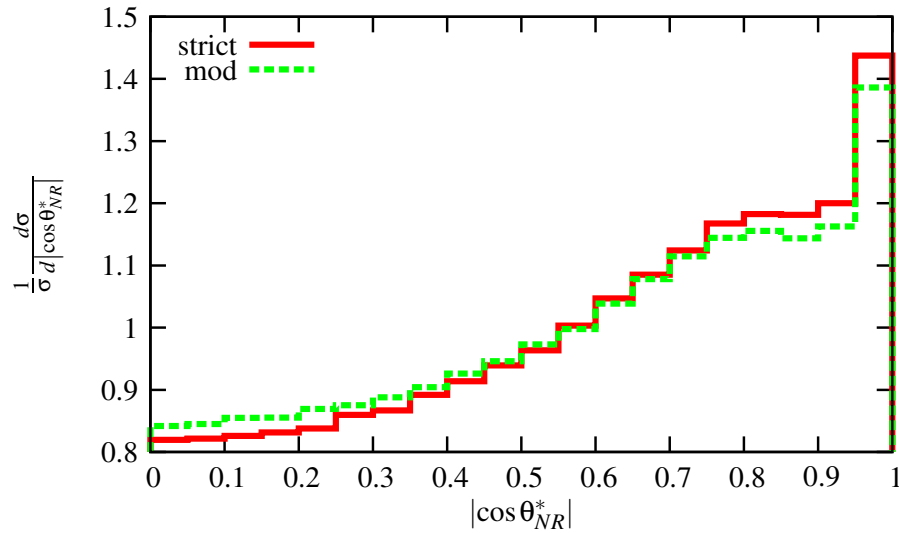
Dipole phase space:

$$\int d\phi_{unres} = \frac{(p_{\tilde{i}} + p_{\tilde{k}})^2}{16\pi^2} \int_0^1 d\kappa \int_{z_-(\kappa)}^{z_+(\kappa)} dz \frac{1}{4z(1-z)} \left( 1 - \frac{\kappa}{4z(1-z)} \right), \quad \kappa = 4 \frac{(-k_{\perp}^2)}{(p_{\tilde{i}} + p_{\tilde{k}})^2}.$$

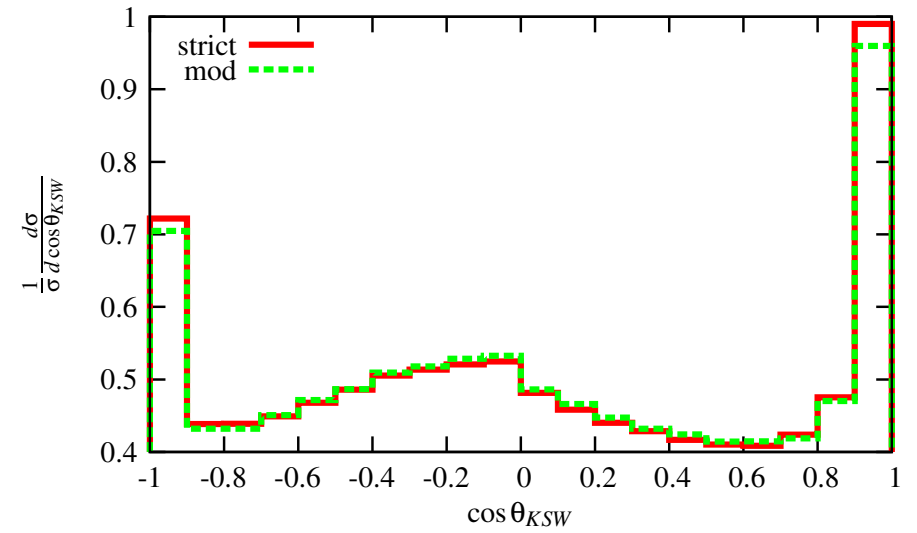
# Numerical results: Electron-positron annihilation



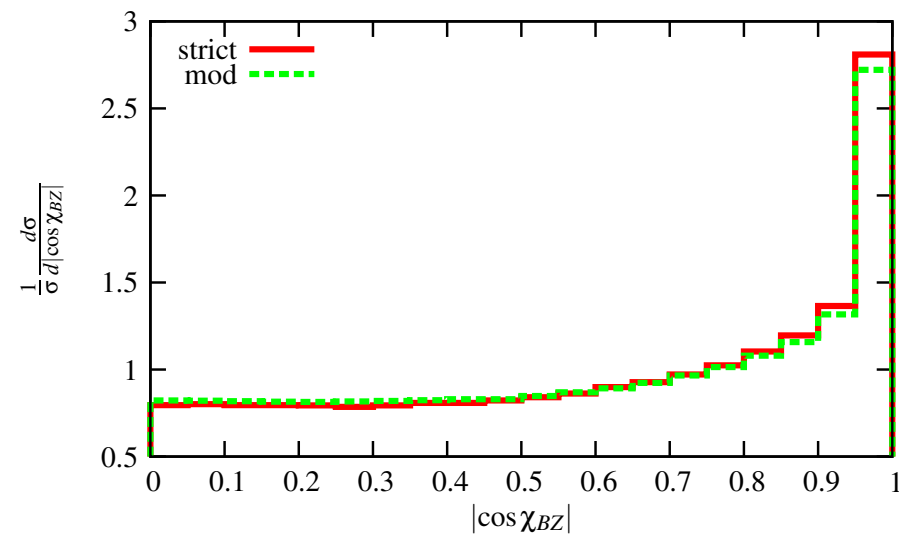
# Numerical results: Four-jet angles



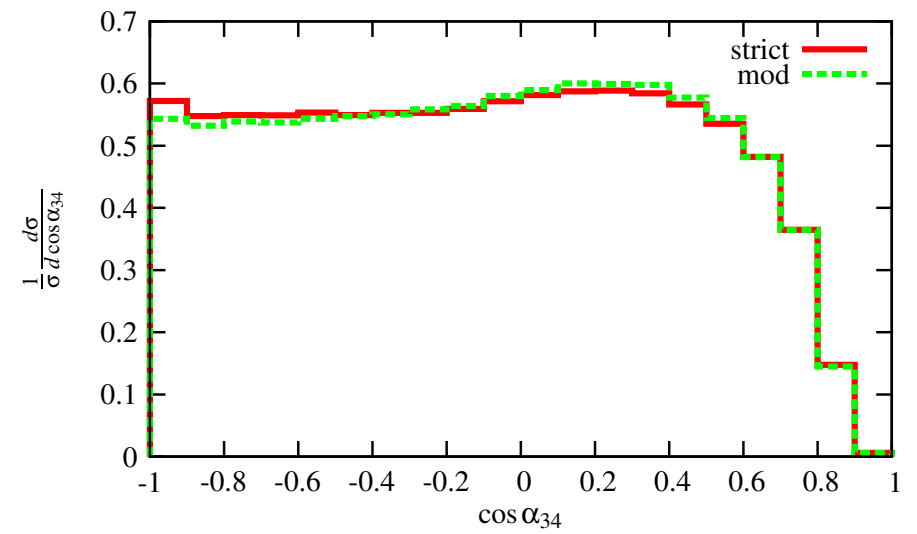
modified Nachtmann-Reiter angle



Körner-Schierholz-Willrodt angle

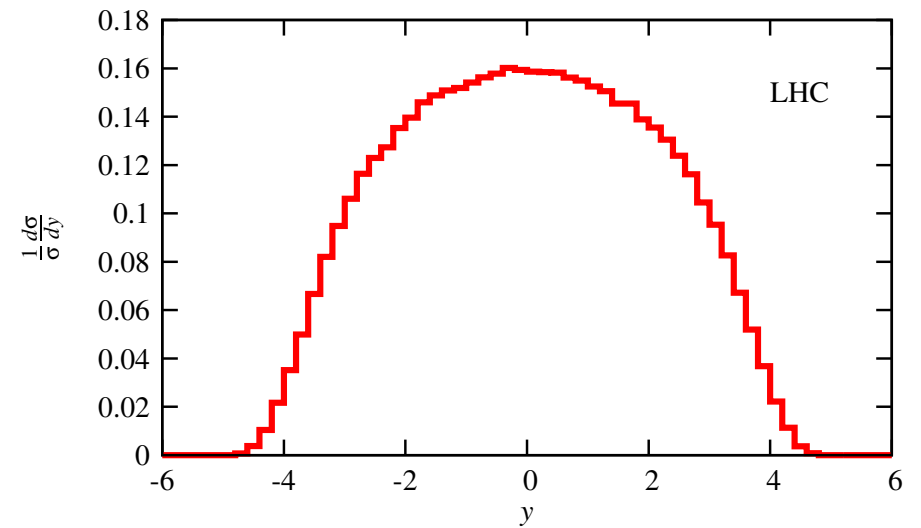
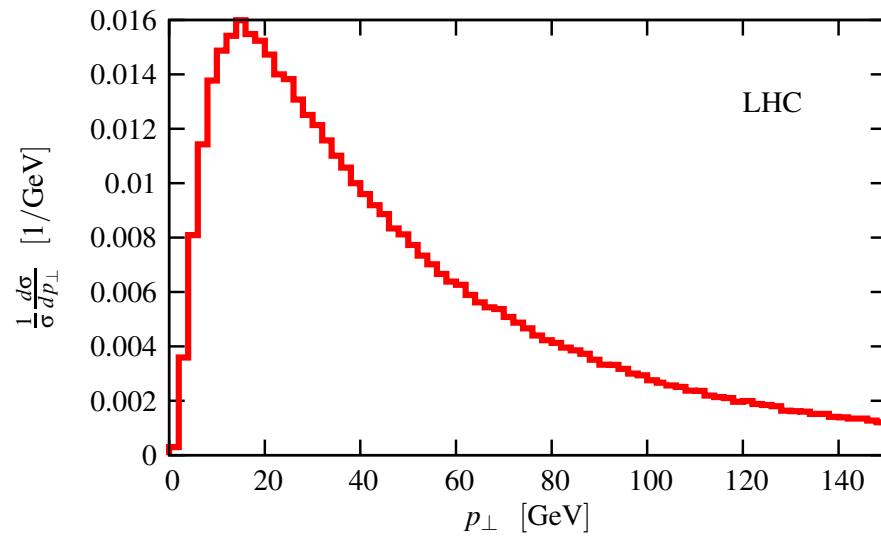
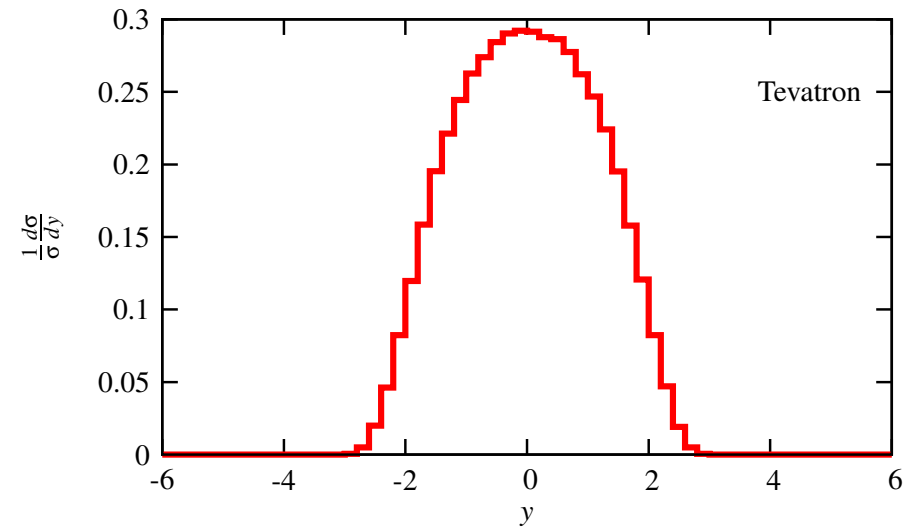
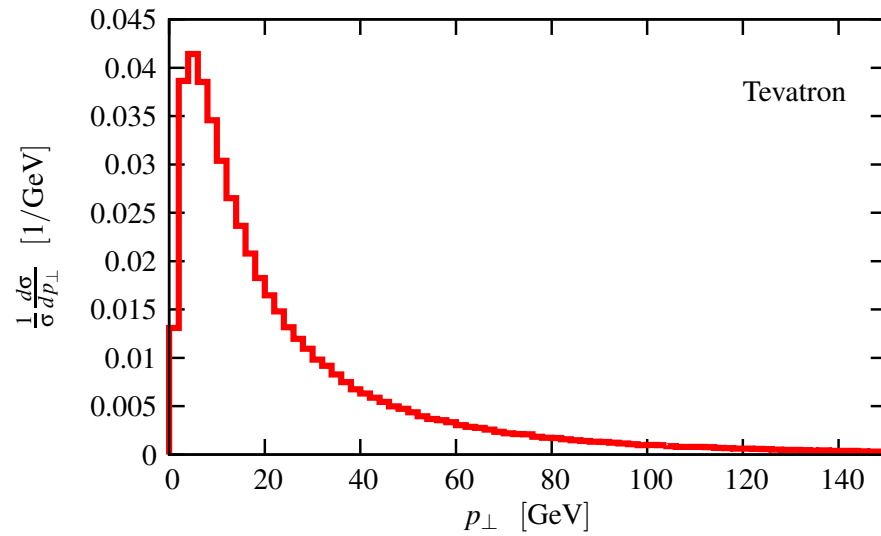


Bengtsson-Zerwas angle



angle  $\alpha_{34}$

# $Z/\gamma^*$ -production at the Tevatron and at the LHC



# Summary

Implementation of a new parton shower algorithm based on the dipole formalism.

Transverse momentum as evolution variable.

Momentum conservation and “angular ordering” are inherent.

Initial- and final-state partons are treated on the same footing.

Natural choice to combine with NLO.