GENEVA: SCET-Friendly Event Generation

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Work with Christian Bauer and Jesse Thaler

Physics: arXiv:0801.4026
Techniques: arXiv:0801.4028

The LHC Challenge

At LHC, need more prediction, less postdiction/tuning

- Precise theoretical calculations
  - Many NLO or even NNLO calculations already available
  - NLL, NNLL resummations, power corrections from SCET
  - Automation of multi-leg matrix elements
    - Tree Level: practically solved
    - One-loop: almost completed (Blackhat, QCDLoop, ...)
- Calculations of backgrounds (almost) useless unless easily available to experiments
  - Need a generic way to turn *parton-level* calculations into *hadron-level* events
  - Need to consistently combine most accurate available descriptions for different parts of phase space

"Universal Monte Carlo": Framework to accomodate any* theory calculation?

Outline

1. Physics
   - Factorization
   - The GENEVA Framework
2. Techniques
   - Analytic Parton Shower Reweighting
3. Results
   - $e^+e^- \to n$ jets
4. The Future
   - The NLO Cascade

Outline

Factorization

Analytic calculations:

$$\mathrm{d}\sigma = \sigma_{\mathrm{UV}}(\mu) \otimes f_{\mathrm{IR}}(\mu)$$ [Collins, Soper, Sterman, ...]

$$= |C_{\mathrm{UV}}(\mu)|^2 \otimes \langle O_{\mathrm{IR}}(\mu)\rangle^2_{\mathrm{SCET}}$$ (SCET)

In Monte Carlo treatment:

$$\mathrm{d}\sigma = \underbrace{|\mathcal{M}|^2}_{\text{matrix element}} \otimes \underbrace{\mathrm{MC}(\mu)}_{\text{parton shower}}$$

- Need to cancel unphysical μ dependence in parton shower $\mathrm{MC}(\mu)$

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In Monte Carlo treatment:

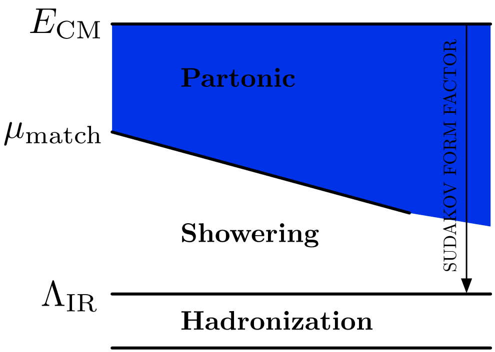
$$\mathrm{d}\sigma = \underbrace{|\mathcal{M}(\mu)|^2}_{\text{matrix element}} \otimes \underbrace{\mathrm{MC}(\mu)}_{\text{parton shower}}$$

- Need to cancel unphysical μ dependence in parton shower $\mathrm{MC}(\mu)$
  - "Sudakov improve" matrix elements $\mathcal{M} \to \mathcal{M}(\mu)$
  - Since $\mathrm{MC}(\mu)$ resums correct double logs, $\mathcal{M}(\mu)$ does also
- At LL [Bauer, Schwartz]

$$|C_{\mathrm{UV}}(\mu)|^2 \simeq |\mathcal{M}(\mu)|^2 + \cdots$$

$$\langle O_{\mathrm{IR}}(\mu)\rangle^2_{\mathrm{SCET}} \simeq \mathrm{MC}(\mu) + \cdots$$

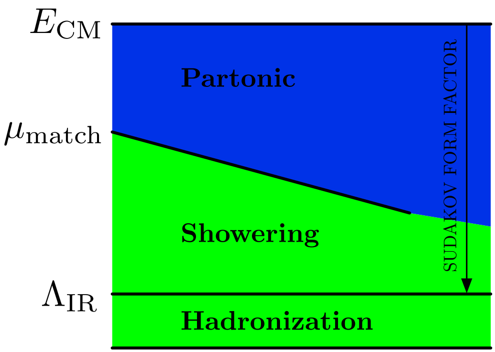
The GENEVA Framework



$$d\sigma_{\rm GENEVA} = |\mathcal{M}(\mu)|^2$$

$|\mathcal{M}(\mu)|^2$: Calculate in perturbation theory

The GENEVA Framework



$$d\sigma_{\text{GENEVA}} = |\mathcal{M}(\mu)|^2 \otimes \mathrm{MC}(\mu)$$

$|\mathcal{M}(\mu)|^2$: Calculate in perturbation theory

$\mathrm{MC}(\mu)$: "Universal" Showering and Hadronization (e.g. PYTHIA, HERWIG, ...)

In some sense, this is already being done ...

PS/ME Merging (LO/LL)

Supplement tree diagrams (LO) with LL Sudakovs to cancel μ dependence

$$|\mathcal{M}^{\mathrm{PS/ME}}(\mu)|^2 = |\mathcal{M}^{\mathrm{tree}}|^2 \times \Delta_{\mathrm{Sud}}(\mu)$$

$$\Delta_{\mathrm{Sud}}(\mu) = \exp\left[-\int_{\mu} Q_{\mathrm{split}}\right]$$

PS/ME Merging (LO/LL)

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$$|\mathcal{M}^{\mathrm{PS/ME}}(\mu)|^2 = |\mathcal{M}^{\mathrm{tree}}|^2 \times \Delta_{\mathrm{Sud}}(\mu) = |\mathcal{M}^{\mathrm{tree}}|^2 + \mathcal{O}(\mathrm{NLO})$$

$$\Delta_{\mathrm{Sud}}(\mu) = \exp\left[-\int_{\mu} Q_{\mathrm{split}}\right]$$

$$\frac{\mathrm{d}}{\mathrm{d}\mu}\Delta_{\mathrm{Sud}}(\mu) = Q_{\mathrm{split}}\Delta_{\mathrm{Sud}}(\mu) = -\frac{\mathrm{d}}{\mathrm{d}\mu}\mathrm{MC}(\mu) + \mathcal{O}(\mathrm{NLL})$$

$\Rightarrow$ Observables correct to LO/LL

Issues

- Physics: Picking intermediate scales and Sudakovs
- Technical: Double-counting between $|\mathcal{M}(\mu)|^2$ and $\mathrm{MC}(\mu)$

Various algorithms (and implementations) do deal with these

- CKKW(-L) [Catani, Krauss, Kuhn, Webber; Lönnblad], MLM [Mangano], Pseudo-Shower [Mrenna, Richardson], VINCIA [Giele, Kosower, Skands]

PS/NLO Merging (NLO/LL)

Supplement tree- and one-loop diagrams (NLO) with LL Sudakovs

$$|\mathcal{M}_n^{\mathrm{MC@NLO}}(\mu)|^2 = \mathrm{d}\sigma_n^{\mathrm{NLO}} \qquad \Delta_{\mathrm{Sud}}(\mu)$$

$$|\mathcal{M}_{n+1}^{\mathrm{MC@NLO}}(\mu)|^2 = |\mathcal{M}_{n+1}^{\mathrm{tree}}|^2$$

PS/NLO Merging (NLO/LL)

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$$|\mathcal{M}_n^{\mathrm{MC@NLO}}(\mu)|^2 = \left[d\sigma_n^{\mathrm{NLO}} + |\mathcal{M}_n^{\mathrm{tree}}|^2 \int_\mu Q_{\mathrm{split}} - \int_\mu |\mathcal{M}_{n+1}^{\mathrm{tree}}|^2 \right] \Delta_{\mathrm{Sud}}(\mu)$$

$$|\mathcal{M}_{n+1}^{\mathrm{MC@NLO}}(\mu)|^2 = |\mathcal{M}_{n+1}^{\mathrm{tree}}|^2 - |\mathcal{M}_n^{\mathrm{tree}}|^2 (Q_{\mathrm{split}} - \Delta'_{\mathrm{Sud}})$$

$\Rightarrow$ Observables correct to NLO/LL

Issues

- Physics: Canceling IR divergences and LL μ dependence at same time
- Technical: More involved double-counting between $|\mathcal{M}(\mu)|^2$ and $\mathrm{MC}(\mu)$ and relationship between $\mathcal{M}_n(\mu)$ and $\mathcal{M}_{n+1}(\mu)$

Fewer algorithms (and even fewer implementations)

- MC@NLO [Frixione, Webber], POWHEG [Nason, et al.], NLO merging based on dipole subtractions [Krämer, Mrenna, Nagy, Soper]

So What Is The Problem?

No easy way to directly implement any $|\mathcal{M}(\mu)|^2$! Traditionally,

$$\mathrm{d}\sigma_{\mathrm{trad}} = \sum_{n}^{n_{\mathrm{max}}} \mathrm{MC}_n(\mu_{\mathrm{shower}}) \left[|\mathcal{M}_n|^2 \, \mathrm{d}\Phi_n \right]$$

Algorithmic (technical) issues

- Phase space double-counting and dead zones between $\mathrm{MC}_i(\mu)$ vs. $\mathrm{d}\Phi_j$
- Analytic vs. automatic generation of $|\mathcal{M}_n|^2$
- Efficiency of $\mathrm{d}\Phi_n$ for large n (currently $n \lesssim 6 - 8$)

Physics issues

- Logarithmic scale ambiguities
- Canceling IR divergences

Different MC techniques/engines for CKKW, MLM, MC@NLO, POWHEG, ...

$\Rightarrow$ Effectively used $|\mathcal{M}(\mu)|^2$ (more or less) dictated by algorithms

GENEVA: Generate Events Analytically

$$d\sigma_{\text{GENEVA}} = \sum_{n}^{n_{\max}} \mathrm{MC}_n(\mu_{\text{shower}}) \left[|\mathcal{M}_n|^2 \, d\Phi_n \right]$$

GENEVA: Generate Events Analytically

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1. Change point of view: Resum logs (instead of canceling μ-dependence)

$$|\mathcal{M}|^2 \to |\mathcal{M}(\mu_{\text{IR}})|^2$$

   - Combines fixed-order expansion with collinear expansion

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2. To avoid large $\ln \mu_{\mathrm{IR}}/\mu_{\mathrm{shower}}$ take

$$\mu_{\mathrm{IR}} \to \mu_{\mathrm{shower}} \equiv \mu_n$$

   - Combines calculation with QCD model (parton shower)

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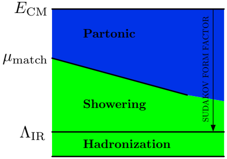
3. Factor out $|\mathcal{M}(\mu)|^2$ and replace

$$\mathrm{MC}_n(\mu_n)\left[d\Phi_n\right] \to \mathrm{dMC}_n(\mu_n)$$

   - Combines different phase space algorithms

Factorization in GENEVA

Scale factorization & "factorization" of physics from (phase-space) algorithms



$$d\sigma_{\text{GENEVA}} = \sum_{n}^{n_{\max}} \underbrace{|\mathcal{M}_n(\mu_n)|^2}_{\text{Physics}} \otimes \underbrace{\mathrm{dMC}_n(\mu_n)}_{\text{Algorithms}}$$

Systematically improvable Monte Carlo

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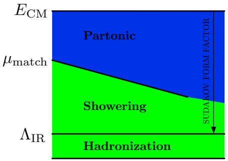

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Systematically improvable Monte Carlo

- Use best $\mathcal{M}_n(\mu_n)$ from any* available $\mathrm{N}^i\mathrm{LL}$ and $\mathrm{N}^j\mathrm{LO}$ calculations
    - At LL can "guess" a correct answer (see PS/ME & PS/NLO merging)
    - Can use (SCET-) calculation for improved NLL or NNLL resummation
    - Choose μ_n as low as validity of calculations allows

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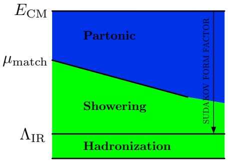

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Systematically improvable Monte Carlo

- Use best $\mathcal{M}_n(\mu_n)$ from any* available N^{i}LL and N^{j}LO calculations
  - At LL can “guess” a correct answer (see PS/ME & PS/NLO merging)
  - Can use (SCET-) calculation for improved NLL or NNLL resummation
  - Choose μ_n as low as validity of calculations allows
- $\mathrm{dMC}_n(\mu_n)$: Phase space with a matching scale
  - By definition no double-counting, negative weights
  - *In principle* could swap in and out different $\mathrm{dMC}(\mu)$ generators

Outline

The GenEvA Algorithm

Use parton shower itself as $\mathrm{dMC}(\mu)$ phase space generator

- It's fast and covers all of multiplicity, flavor, phase space
- Has symmetries and singularities of QCD automatically built in

$$\mathrm{d}\sigma_{\text{shower}} \equiv \mathrm{dMC}(E_{\text{CM}})$$

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$$d\sigma_{\text{shower}} \equiv \mathrm{dMC}(E_{\text{CM}}) \equiv |\mathcal{M}_n^{\text{shower}}(\mu_n)|^2\, \mathrm{dMC}_n(\mu_n)$$

$$\Rightarrow \qquad \mathrm{dMC}_n(\mu_n) = \frac{d\sigma_{\text{shower}}}{|\mathcal{M}_n^{\text{shower}}(\mu_n)|^2}$$

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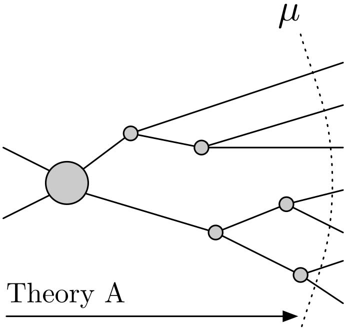
$$\Rightarrow \qquad \mathrm{dMC}_n(\mu_n) = \frac{d\sigma_{\mathrm{shower}}}{|\mathcal{M}_n^{\mathrm{shower}}(\mu_n)|^2}$$

$$\Rightarrow \qquad d\sigma_{\mathrm{GENEVA}} = d\sigma_{\mathrm{shower}} \sum_n^{n_{\max}} \frac{|\mathcal{M}_n(\mu_n)|^2}{|\mathcal{M}_n^{\mathrm{shower}}(\mu_n)|^2}$$

Non-trivial: Analytically calculate $|\mathcal{M}_n^{\mathrm{shower}}(\mu_n)|^2$

- Possible for analytic parton shower algorithm [Bauer, FT, arXiv:0705.1719]
- Need to account for parton shower covering phase space multiple times
  - Using numerical tricks from ALPGEN and MADEVENT for speed

Combining Different Theories



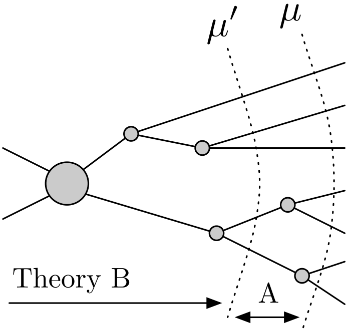
$$|\mathcal{M}^{\text{Best}}(\mu)|^2 = |\mathcal{M}^{\text{A}}(\mu)|^2$$

Matching and Running (backwards)

GENEVA Best: NLO/LO/LL

A = LL (Shower)

Combining Different Theories



Matching and Running (backwards)

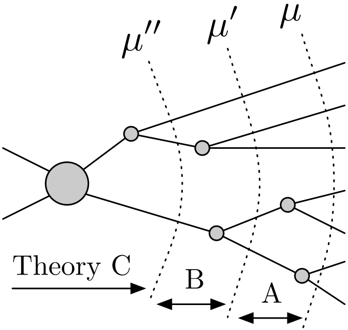
GENEVA Best: NLO/LO/LL

A = LL (Shower)

B = LO/LL (CKKW-L*)

$$|\mathcal{M}^{\text{Best}}(\mu)|^2 = |\mathcal{M}^{\text{A}}(\mu)|^2 \times \frac{|\mathcal{M}^{\text{B}}(\mu')|^2}{|\mathcal{M}^{\text{A}}(\mu')|^2}$$

Combining Different Theories



Matching and Running (backwards)

GENEVA Best: NLO/LO/LL

A = LL (Shower)

B = LO/LL (CKKW-L*)

C = NLO/LL (POWHEG*)

$$|\mathcal{M}^{\mathrm{Best}}(\mu)|^2 = |\mathcal{M}^{\mathrm{A}}(\mu)|^2 \times \frac{|\mathcal{M}^{\mathrm{B}}(\mu')|^2}{|\mathcal{M}^{\mathrm{A}}(\mu')|^2} \times \frac{|\mathcal{M}^{\mathrm{C}}(\mu'')|^2}{|\mathcal{M}^{\mathrm{B}}(\mu'')|^2}$$

Theory (SCET-) friendly

- Shower already resums leading logarithms: /LL comes "for free"
- Events come with an entire "scale history"
- Reusable MC: Events with multiple weights for different theory predictions

Outline

Proof of Concept Version

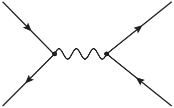
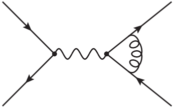
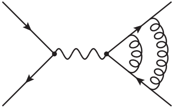
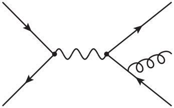
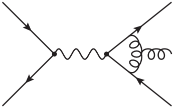
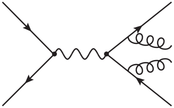
GenEvA prototype: $e^+e^- \to n$ (massless) jets

- Use tree-level matrix elements from MadGraph (currently up to $n = 6$)
- Analytic NLO matrix elements
- $\mathrm{dMC}(\mu)$ with virtuality as evolution variable and running matching scale
- Showering regime covered by underlying analytic parton shower
  - Fixed α_s
  - No Hadronization

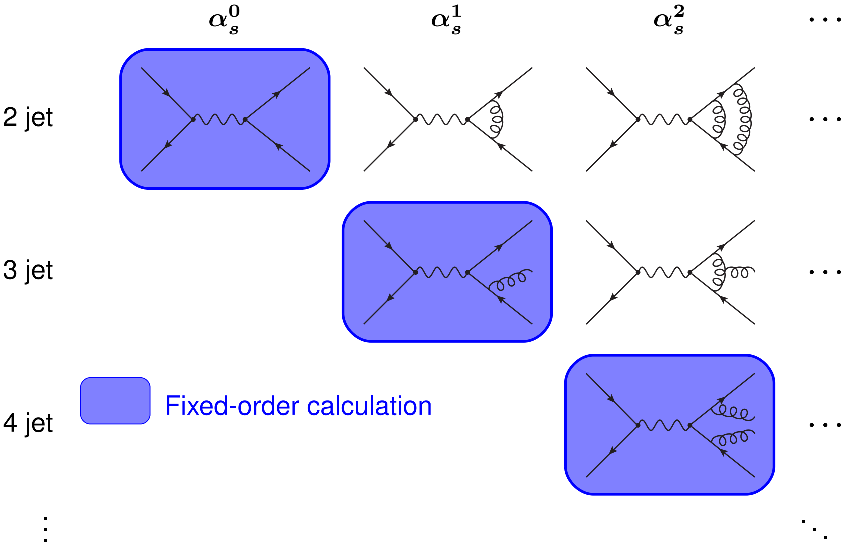
Numbers used for plots:

$$E_{\mathrm{CM}} = 1000\,\mathrm{GeV}$$
$$\mu_{\mathrm{match}} = 50\,\mathrm{GeV}$$
$$\Lambda_{\mathrm{IR}} = 10\,\mathrm{GeV}$$

Relevant Diagrams

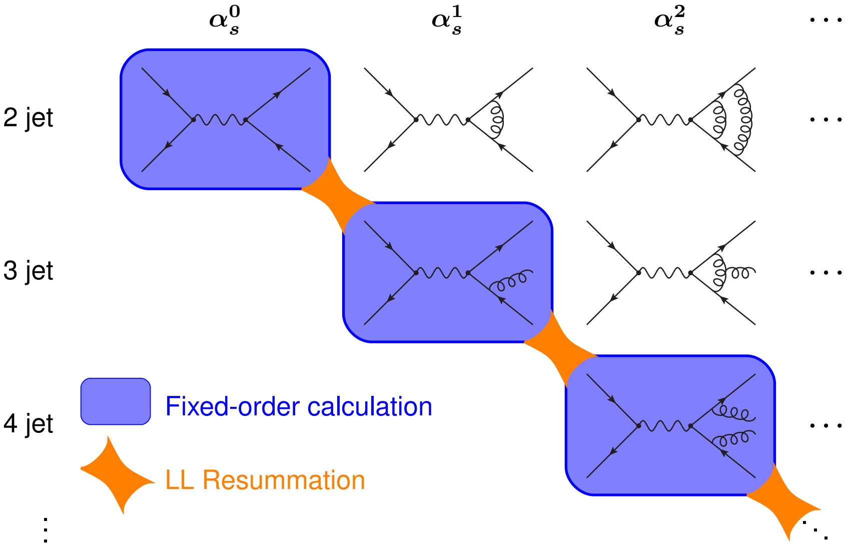
	α_s^0	α_s^1	α_s^2	$\cdots$
2 jet				$\cdots$
3 jet				$\cdots$
4 jet				$\cdots$
$\vdots$				$\ddots$

LO: Tree-level Generators



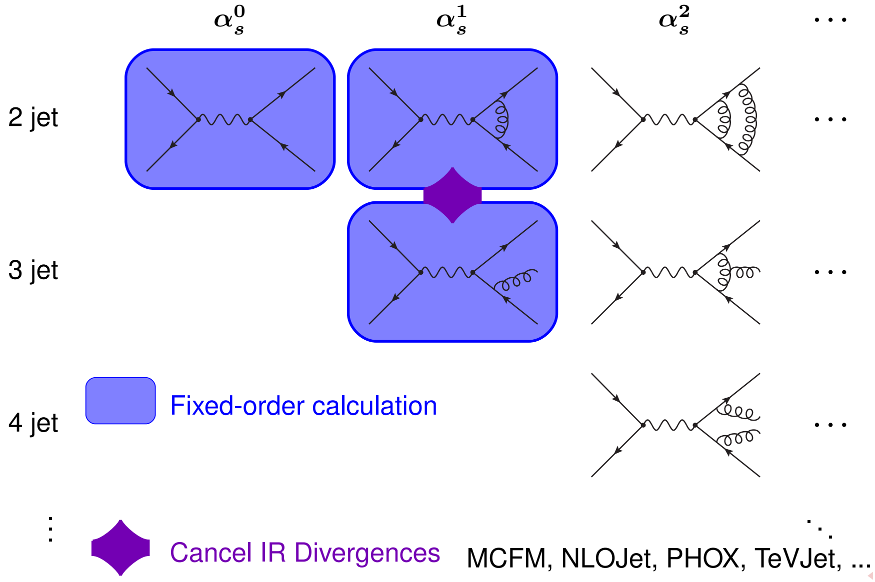
ALPGEN, MADGRAPH, SHERPA, WHIZARD, ...

LO/LL: PS/ME Merging

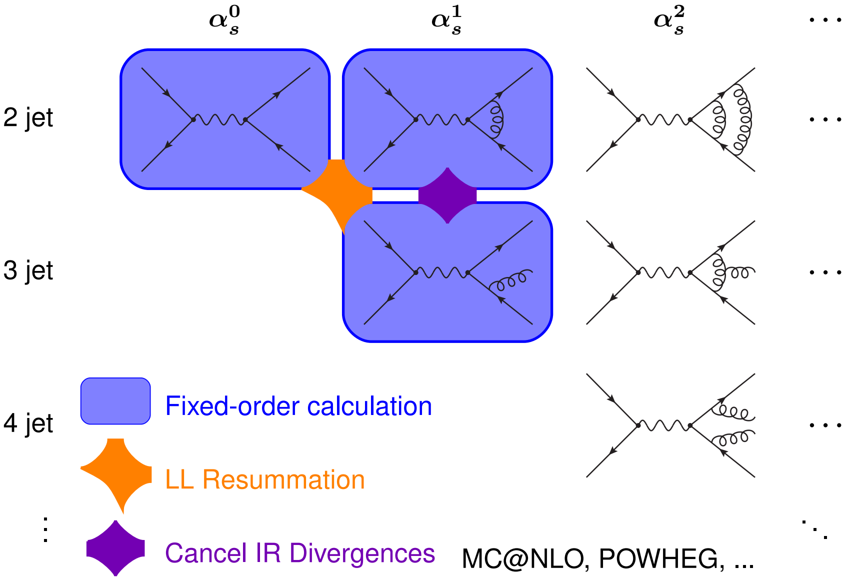


CKKW(-L), MLM, ...

NLO: Loop-Level Generators

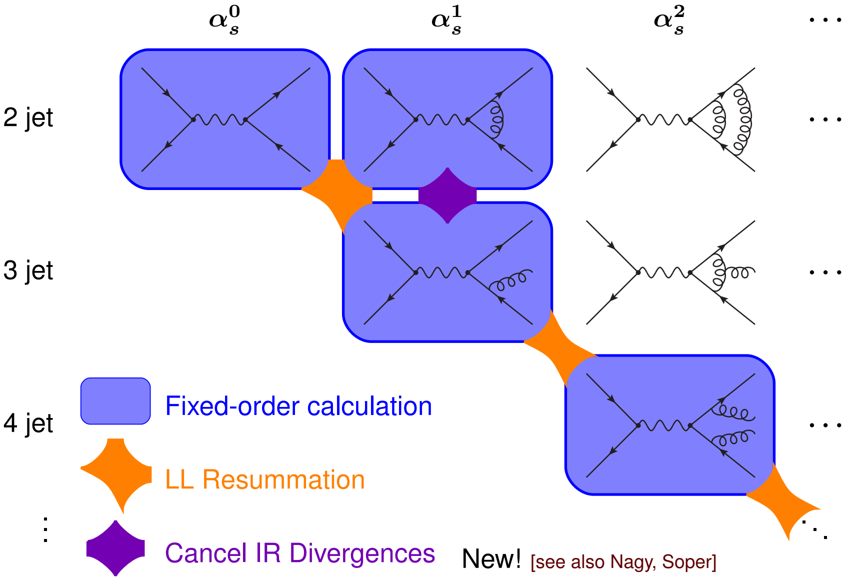


NLO/LL: PS/NLO Merging

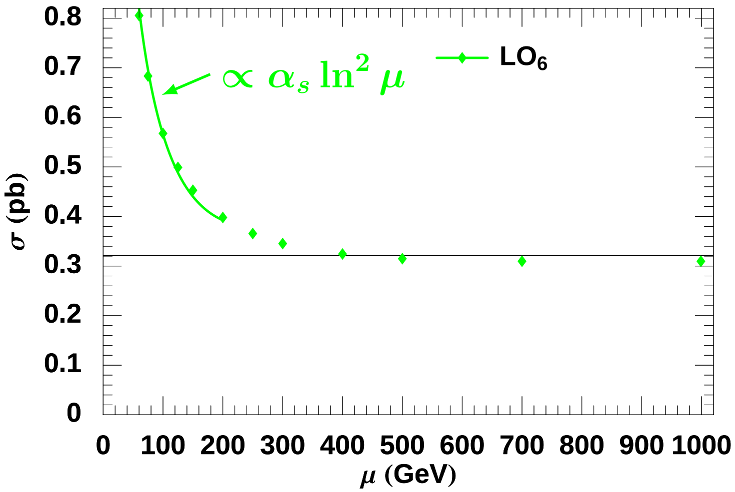


MC@NLO, POWHEG, ...

NLO/LO/LL: GENEVA Best



Total Cross Section Scale Dependence



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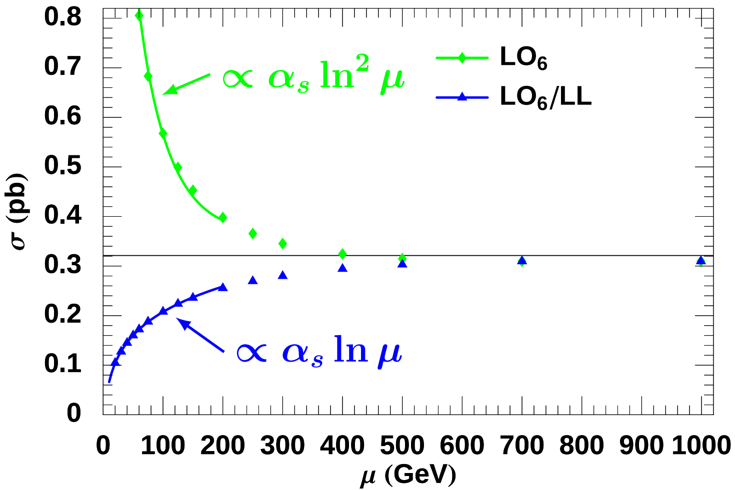

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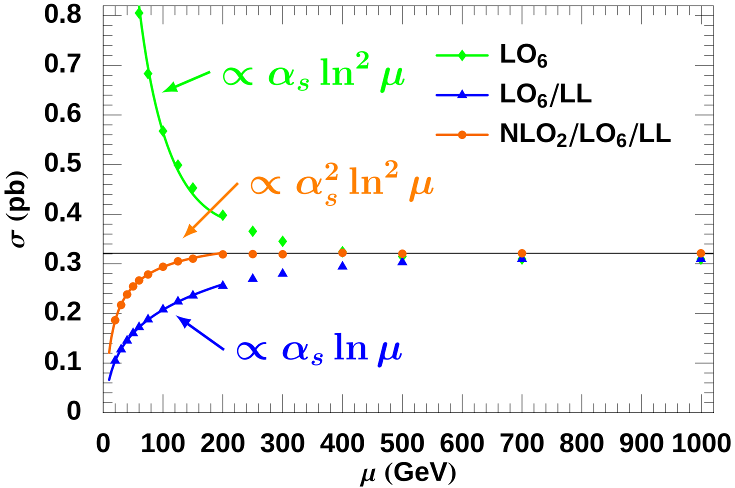
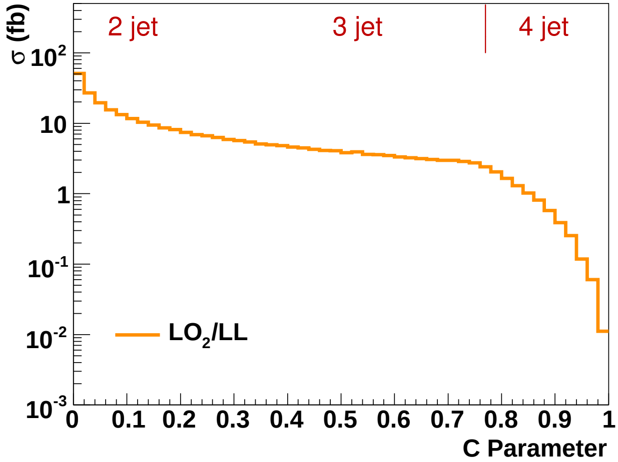


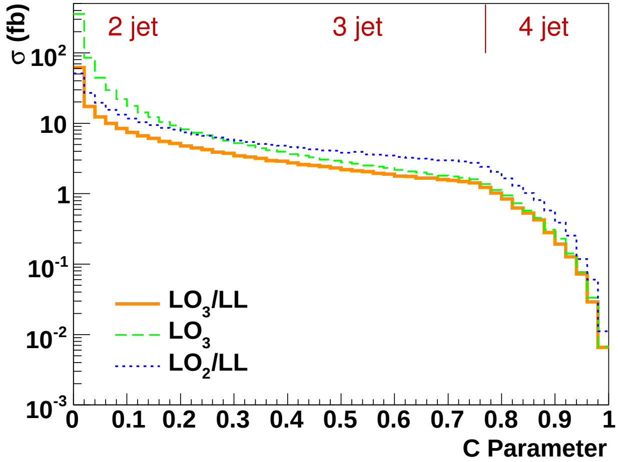
μ dependence scales exactly how it should

LO_n/LL Calculation



LO_2/LL: Shower only

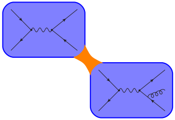
LO$_n$/LL Calculation



LO$_2$/LL: Shower only

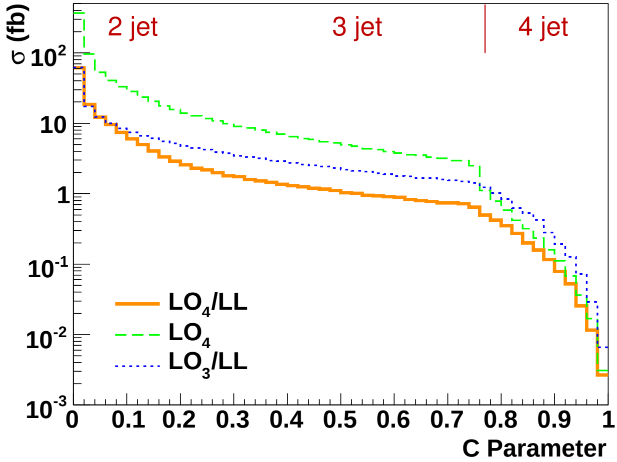
LO$_3$: Fixed-order ME

LO$_3$/LL:



LO/LL answer is smaller than either LO or LL alone

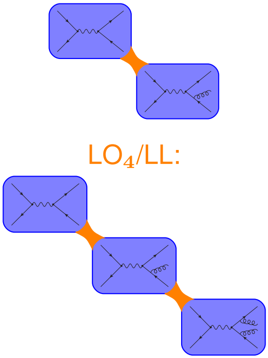
LO_n/LL Calculation



LO_4: Fixed-order ME

LO_3/LL:

LO_4/LL:



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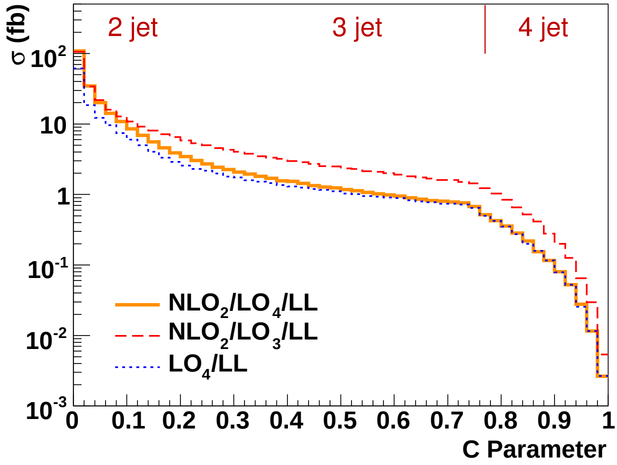
$NLO_2/LO_n/LL$ Calculation



LO_4/LL: ME/PS

$NLO_2/LO_3/LL$:

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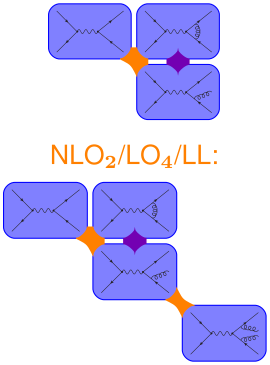
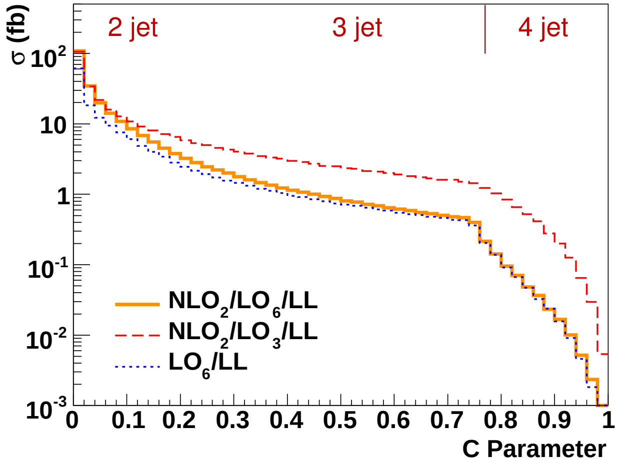


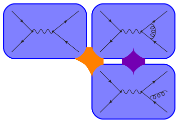
Interpolation between PS/ME and PS/NLO, more than a simple k-factor

$NLO_2/LO_n/LL$ Calculation

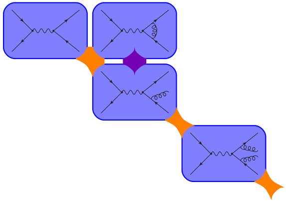


LO_6/LL: ME/PS

$NLO_2/LO_3/LL$:

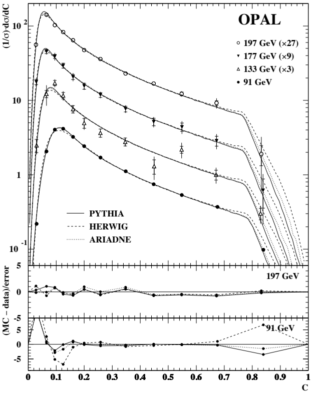


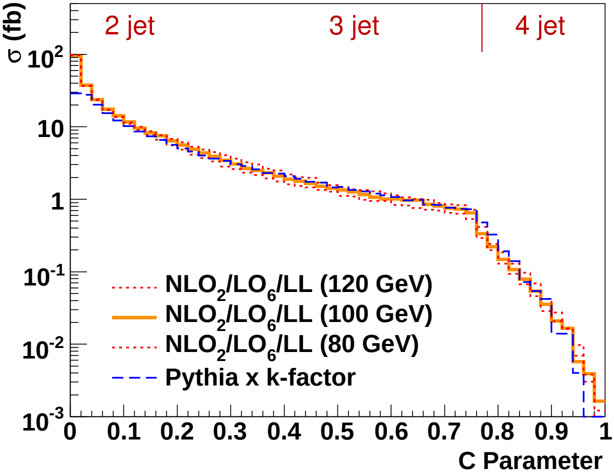
$NLO_2/LO_6/LL$:



Interpolation between PS/ME and PS/NLO, more than a simple k-factor

Pseudo-Comparison with Data





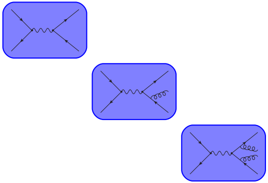
- LEP ≈ Pythia, Pythia* ≈ GenEvA (for large C where we trust it)
- What is going on at low C? Hadronization? Evolution variable?

Outline

The NLO Cascade

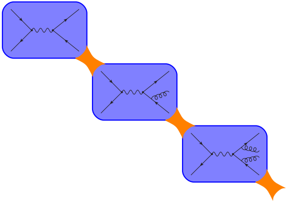
Tevatron: Arbitrary tree diagrams

[ALPGEN, MADGRAPH, ...]

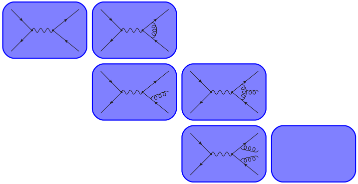


$\Rightarrow$

Best used with PS/ME Merging



LHC? Arbitrary one-loop diagrams (Blackhat, QCDLoop, ...)

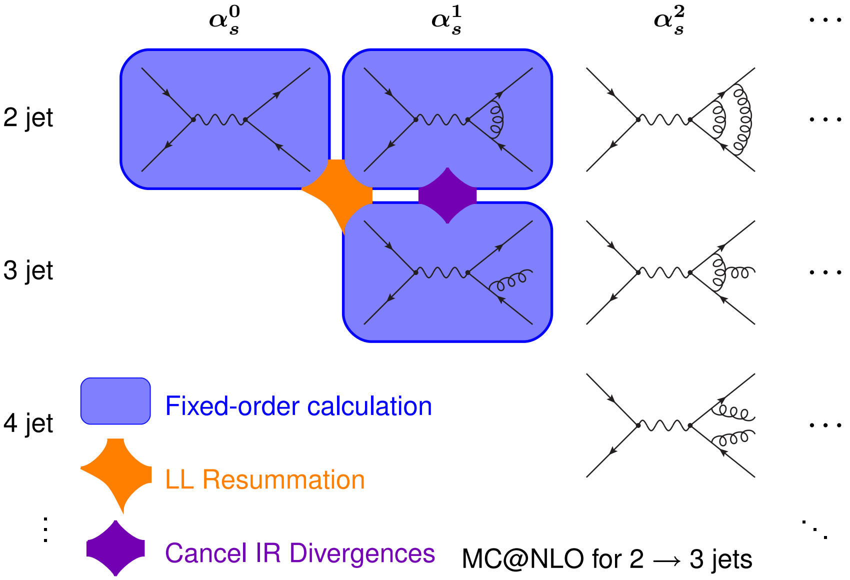


$\Rightarrow$

Best use? PS/NLO alone?

?

NLO$_2$/LL: PS/NLO Merging only for $2 \longrightarrow 3$ jets



NLO$_3$/LL: PS/NLO Merging only for 3 $\longrightarrow$ 4 jets

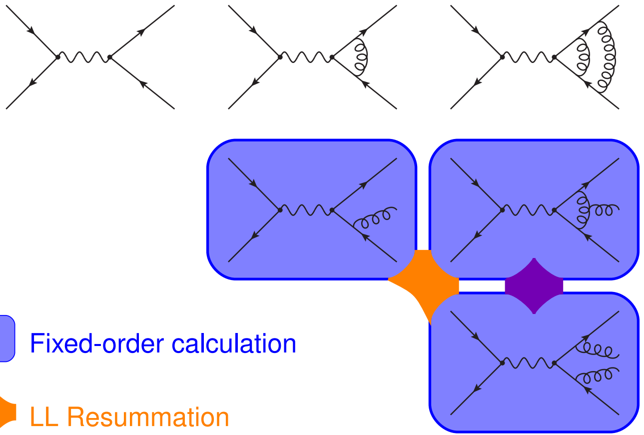
α_s^0 α_s^1 α_s^2 $\cdots$

2 jet $\cdots$

3 jet $\cdots$

4 jet $\cdots$

$\vdots$ $\ddots$



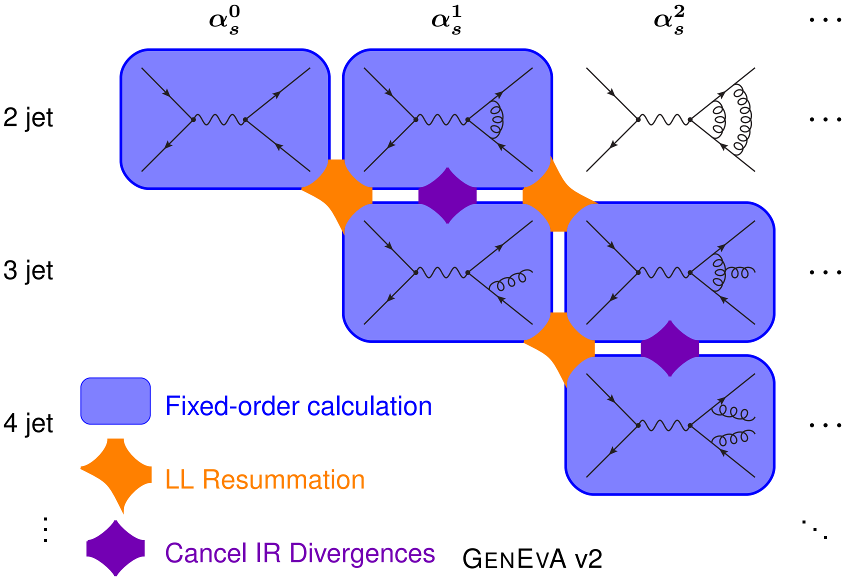
Fixed-order calculation

LL Resummation

Cancel IR Divergences

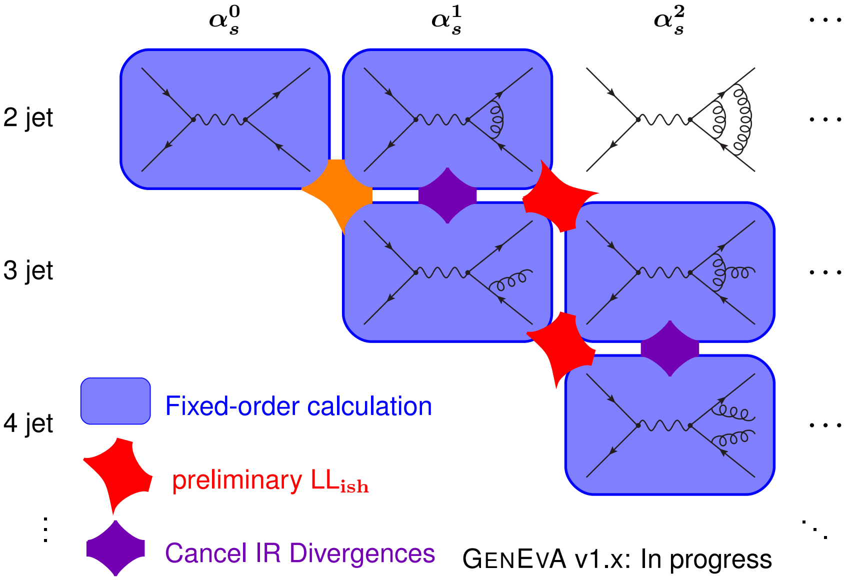
MC@NLO for 3 $\longrightarrow$ 4 jets

$NLO_{2,3}/LO_4/LL$: NLO Cascade



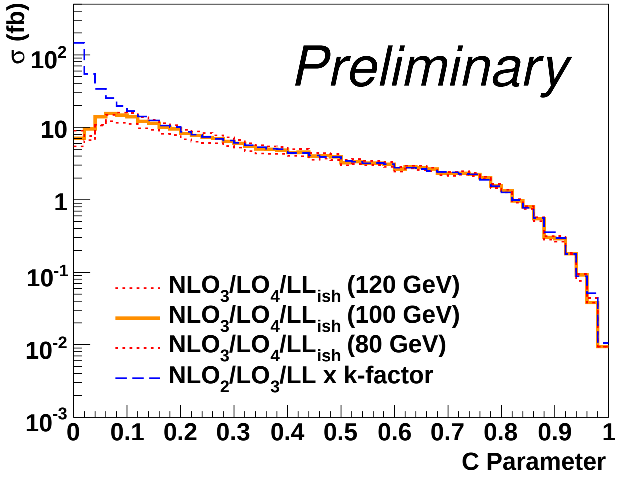
GENEVA v2

NLO$_{2,3}$/LO$_4$/LL: NLO Cascade

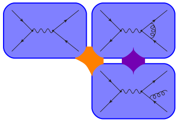


GENEVA v1.x: In progress

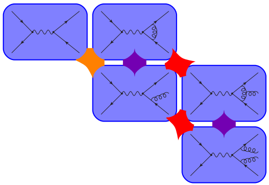
Preliminary $NLO_{2,3}/LO_4/LL_{ish}$



$NLO_2/LO_3/LL$:

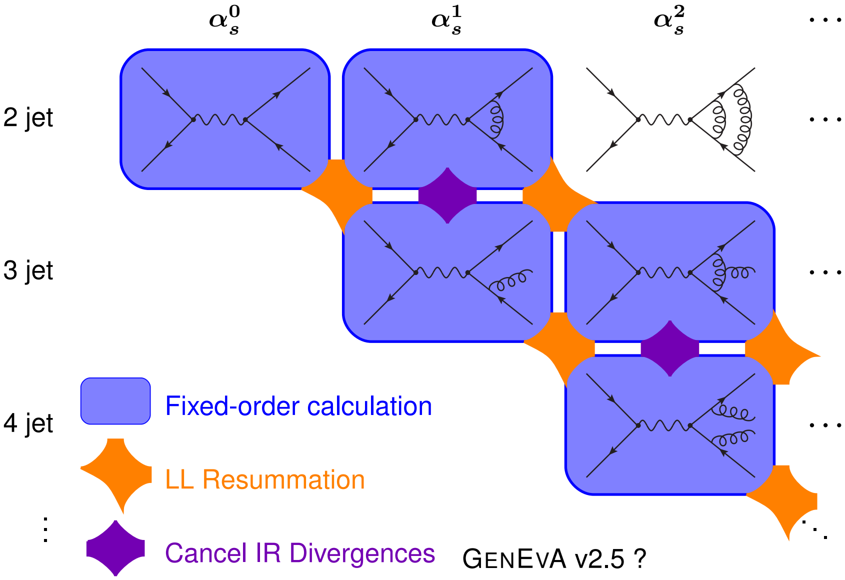


$NLO_{2,3}/LO_4/LL_{ish}$:



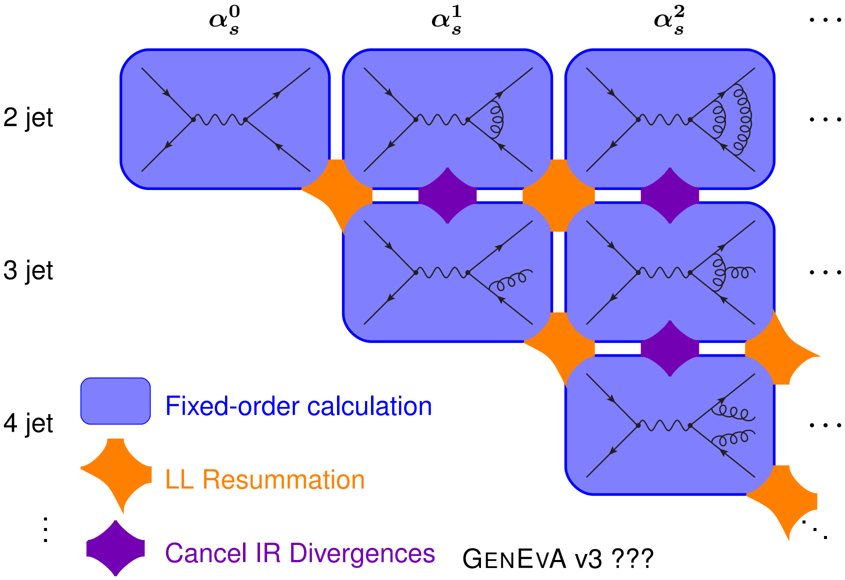
- Dramatic shape variation, goes in right direction
- NLO corrections should resolve ambiguity in evolution variable

The Future: NLO$_m$/LO$_n$/LL ?



GENEVA v2.5 ?

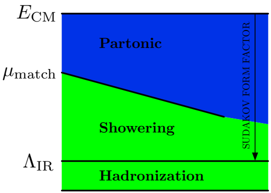
The Future: $NNLO_2/NLO_m/LO_n/LL$???



GENEVA v3 ???

Conclusions

GenEvA Framework: Systematically improvable and reusable Monte Carlo



$$d\sigma_{\rm GenEvA} = \sum_{n}^{n_{\max}} \underbrace{|\mathcal{M}_n(\mu_n)|^2}_{\text{Physics}} \otimes \underbrace{d\mathrm{MC}_n(\mu_n)}_{\text{Algorithms}}$$

Challenge for Theory: Find $|\mathcal{M}^{\rm Best}(\mu)|^2$

- SCET: subleading-log treatment of multiple scales?
- NNLO/NLO/LO/NLL/LL merged sample?

Technical challenges: Extension to pp collisions

- Heavy resonances, massive quarks, initial-state shower, PDFs, ...
- Interface to $p_\perp$ showers

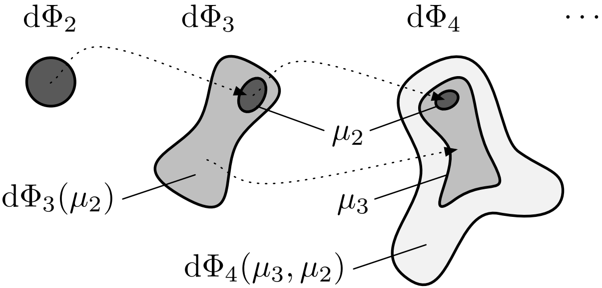
Outline

Backup Slides

Phase Space with a Matching Scale

In principle, double counting is trivial to solve

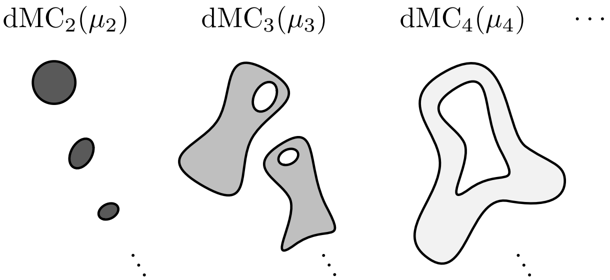
1. Define $\mathbf{d\Phi}_n(\mu)$ by action of pheno. model (parton shower) started at μ



Phase Space with a Matching Scale

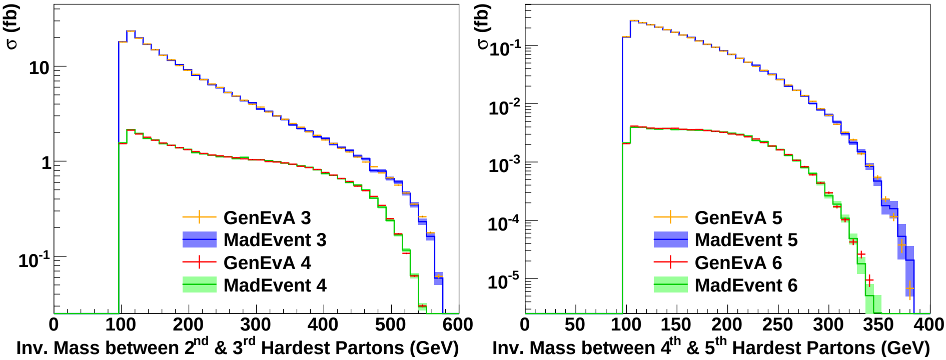
In principle, double counting is trivial to solve

1. Define $\mathrm{d}\Phi_n(\mu)$ by action of pheno. model (parton shower) started at μ
2. Assemble pieces into "Monte Carlo" space $\mathrm{dMC}_n(\mu_n)$



Complete mapping of phase space $\sum_{n=2}^{n_{\max}} \mathrm{dMC}_n(\mu_n) \to \sum_{n=2}^{\infty} \mathrm{d}\Phi_n$

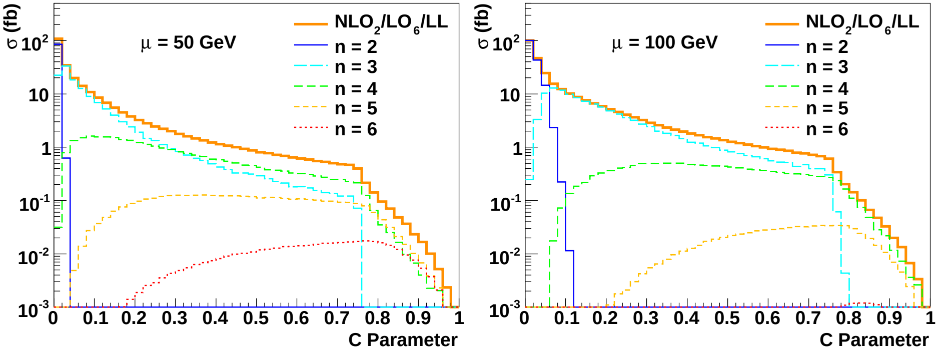
Perfect Agreement with MadGraph/MadEvent



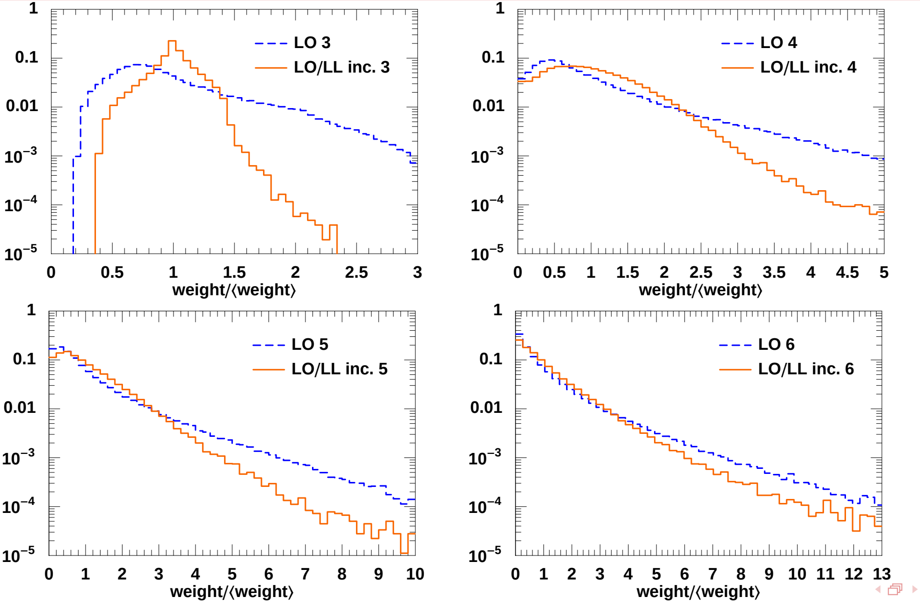
Fully Inclusive Sample

Inclusive sample merged up to $6j$

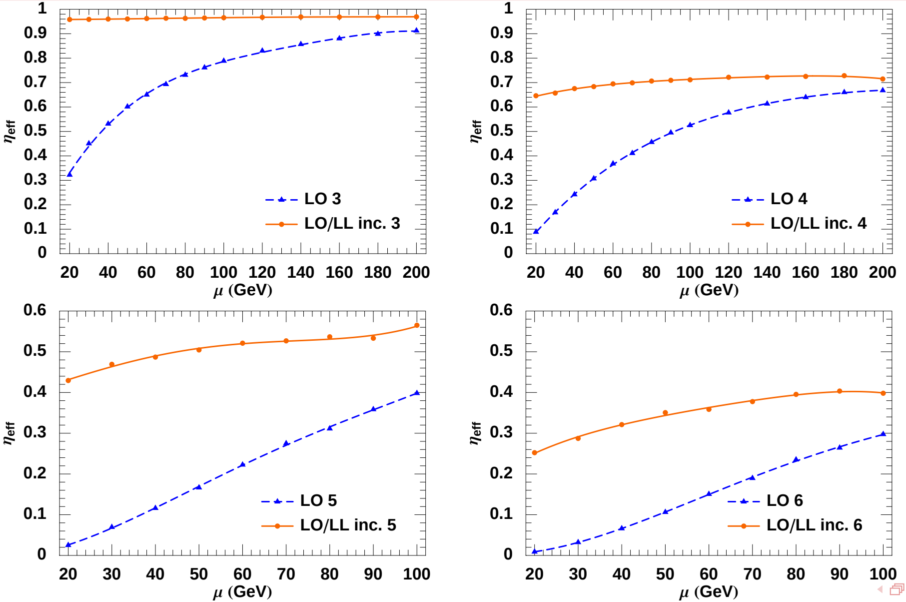
Contributions of individual $2j, 3j, 4j, 5j, 6j$



Distribution of Weights



Statistical Efficiency



The Event Weight

$$w = \frac{\mathrm{d}\sigma}{\mathrm{d}P}$$

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- $\mathrm{d}P = \mathcal{P}(\Sigma)\,\mathrm{d}\Sigma$ is function of full parton shower history $\Sigma = \{t_i, z_i\}$

The Event Weight

$$w \equiv w(\Phi) = \frac{\sigma(\Phi)}{\mathcal{P}[\Sigma(\Phi)]J[\Sigma(\Phi)]}$$

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- Mapping $\Sigma \to \Phi \equiv \Phi(\Sigma)$, Jacobian $J(\Sigma) = \mathrm{d}\Sigma/\mathrm{d}\Phi$

The Event Weight

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- $\mathrm{d}P = \mathcal{P}(\Sigma)\,\mathrm{d}\Sigma$ is function of full parton shower history $\Sigma = \{t_i, z_i\}$
- Mapping $\Sigma \to \Phi \equiv \Phi(\Sigma)$, Jacobian $J(\Sigma) = \mathrm{d}\Sigma/\mathrm{d}\Phi$
- Each Φ can have multiple $\Sigma_i(\Phi)$ that map to it
  - Have to sum over all parton shower histories $\Sigma_i(\Phi)$ that map to the same point Φ in phase space.

Overcounting

$$w \equiv w(\Phi) = \frac{\sigma(\Phi)}{\sum_i \mathcal{P}[\Sigma_i(\Phi)] J[\Sigma_i(\Phi)]}$$

Summing over $\mathcal{P}[\Sigma_i(\Phi)]$ is hard

- Requires to explicitly construct all $\Sigma_i(\Phi)$ for given Φ
- Naively grows like $n!$
- Same problem in subtraction methods

Overcounting

$$w \equiv w(\Sigma) = \frac{\sigma(\Sigma)}{\mathcal{P}(\Sigma)}$$

Instead make weight function of Σ

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$$w \equiv w(\Sigma) = \frac{\sigma[\Phi(\Sigma)]}{\mathcal{P}(\Sigma) J(\Sigma)}\, \hat{\alpha}(\Sigma)$$

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- Define $\sigma(\Sigma) = \frac{\sigma[\Phi(\Sigma)]}{J(\Sigma)}\, \hat{\alpha}(\Sigma)$ with $\hat{\alpha}(\Sigma) = \frac{\alpha(\Sigma)}{\sum_i \alpha(\Sigma_i)}$

Overcounting

$$w \equiv w(\Sigma) = \frac{\sigma[\Phi(\Sigma)]}{\mathcal{P}(\Sigma)J(\Sigma)}\,\hat{\alpha}(\Sigma)$$

Instead make weight function of Σ

- Define $\sigma(\Sigma) = \dfrac{\sigma[\Phi(\Sigma)]}{J(\Sigma)}\,\hat{\alpha}(\Sigma)$ with $\hat{\alpha}(\Sigma) = \dfrac{\alpha(\Sigma)}{\sum_i \alpha(\Sigma_i)}$
- $\hat{\alpha}(\Sigma)$ distributes $\sigma(\Phi)$ among $\Sigma_i(\Phi)$, can be chosen freely
  - Trivial but inefficient: $\alpha(\Sigma) = 1$
  - Ideal but hard: $\alpha(\Sigma) = \mathcal{P}(\Sigma)J(\Sigma)$
- Pick $\alpha(\Sigma) \approx \mathcal{P}(\Sigma)J(\Sigma)$ such that $\sum_i \alpha(\Sigma_i)$ looks like sum of graphs
  - Compute with ALPHA algorithm [Caravaglios, Moretti; Mangano, Pittau (ALPGEN)]
  - Only grows like 2^n