How well can we hope to know $\bar{B} \to X_s\gamma$?

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and *in preparation*

Why $\bar{B} \to X_s\gamma$?

- Good knowledge of $d\Gamma/dE_\gamma$ helps measure $|V_{ub}|$

- $\Gamma(\bar{B} \to X_s\gamma)$ can constrain New Physics

Why $\bar{B} \to X_s\gamma$?

$|V_{ub}|$ Leading order

- At the endpoint region (**at LO in $1/m_b$**)

$$\frac{d\Gamma_u}{dP_+} \sim H_u \cdot J \otimes S + \dots$$

$$\frac{d\Gamma_s}{dE_\gamma} \sim H_s \cdot J \otimes S + \dots$$

H_u, H_s, J calculable in PT

- Extract S from $\bar{B} \to X_s\,\gamma$ and measure $|V_{ub}|$ from $\bar{B} \to X_u\,l\,\bar{\nu}$
- Need to understand $1/m_b$ corrections
  - Contributions $\propto S$ (calculable subleading hard and jet functions)
  - New hadronic functions

Why $\bar{B} \to X_s \gamma$?

$|V_{ub}|$ $1/m_b$ Corrections

- New hadronic functions:

  Known for $\bar{B} \to X_u \, l \, \bar{\nu}$ and $Q_{7\gamma} - Q_{7\gamma}$ contribution to $\bar{B} \to X_s \, \gamma$ (K. Lee, Stewart '04; Bosch, Neubert, GP '04; Beneke, Campanario, Mannel, Pecjak '04)

  "Kinetic"

  – u F.T. of $\int_0^{\bar{n}\cdot x/2} dt \, \langle \bar{h}(0) \ldots (iD_\perp)^2(tn) \ldots h(x_-) \rangle$

  "Chromo-Electro-Magnetic"

  – t F.T. of $\int_0^{\bar{n}\cdot x/2} dt \, \langle \bar{h}(0) \, \frac{\not{n}}{2} \ldots \gamma_\perp^\mu n^\nu \, g \, G_{\mu\nu}(tn) \ldots h(x_-) \rangle$

  – v F.T. of $\int_0^{\bar{n}\cdot x/2} dt \, \langle \bar{h}(0) \, \frac{\not{n}}{2} \ldots \sigma_{\mu\nu}^\perp \, g \, G_\perp^{\mu\nu}(tn) \ldots h(x_-) \rangle$

  "four-quark" shape functions

  – F.T. of $\pi\alpha_s \int_0^{\bar{n}\cdot x/2} dt \int_t^{\bar{n}\cdot x/2} ds \, \langle \bar{h}(0) \ldots q(tn) \, \bar{q}(sn) \ldots h(x_-) \rangle$

Why $\bar{B} \to X_s\gamma$?

$|V_{ub}|$ Complete SSF

- For $\bar{B} \to X_u\, l\, \bar{\nu}$ only need one operator
- for $\bar{B} \to X_s\, \gamma$ need Effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_{p=u,c} \lambda_p \left(C_1 Q_1^p + C_2 Q_2^p + \sum_{i=3,\ldots,10} C_i Q_i + C_{7\gamma} Q_{7\gamma} + C_{8g} Q_{8g} \right) + \text{h.c.}$$

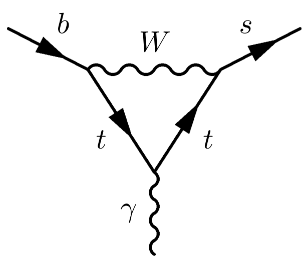
  - At leading order in $1/m_b$ only $Q_{7\gamma} - Q_{7\gamma}$ contribute
  - At higher orders need other $Q_i - Q_j$ contributions
  - Most important: $Q_{7\gamma}, Q_{8g}$, and Q_1

$$\begin{aligned} Q_{7\gamma} &= \frac{-e}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1+\gamma_5) F^{\mu\nu} b \\ Q_{8g} &= \frac{-g_s}{8\pi^2} m_b \bar{s} \sigma_{\mu\nu} (1+\gamma_5) G^{\mu\nu} b \\ Q_1^p &= (\bar{p} b)_{V-A} (\bar{s} p)_{V-A} \quad (p = u, c) \end{aligned}$$

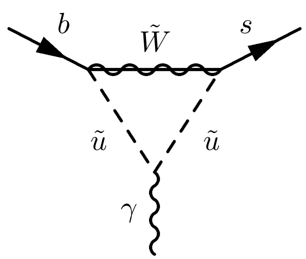
Why $\bar{B} \to X_s\gamma$?
New Physics

- Knowledge of $\Gamma(\bar{B} \to X_s\gamma)$ can constrain New Physics

SM



SUSY



- BSM calculations are becoming advanced e.g.

  Complete NLO SUSY MFV

  SusyBSG package (Degrassi, Gambino, Slavich '07)

- Need to think seriously about errors!

Why $\bar{B} \to X_s\gamma$?

$\Gamma(\bar{B} \to X_s\gamma)$ in SM

- Experiment
  - Experimental value of $\text{Br}(\bar{B} \to X_s\gamma)$:
    Extrapolated from measured $E_\gamma \sim 1.9$ GeV to $E_\gamma > 1.6$ GeV
    (HFAG Average '06)
    $\text{Br}(\bar{B} \to X_s\gamma, E_\gamma > 1.6 \text{ GeV}) = (3.55 \pm 0.26) \cdot 10^{-4}$ (error 7%)
- Theory NNLO:
  - OPE: Assume 1.6 GeV is in the OPE region
    (Misiak et. al. '06)
    $\text{Br}(\bar{B} \to X_s\gamma, E_\gamma > 1.6 \text{ GeV}) = (3.15 \pm 0.23) \cdot 10^{-4}$ (error 7%)
  - MSOPE: 1.6 GeV is still in MSOPE region
    (Becher, Neubert '06)
    $\text{Br}(\bar{B} \to X_s\gamma, E_\gamma > 1.6 \text{ GeV}) = (2.98 \pm 0.26) \cdot 10^{-4}$ (error 9%)
- **Largest error "non perturbative": estimated 5%**

Why $\bar{B} \to X_s \gamma$?
Non Perturbative error

- Common lore:

  like $\Gamma(\bar{B} \to X_u \, l \, \bar{\nu})$ non perturbative effects arise at $1/m_b^2$

- Hints that not all is well
  - $Q_{8g} - Q_{8g}$ (Ali, Greub '95; Kapustin, Ligeti, Politzer '95)
  - $Q_1 - Q_{7\gamma}$ (Voloshin '96; Ligeti,Randall, Wise '97; Grant, Morgan, Nussinov,Peccei '97; Buchalla, Isidori, Rey '97)

- **Never** a systematic study!

  In fact largest uncertainty from $Q_{7\gamma} - Q_{8g}$ was **missed**!

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- **Conclusion:**

  understanding "non perturbative" effects in $\bar{B} \to X_s\gamma$

  important for $|V_{ub}|$ and for New Physics

What's new in this talk?

- **New** factorization formula for $\bar{B} \to X_s\gamma$

- **New** contributions to CP asymmetry in $\bar{B} \to X_s\gamma$

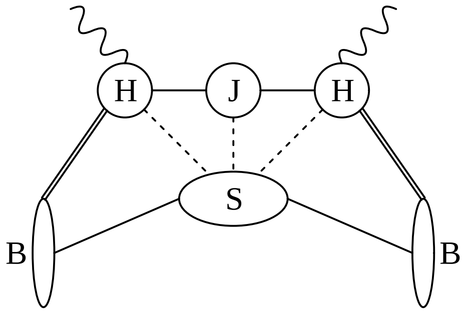
New Factorization Formula

At the endpoint region

- Considering only $Q_{7\gamma} - Q_{7\gamma}$... factorization formula for $d\Gamma/dE_\gamma$

  (Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01)

$$H \cdot J \otimes S$$



New Factorization Formula

At the endpoint region

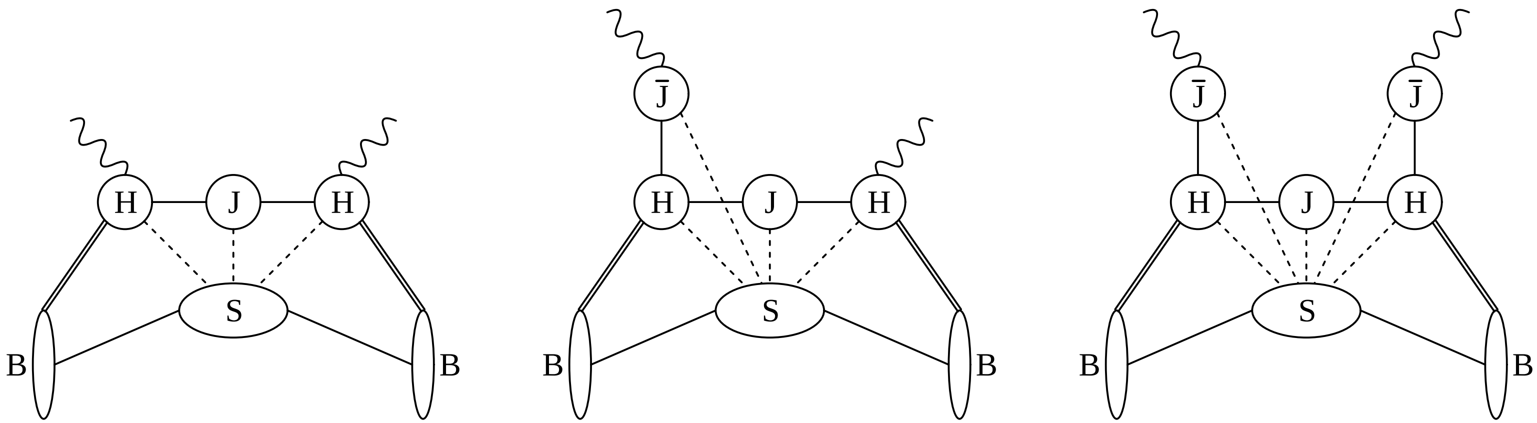
- Considering only $Q_{7\gamma} - Q_{7\gamma}$ factorization formula for $d\Gamma/dE_\gamma$

  (Korchemsky, Sterman '94; Bauer, Pirjol, Stewart '01)

- Considering also other operators **new** factorization formula

  for $d\Gamma/dE_\gamma$ (Lee, Neubert, GP)

$H \cdot J \otimes S$ $\qquad$ $H \cdot J \otimes S \otimes \bar{J}$ $\qquad$ $H \cdot J \otimes S \otimes \bar{J} \otimes \bar{J}$



- No analog in semileptonic decays

Resolved Photon Contribution

- For $Q_i \neq Q_{7\gamma}$, γ does not couple directly to weak vertex

  γ coupling to light partons $\Rightarrow$ new "resolved photon contribution"

  Probe hadronic structure at the scale $\sqrt{E_\gamma \Lambda_{\rm QCD}}$

  Need new jet function $\bar{J}$!

- Resolved photon contribution for $d\Gamma/dE_\gamma$
  - arise at order $1/m_b$ (and higher)
  - "single" resolved contribution (one $\bar{J}$):

    $Q_{7\gamma} - Q_{8g}$, $Q_{7\gamma} - Q_1$
  - "double" resolved contribution (two $\bar{J}$'s):

    $Q_{8g} - Q_{8g}$, $Q_{8g} - Q_1$, $Q_1 - Q_1$

- New soft functions (S)
  - Contain non localities in two light-cone directions
  - Have non zero normalization $\Rightarrow$ No local OPE for total rate

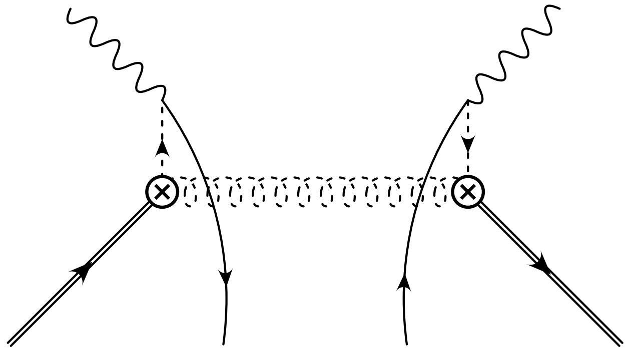
Resolved Photon Contribution

- New factorization formula :

  Lowest order terms arise at $1/m_b$ and at tree level

- We need
  - $Q_{7\gamma} - Q_{7\gamma}$ $\surd$
  - $Q_{8g} - Q_{8g}$
  - $Q_{7\gamma} - Q_{8g}$ ($\overline{\text{hc}}$ gluon, hc gluon)
  - $Q_{7\gamma} - Q_1$
  - $Q_{8g} - Q_1$ $1/m_b^2$
  - $Q_1 - Q_1$ $1/m_b^2$

$Q_{8g} - Q_{8g}$



$$\frac{d\Gamma_{88}^{\rm subl}}{dE_\gamma} = \frac{G_F^2\alpha|V_{tb}V_{ts}^*|^2|C_{8g}|^2\, m_b^2}{\pi^4}(d-2)^2 E_\gamma\, \bar{n}\cdot p\, e_s^2\pi\alpha_s$$

$$\times \int d\omega\, \delta(n\cdot p+\omega) \int \frac{dt}{2\pi}\, e^{i\omega t} \int_{-\infty}^{0} ds \int_0^\infty du$$

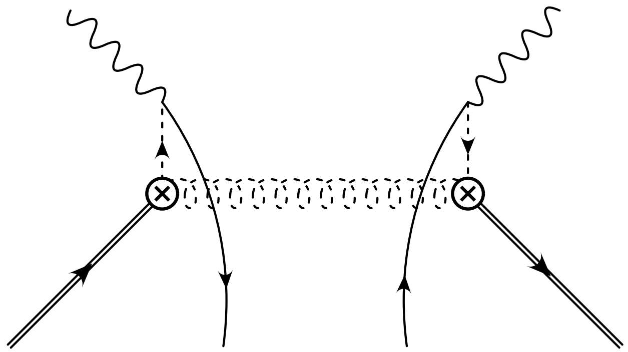
$$\times \frac{1}{2M_B}\langle \bar{B}|\left(\bar{h}S_n\right)_0 \Gamma_{\bar{n}}\, T^A \left(S_n^\dagger S_{\bar{n}}\right)_0 \left(S_{\bar{n}}^\dagger q\right)_{s\bar{n}} \left(\bar{q}S_{\bar{n}}\right)_{u\bar{n}+tn} \left(S_{\bar{n}}^\dagger S_n\right)_{tn} T^A\, \overline{\Gamma}_{\bar{n}} \left(S_n^\dagger h\right)_{tn}|\bar{B}\rangle$$

where $q = s$ and $\Gamma_{\bar{n}} = \not{n}\,\not{\bar{n}}\,(1-\gamma_5)/8$

- Define $g_{88}(\omega,\omega_1,\omega_2)$ as F.T. of matrix element

  g_{88} has support for $-\infty < \omega \le \bar{\Lambda}$ and $0 \le \omega_{1,2} \le \infty$

$Q_{8g} - Q_{8g}$



$$\frac{d\Gamma_{88}^{\rm subl}}{dE_\gamma} \propto \int d\omega\, \delta(n\cdot p+\omega) \int \frac{d\omega_1}{\omega_1} \int \frac{d\omega_2}{\omega_2}\, g_{88}(\omega,\omega_1,\omega_2)$$

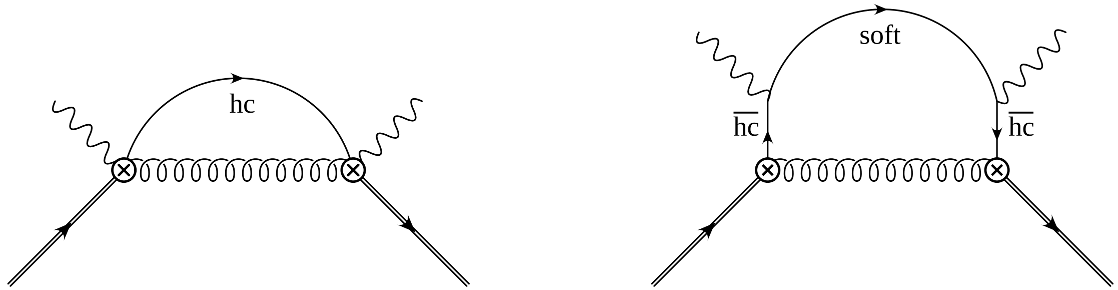
- New factorization formula

$$\begin{aligned} & \frac{1}{4m_b E_\gamma^2} \int d\omega\, \delta(n\cdot p+\omega) \int \frac{d\omega_1}{\omega_1} \int \frac{d\omega_2}{\omega_2}\, g_{88}(\omega,\omega_1,\omega_2) \\ = \quad & H \int d\omega\, J\big(m_b(n\cdot p+\omega)\big) \\ \times \quad & \int d\omega_1\, \bar{J}(2E_\gamma\omega_1) \int d\omega_2\, \bar{J}(2E_\gamma\omega_2)\, g_{88}(\omega,\omega_1,\omega_2) \end{aligned}$$

$Q_{8g} - Q_{8g}$

- Understanding $\ln[m_b(m_b - 2E_\gamma)/m_s^2]$ in parton calculation

  (Ali, Greub '95; Kapustin, Ligeti, Politzer '95)



$\ln[m_b(m_b - 2E_\gamma)/\mu^2] \quad + \quad \ln[\mu^2/m_s^2]$

$Q_{8g} - Q_{8g}$

- Understanding $\ln[m_b(m_b - 2E_\gamma)/m_s^2]$ in parton calculation

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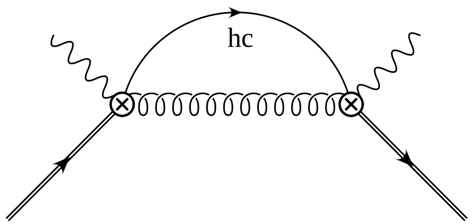
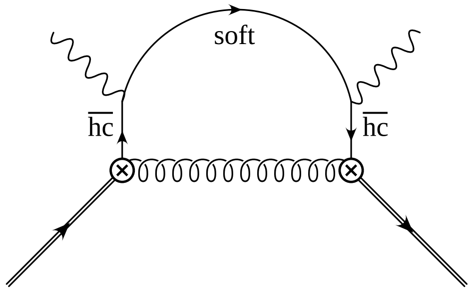
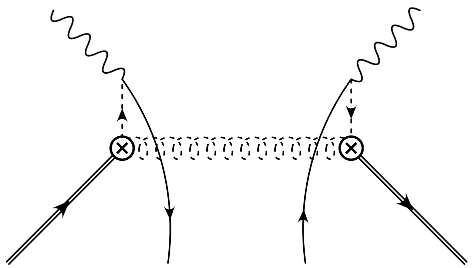




$$\ln[m_b(m_b - 2E_\gamma)/\mu^2] \quad + \quad \ln[\mu^2/m_s^2]$$

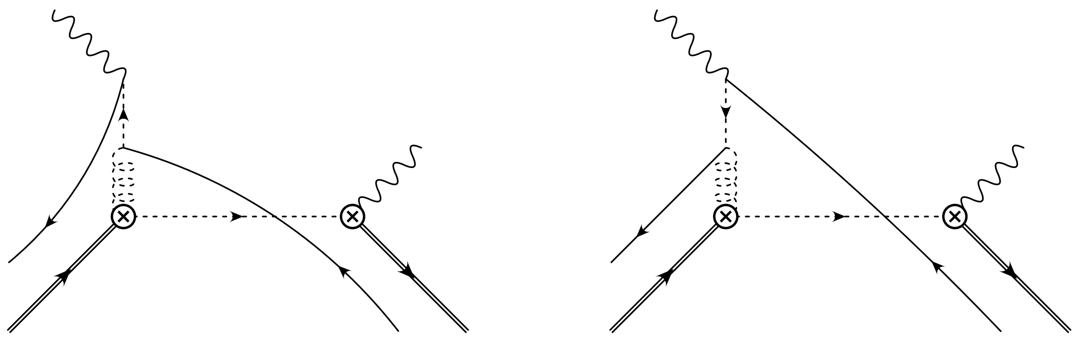
$\downarrow$ $\quad\quad$ $\downarrow$



subleading jet function

subleading shape function

$Q_{7\gamma} - Q_{8g}$: $\overline{\text{hc}}$ gluon



$$\frac{d\Gamma_{78a}^{\text{subl}}}{dE_\gamma} = \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2 m_b^2}{2\pi^4} 4E_\gamma^2 \pi \alpha_s \int d\omega\, \delta(n\cdot p + \omega) \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \sum_q e_q$$

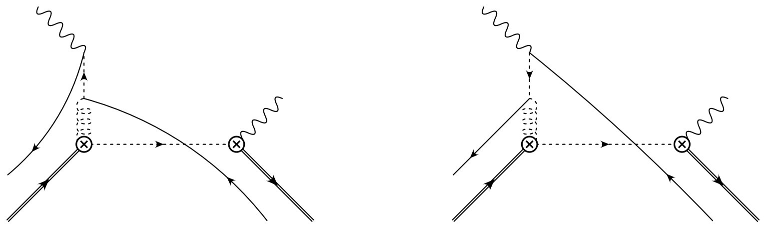
$$\Bigg\{ \text{Re}\Big(C_7^* C_8\Big) \cdot \left[\frac{1}{\omega_1 - \omega_2} \left(\frac{1}{\omega_2} + \frac{1}{\omega_1}\right) \text{Re}\Big(g_{78q}^{RR} + g_{78q}^{RL}\Big) - \frac{1}{\omega_1}\frac{1}{\omega_2} \text{Re}\Big(g_{78q}^{RR} - g_{78q}^{RL}\Big) \right]$$

$$+ \quad \text{Im}\Big(C_7^* C_8\Big) \cdot \left[\frac{1}{\omega_1 - \omega_2} \left(\frac{1}{\omega_2} + \frac{1}{\omega_1}\right) \text{Im}\Big(g_{78q}^{RR} + g_{78q}^{RL}\Big) - \frac{1}{\omega_1}\frac{1}{\omega_2} \text{Im}\Big(g_{78q}^{RR} - g_{78q}^{RL}\Big) \right] \Bigg\}$$

- Where $q = u, d, s$ and $g_{78q}^{ij}(\omega, \omega_1, \omega_2)$ F.T. of $\Big(\Gamma^{R,L} = \bar{\not n}(1 \pm \gamma_5)/2\Big)$

$$\frac{1}{2M_B} \langle \bar{B} | \Big(\bar{h} S_{\bar{n}}\Big)_0 \Gamma^i\, T^A \Big(S_{\bar{n}}^\dagger S_n\Big)_0 \Big(S_n^\dagger h\Big)_{tn} \Big(\bar{q} S_{\bar{n}}\Big)_{u\bar{n}} \Gamma^j\, T^A \Big(S_{\bar{n}}^\dagger q\Big)_{s\bar{n}} |\bar{B}\rangle$$

$Q_{7\gamma} - Q_{8g}$: $\overline{\text{hc}}$ gluon



$$\frac{d\Gamma^{\rm subl}_{78a}}{dE_\gamma} = \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2 m_b^2}{2\pi^4} 4E_\gamma^2 \pi\alpha_s \int d\omega\, \delta(n\cdot p+\omega) \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \sum_q e_q$$

$$\left\{ \mathrm{Re}\left(C_7^* C_8\right) \cdot \left[\frac{1}{\omega_1-\omega_2}\left(\frac{1}{\omega_2}+\frac{1}{\omega_1}\right) \mathrm{Re}\left(g_{78q}^{RR}+g_{78q}^{RL}\right) - \frac{1}{\omega_1}\frac{1}{\omega_2}\mathrm{Re}\left(g_{78q}^{RR}-g_{78q}^{RL}\right)\right] \right.$$

$$\left. + \quad \mathrm{Im}\left(C_7^* C_8\right) \cdot \left[\frac{1}{\omega_1-\omega_2}\left(\frac{1}{\omega_2}+\frac{1}{\omega_1}\right) \mathrm{Im}\left(g_{78q}^{RR}+g_{78q}^{RL}\right) - \frac{1}{\omega_1}\frac{1}{\omega_2}\mathrm{Im}\left(g_{78q}^{RR}-g_{78q}^{RL}\right)\right] \right\}$$

- **New** ingredient: dependence on weak phase of $C_7^* C_8$!

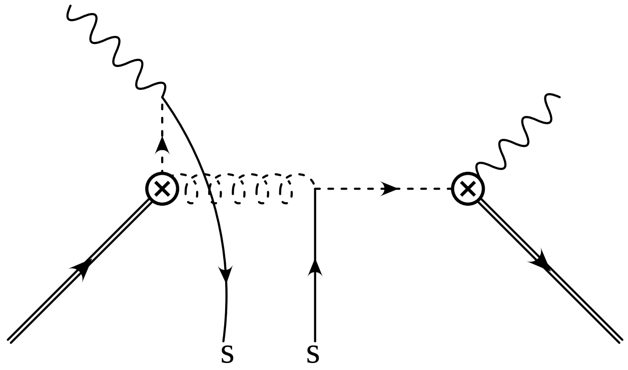
  Strong phase comes from soft function $g_{78q}^{ij}(\omega,\omega_1,\omega_2)$

  Similar to strong phase in color suppressed $\bar{B}^0 \to D^{(*)0} M^0$

  (Mantry, Pirjol, Stewart '03)

- New source of CP asymmetry in $\bar{B} \to X_s\gamma$

$Q_{7\gamma} - Q_{8g}$:   hc gluon



$$\frac{d\Gamma^{\rm subl}_{78b}}{dE_\gamma} = \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2 m_b^2}{2\pi^4}\,\frac{E_\gamma^2 \pi \alpha_s}{2}$$

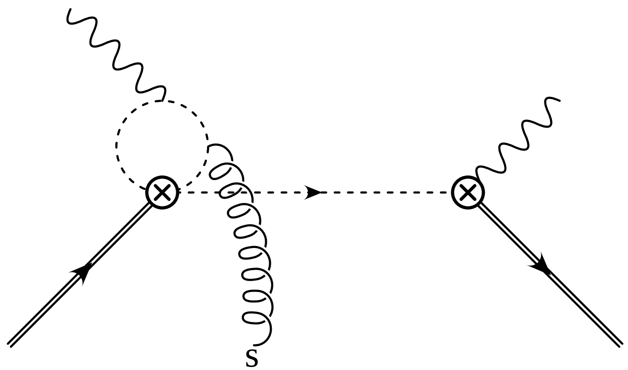
$$\times \quad \int d\omega\, \delta(n\cdot p+\omega) \int \frac{dt}{2\pi}\, e^{i\omega t} \int_t^0 ds \int du\, \theta(-u)$$

$$\times \quad \left\{ \mathrm{Im}\Big(C^*_{7\gamma}C_{8g}\Big)\mathrm{Im}\Big[\tilde{g}^{RR}_{78b}(s,u,t)\Big] - \mathrm{Re}\Big(C^*_{7\gamma}C_{8g}\Big)\mathrm{Re}\Big[\tilde{g}^{RR}_{78b}(s,u,t)\Big] \right\}$$

where $\tilde{g}^{RR}_{78b}(s,u,t)$ is

$$\frac{1}{2M_B}\langle \bar{B}| \Big(\bar{h}S_{\bar{n}}\Big)_0 \Gamma^R\, T^A\, \Big(S_n^\dagger S_{\bar{n}}\Big)_0 \Big(S_{\bar{n}}^\dagger q\Big)_{u\bar{n}} \Big(\bar{q}S_n\Big)_{sn} \Gamma^R\, T^A\, \Big(S_n^\dagger h\Big)_{tn} |\bar{B}\rangle$$

$Q_{7\gamma} - Q_1$



$$\frac{d\Gamma_{71c}}{dE_\gamma} = \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2 m_b^2}{2\pi^4} \cdot E_\gamma^3 \frac{e_c}{m_b} \int d\omega\, \delta(n\cdot p + \omega)$$

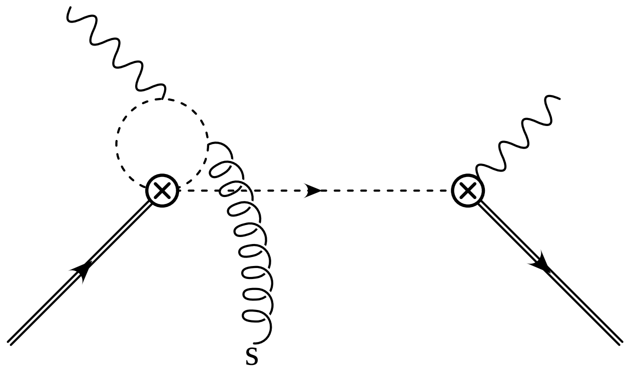
$$\int_0^\infty \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2}{2E_\gamma\,\omega_1}\right)\right] \mathrm{Re}(C_1 C_{7\gamma}^*)\, g_{71c}(\omega,\omega_1)$$

$g_{71c}(\omega,\omega_1)$ F.T. of

$$\frac{1}{2M_B}\langle \bar{B}| \left(\bar{h} S_{\bar{n}}\right)_0 i\gamma_\perp^\mu \not{\bar{n}} \bar{n}^\alpha \left(S_n^\dagger S_{\bar{n}}\right)_{tn} \left(S_{\bar{n}}^\dagger g G_{\alpha\mu} S_{\bar{n}}\right)_{u\bar{n}} \left(S_{\bar{n}}^\dagger h\right)_{tn} |\bar{B}\rangle$$

- Non trivial $\bar{J}$ from loop: $F(r) = 4r \arctan^2\left[(4r-1)^{-1/2}\right]$
- No $\mathrm{Im}(C_1 C_{7\gamma}^*)$ contribution since all moments of soft function vanish

$Q_{7\gamma} - Q_1$



$$\frac{d\Gamma_{71c}}{dE_\gamma} = \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2 m_b^2}{2\pi^4} \cdot E_\gamma^3 \frac{e_c}{m_b} \int d\omega\, \delta(n\cdot p + \omega)$$

$$\int_0^\infty \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2}{2E_\gamma\, \omega_1}\right)\right] \mathrm{Re}(C_1 C_{7\gamma}^*)\, g_{71c}(\omega, \omega_1)$$

- Taking $m_c \sim m_b$, can expand $\bar{J}$

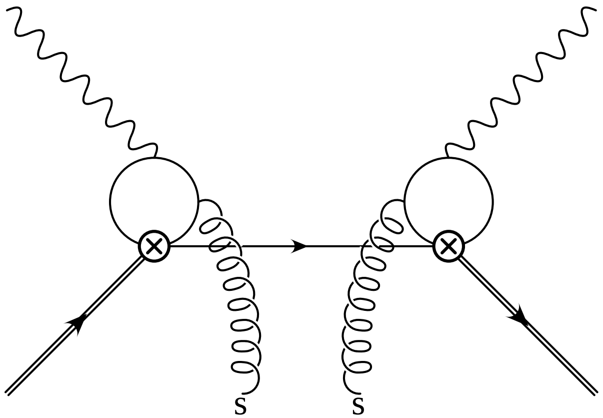
  (Voloshin '96; Ligeti,Randall, Wise '97; Grant, Morgan, Nussinov,Peccei '97; Buchalla, Isidori, Rey '97)

  Reproduce the $\lambda_2/9m_c^2$ term in the total rate

- Taking $m_c^2 \sim m_b \Lambda_{\mathrm{QCD}}$, $\bar{J}$ should not be expanded

$Q_{8g} - Q_1$ and $Q_1 - Q_1$

- $Q_{8g} - Q_1$ and $Q_1 - Q_1$ give $1/m_b^2$ suppressed contribution, e.g.



New Contributions - Summary

- At order $1/m_b$ and at tree level find 4 contributions:

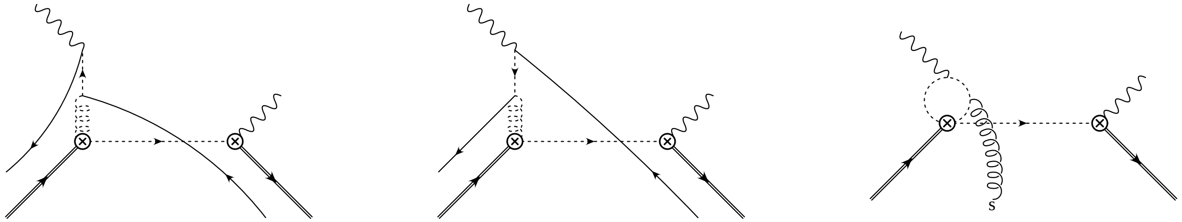
Pair	Factorization	S
$Q_{8g} - Q_{8g}$	$H \cdot J \otimes S \otimes \bar{J} \otimes \bar{J}$	$\langle \bar{h}...q\,\bar{q}...h \rangle$
$Q_{7\gamma} - Q_{8g}$ ($\overline{\text{hc}}$ gluon)	$H \cdot J \otimes S \otimes \bar{J}$	$\langle \bar{h}...h\,\bar{q}...q \rangle$
$Q_{7\gamma} - Q_{8g}$ (hc gluon)	$H \cdot J \otimes S \otimes \bar{J}$	$\langle \bar{h}...q\,\bar{q}...h \rangle$
$Q_{7\gamma} - Q_1$	$H \cdot J \otimes S \otimes \bar{J}$	$\langle \bar{h}...G_{\mu\nu}...h \rangle$

4-q soft functions differ by position and Dirac structure

- What about the “total rate”?

"Total Rate"

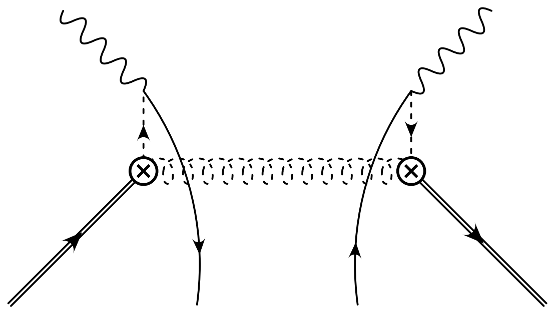
- $Q_{7\gamma} - Q_{8g}$, $Q_{7\gamma} - Q_1$, interference with $Q_{7\gamma}$ at tree level



$\Rightarrow$ $\overline{\text{hc}}$ photon even for total rate

q in $\langle \bar{h}...h\, \bar{q}...q \rangle$ and $G_{\mu\nu}$ in $\langle \bar{h}...G_{\mu\nu}...h \rangle$ are soft

- $Q_{8g} - Q_{8g}$



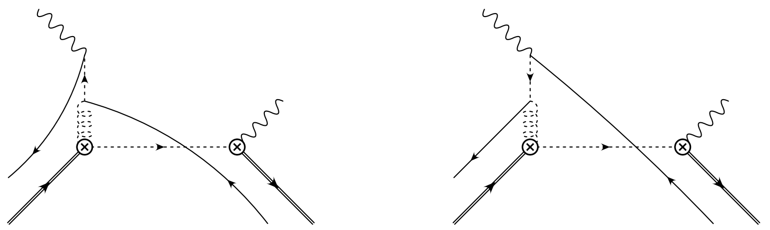
Outside the endpoint region photon no longer $\overline{\text{hc}}$

s quark might not be soft

Can't extrapolate from endpoint region

"Total Rate": $Q_{7\gamma} - Q_{8g}$ $\overline{\text{hc}}$ gluon

- $Q_{7\gamma} - Q_{8g}$ with $\overline{\text{hc}}$ gluon



Out of the **four** contributions

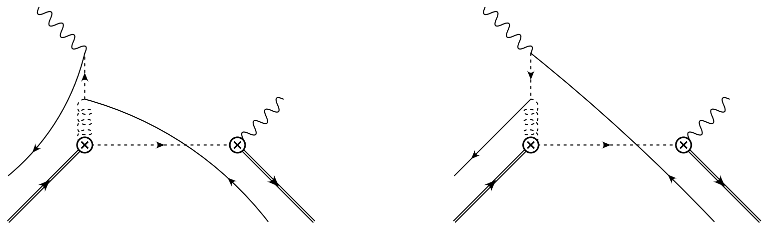
$$\frac{d\Gamma^{\rm subl}_{78a}}{dE_\gamma} = \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2 m_b^2}{2\pi^4} \, 4E_\gamma^2 \pi \alpha_s \int d\omega \, \delta(n\cdot p + \omega) \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \sum_q e_q$$

$$\left\{ {\rm Re}\,(C_7^* C_8) \cdot \left[\frac{1}{\omega_1 - \omega_2} \left(\frac{1}{\omega_2} + \frac{1}{\omega_1} \right) {\rm Re}\left(g_{78q}^{RR} + g_{78q}^{RL} \right) - \frac{1}{\omega_1}\frac{1}{\omega_2} {\rm Re}\left(g_{78q}^{RR} - g_{78q}^{RL} \right) \right] \right.$$

$$\left. + \quad {\rm Im}\,(C_7^* C_8) \cdot \left[\frac{1}{\omega_1 - \omega_2} \left(\frac{1}{\omega_2} + \frac{1}{\omega_1} \right) {\rm Im}\left(g_{78q}^{RR} + g_{78q}^{RL} \right) - \frac{1}{\omega_1}\frac{1}{\omega_2} {\rm Im}\left(g_{78q}^{RR} - g_{78q}^{RL} \right) \right] \right\}$$

"Total Rate": $Q_{7\gamma} - Q_{8g}$ $\overline{\text{hc}}$ gluon

- $Q_{7\gamma} - Q_{8g}$ with $\overline{\text{hc}}$ gluon



Only **two** survive in the total rate

$$\Gamma^{\text{subl}}_{78a} = \int dE_\gamma \frac{G_F^2 \alpha |V_{tb}V_{ts}^*|^2 m_b^2}{2\pi^4} 4E_\gamma^2 \pi \alpha_s \int d\omega\, \delta(n\cdot p + \omega) \int_0^\infty d\omega_1 \int_0^\infty d\omega_2 \sum_q e_q$$
$$\Big\{ \text{Re}\,(C_7^* C_8) \cdot \Big[\qquad - \frac{1}{\omega_1}\frac{1}{\omega_2} \text{Re}\left(g_{78q}^{RR} - g_{78q}^{RL}\right) \Big]$$
$$+ \quad \text{Im}\,(C_7^* C_8) \cdot \Big[\frac{1}{\omega_1 - \omega_2}\left(\frac{1}{\omega_2} + \frac{1}{\omega_1}\right) \text{Im}\left(g_{78q}^{RR} + g_{78q}^{RL}\right) \Big] \Big\}$$

- $\text{Im}\,(C_7^* C_8)$ gives new contribution to CP asymmetry
- $\text{Re}\,(C_7^* C_8)$ was analyzed previously (Lee, Neubert, GP '06)

"Total Rate": $Q_{7\gamma} - Q_{8g}$ $\overline{\text{hc}}$ gluon

- Naive dimensional analysis

$$\frac{\Delta\Gamma}{\Gamma_{77}} \approx \frac{C_{8g}}{C_{7\gamma}}\, \pi\alpha_s \frac{\Lambda_{\text{QCD}}}{m_b} \approx 5\%$$

- Using VIA

$$\frac{\Delta\Gamma_{\text{VIA}}}{\Gamma_{77}} \approx (-0.3\%, -3\%)$$

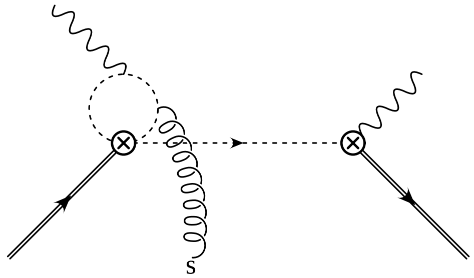
- Same terms also give **leading contribution** to isospin asymmetry

$$\Delta_{0-} \equiv \frac{\Gamma(\bar{B}^0 \to X_s\gamma) - \Gamma(B^- \to X_s\gamma)}{\Gamma(\bar{B}^0 \to X_s\gamma) - \Gamma(B^- \to X_s\gamma)} \approx (1\%,\ 9.5\%) \textbf{ in VIA}$$

- Asymmetry measured by BaBar
  - [PRD **72** 052004 (2005)]: $\Delta_{0-} = -0.6\% \pm 6.3\%$, $E_\gamma > 1.9$ GeV
  - [PRD **77** 051103(R) (2008)]: $\Delta_{0-} = -6\% \pm 16.3\%$, $E_\gamma > 2.2$ GeV

"Total Rate": $Q_{7\gamma} - Q_1$

- $Q_{7\gamma} - Q_1$ contribution (in $\bar{n} \cdot A = 0$ gauge)



$$\Gamma_{71c} = -\Gamma_{77} \frac{e_c}{m_b} \int_0^\infty \frac{d\omega_1}{\omega_1} \left[1 - F\left(\frac{m_c^2}{m_b\,\omega_1}\right)\right] \frac{1}{2M_B} \int du \frac{e^{i\omega_1 u}}{2\pi} \frac{\mathrm{Re}(C_1 C_{7\gamma}^*)}{|C_{7\gamma}|^2} \langle \bar{B}|\bar{h}(0)\gamma_\perp^\mu \not{\bar{n}} i\bar{n}^\alpha g G_{\mu\alpha}(u\bar{n}) h(0)|\bar{B}\rangle$$

- Taking $m_c^2 \sim m_b \Lambda_{\text{QCD}}$,

  Parametrically F should not be expanded

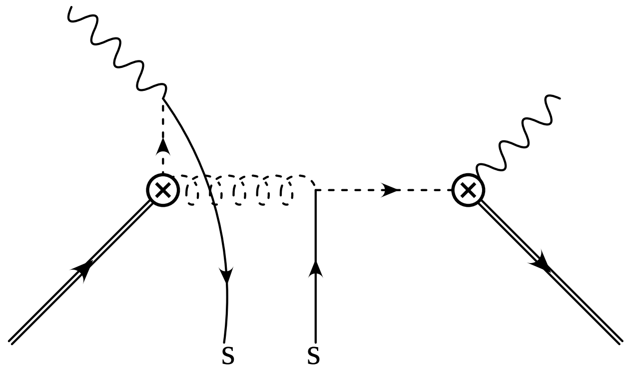
  But **numerically** F is well approximated by very few terms

  $\Rightarrow$ can safely approximate

$$\Gamma_{71c} \approx -\Gamma_{77} \frac{\mathrm{Re}(C_1 C_{7\gamma}^*)}{|C_{7\gamma}|^2} \left(\frac{\lambda_2}{9m_c^2} + \# \frac{m_b \rho_2}{m_c^4}\right)$$

“Total Rate”: $Q_{7\gamma} - Q_{8g}$ hc gluon

- No contribution from $Q_{7\gamma} - Q_{8g}$ with hc gluon



- Similar to $Q_{7\gamma} - Q_{7\gamma}$ 4q SSF, integrate to zero

"Total Rate": Summary

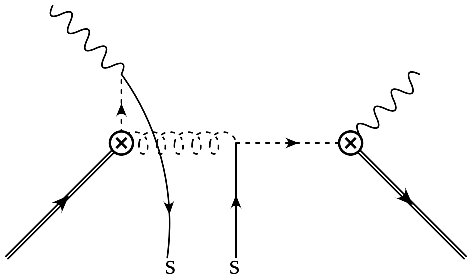
- $Q_{8g} - Q_{8g}$: ?
- $Q_{7\gamma} - Q_1$: rapid **numerical** convergence of $\bar{J}$

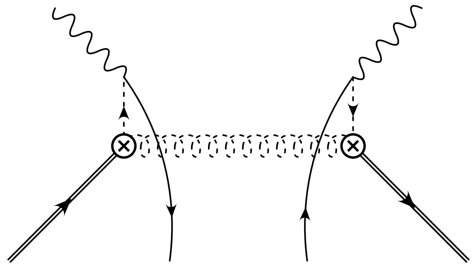
  $\Rightarrow$ contribution well approximated by few HQET parameters
- $Q_{7\gamma} - Q_{8g}$ hc gluon: no contribution to total rate
- $Q_{7\gamma} - Q_{8g}$ $\overline{\text{hc}}$ gluon: most interesting contribution
  - Leading source of isospin asymmetry
  - New source of CP violation
  - Uncertainty on total rate hard to estimate
  - In **VIA**: $\Delta\Gamma_{\text{VIA}}/\Gamma_{77} \approx (-0.3\%, -3\%)$

  - What about the photon spectrum?

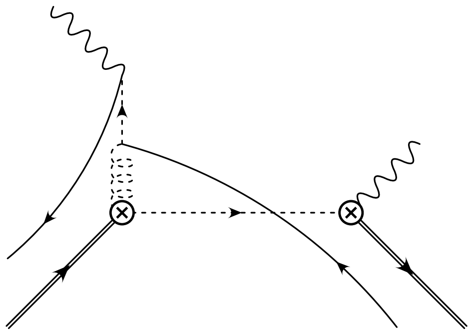
Effect on the Photon Spectrum

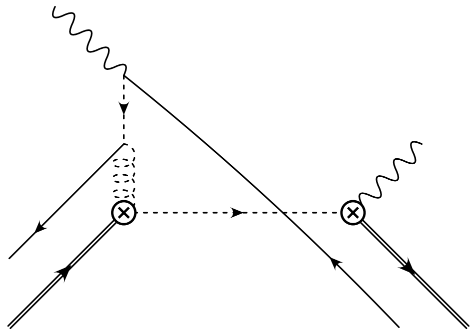
- Even harder to estimate than total rate
  - More non vanishing soft functions g_{ij}
  - How to estimate 4q matrix elements?
- Even when using VIA
  - Many vanish due to color (and/or flavor)





  - or cannot be related to product of LCDAs





Effect on Photon Spectrum - Preliminary!

- Try to model $S \otimes \bar{J}$ as a whole: $f_{ij}(\omega)$
- Estimate zeroth moment of $f_{ij}(\omega)$:

$$
\begin{aligned}
f_{88}(\omega)\Big|_{\rm VIA} &= f_{78}^{\rm hc}(\omega)\Big|_{\rm VIA} = 0 \\
f_{78}^{\overline{\rm hc}}(\omega)\Big|_{\rm VIA} &= \frac{e_q(N_c^2-1)}{8N_c^2}\,\frac{f_B^2 m_B}{\lambda_B^2}\,\delta(\omega)+... \\
f_{71c}^{R}(\omega, 2E_\gamma) &= -\frac{E_\gamma \lambda_2}{3m_c^2}\,\delta(\omega)+...
\end{aligned}
$$

- Replace $\delta(\omega)$ by a normalized function $n(\omega)$
- Taking $\hat{n}(\hat{\omega}) = n(\bar{\Lambda}-\hat{\omega})$ to be

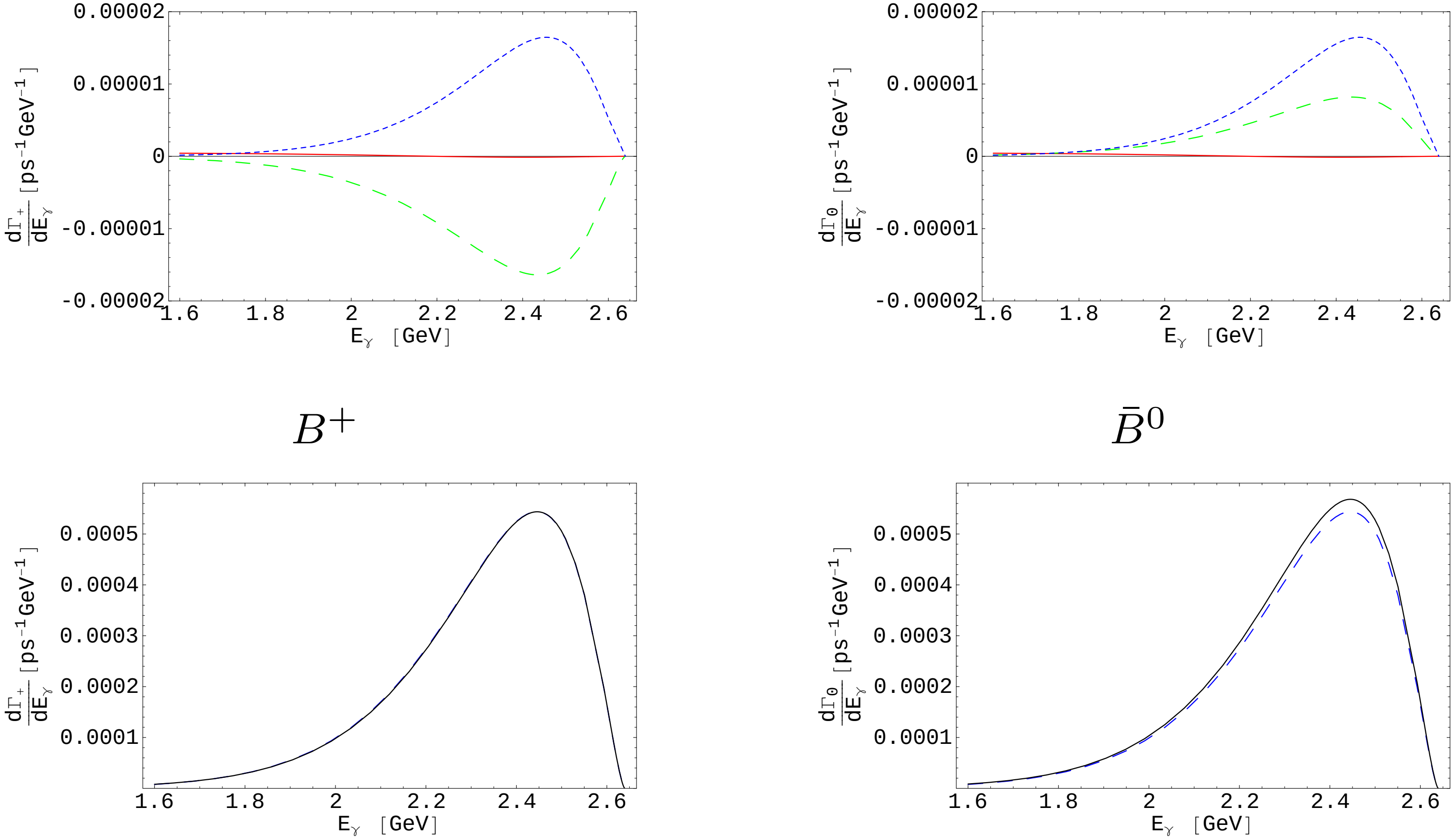
$$
\hat{n}(P_+) = \frac{b^b}{\Gamma(b)\Lambda^b}(P_+)^{b-1}\exp\left(-b\frac{P_+}{\Lambda}\right),
$$

with $\Lambda = 0.77$ GeV and $b = 2.5$

$\hat{n}(P_+)$ also model normalized tree level $Q_{7\gamma} - Q_{7\gamma}$ contribution

Effect on Photon Spectrum - Preliminary!

- New contributions: $\hat{f}_{78}^{\overline{\text{hc}}}$, $\hat{f}_{71c}$, $j_{88} \otimes$ LO SF



- Photon spectrum: dashed: $Q_{7\gamma} - Q_{7\gamma}$ at lowest order only

  Solid: $Q_{7\gamma} - Q_{7\gamma}$ at lowest order + new contributions

Conclusions

- **New factorization formula** for $\bar{B} \to X_s\gamma$ at the endpoint region

$$d\Gamma = H \cdot J \otimes S + H \cdot J \otimes S \otimes \bar{J} + H \cdot J \otimes S \otimes \bar{J} \otimes \bar{J}$$

- New "resolved photon contributions"
  - New jet function $\bar{J}$
  - New soft function: non local in two light cone directions
- Lowest order terms arise at $1/m_b$ and at tree level

  $Q_{8g} - Q_{8g}$, $Q_{7\gamma} - Q_{8g}$, $Q_{7\gamma} - Q_1$
  - **New** source of CP asymmetry
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  In VIA around 5%

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Can we do better?