Power Corrections to the Pion Form Factor in SCET

Emanuele Mereghetti
(ongoing work with Sean Fleming)

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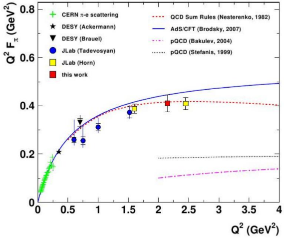
Motivation

Well understood in pQCD

- dimensional counting rule $F_\pi \sim 1/Q^2$
- exact limit $F_\pi(Q^2) \to 8\pi\alpha_s(Q^2)\frac{f_\pi^2}{Q^2}$.
- logarithmic evolution with Q^2



T. Horn et al., nucl-ex/0607005

However

- discrepancy between asymptotic behavior and present data
- discrepancy not seen in $F_{\gamma\pi}$ transition form factor at the same Q^2

Estimate of the power corrections

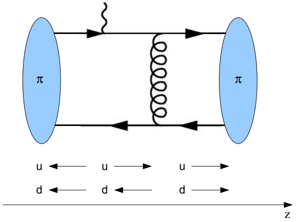
- Brodsky and Lepage: end-point contributions $\frac{\Lambda_{QCD}}{Q}$

  "Exclusive processes in perturbative QCD" Phys.Rev.D22: 2157, 1980

- Chernyak and Zhitnitsky: higher twist wavefunctions $\frac{\Lambda^2_{QCD}}{Q^2}$

  "Asymptotic behaviour of exclusive process in QCD" Phys.Rept.112:173,1984.

$F_\pi(Q^2)$ in SCET. Leading Order



Factorization

- at leading order in η, all orders in α_s

$$F_\pi(Q^2,\mu) = \phi_{\pi,\bar{n}} \otimes \mathrm{T}_H \otimes \phi_{\pi,n}$$

- T_H hard kernel, perturbative
- $\phi_{\pi,n}$ pion lightcone wavefunction, non perturbative.

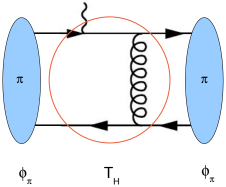
- ϕ_π defined by

$$\begin{aligned}&\langle \pi^a | \bar{\xi}_{n,p_1} W_n \frac{\not{\bar{n}}}{2} \gamma^5 \frac{\tau^b}{\sqrt{2}} \delta\left(\omega_+ - \bar{n}\cdot\mathcal{P}_+\right) W_n^\dagger \xi_{n,p_2} |0\rangle \\ &\qquad = -\, i f_\pi \bar{n}\cdot p\, \delta^{ab} \int_0^1 dz\, \delta\left(\omega_+ - (2z-1)\bar{n}\cdot p\right) \phi_{\pi,n}(\mu,z)\end{aligned}$$

- at tree level

$$F_\pi(Q^2,\mu) = \frac{f_\pi^2}{Q^2} \frac{8\pi\alpha_s(Q)}{9} \int_0^1 dz_1 \frac{\phi_\pi(z_1)}{z_1} \int_0^1 dz_2 \frac{\phi_\pi(z_2)}{z_2}$$

$F_\pi(Q^2)$ in SCET. Leading Order



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Pion Form Factor in SCET. Power Corrections

based on: I. Rothstein, Phys.Rev.D70:054024, 2005

SCET$_I$

hard-collinear particles $p_c \sim Q\left(\lambda^2, \lambda^0, \lambda\right)$, $p_c^2 \sim Q\Lambda_{QCD}$

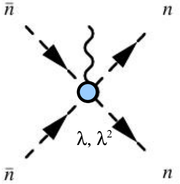
usoft particles $p_{us} \sim Q\left(\lambda^2, \lambda^2, \lambda^2\right)$, $p_{us}^2 \sim \Lambda_{QCD}^2$,

$$\lambda \sim \sqrt{\frac{\Lambda_{QCD}}{Q}}$$

- subleading terms in the matching of electromangetic current J^μ

order λ $\bar{\chi}_{n,\omega_1} \not{\bar{n}} \gamma^5 \mathcal{P}_\perp^\mu \chi_{n\omega_4} \;\bar{\chi}_{\bar{n},\bar{\omega}_3} \not{n} \gamma^5 \chi_{\bar{n}\bar{\omega}_2}$,

order λ^2 $(n^\mu + \bar{n}^\mu)\, \bar{\chi}_{n,\omega} \not{\bar{n}} \gamma^5 \mathcal{P}_\perp^2 \chi_{n\omega_4} \;\bar{\chi}_{\bar{n},\bar{\omega}_3} \not{n} \gamma^5 \chi_{\bar{n}\bar{\omega}_2}$



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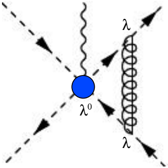
usoft particles	$p_{us} \sim Q\left(\lambda^2, \lambda^2, \lambda^2\right)$,	$p_{us}^2 \sim \Lambda_{QCD}^2$,	

- subleading terms in the matching of electromangetic current J^μ

order λ^2 $\quad (n^\mu + \bar{n}^\mu)\,\chi_{n,\omega}\not{\bar{n}}\gamma^5\mathcal{P}_\perp^2\chi_{n\omega_4}\;\bar{\chi}_{\bar{n},\bar{\omega}_3}\not{n}\gamma^5\chi_{\bar{n}\bar{\omega}_2}$



- TOP with subleading collinear lagrangians

$$T\left[\mathcal{O}\mathcal{L}_{c,\,n}^{(1)}\mathcal{L}_{c,\,n}^{(0)}\right], \quad T\left[\mathcal{O}\mathcal{L}_{c,\,n}^{(1)}\mathcal{L}_{c,\,n}^{(1)}\right]$$

order λ^2,

Pion Form Factor in SCET. Power Corrections

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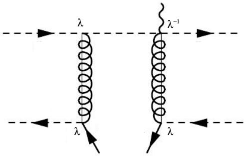
SCET$_I$

hard-collinear particles	$p_c \sim Q\left(\lambda^2, \lambda^0, \lambda\right)$,	$p_c^2 \sim Q\Lambda_{QCD}$	$\lambda \sim \sqrt{\frac{\Lambda_{QCD}}{Q}}$
usoft particles	$p_{us} \sim Q\left(\lambda^2, \lambda^2, \lambda^2\right)$,	$p_{us}^2 \sim \Lambda_{QCD}^2$,	

- subleading terms in the matching of electromagnetic current J^μ

order λ^2 $$(n^\mu + \bar{n}^\mu)\,\chi_{n,\omega}\bar{\not{n}}\gamma^5\mathcal{P}_\perp^2\chi_{n\omega_4}\;\bar{\chi}_{\bar{n},\bar{\omega}_3}\not{n}\gamma^5\chi_{\bar{n}\bar{\omega}_2}$$



- TOP with subleading collinear lagrangians

order λ^2, higher twist wavefunctions

- TOP with usoft collinear interactions

$$T\left[J^\mu\mathcal{L}_{c,\,n}^{(1)}\,\mathcal{L}_{us\,c,\,n}^{(1)}\,\mathcal{L}_{us\,c,\,\bar{n}}^{(1)}\right], \quad T\left[J^\mu\mathcal{L}_{c,\,n}^{(0)}\,\mathcal{L}_{us\,c,\,n}^{(2)}\,\mathcal{L}_{us\,c,\,\bar{n}}^{(1)}\right]$$

order λ^2

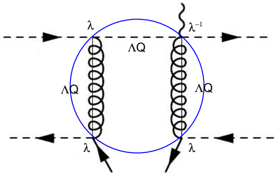
Pion Form Factor in SCET. Power Corrections

SCET$_{II}$:

collinear particles	$p_c \sim Q\left(\eta^2, \eta^0, \eta\right)$,	$p_c^2 \sim \Lambda_{QCD}^2$	$\eta \sim \frac{\Lambda_{QCD}}{Q}$
soft particles	$p_{us} \sim Q\left(\eta, \eta, \eta\right)$,	$p_{us}^2 \sim \Lambda_{QCD}^2$,	

Matching

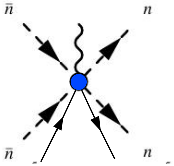
- collinear $p_c = (p^+ + r^+, p^-, p_\perp + r_\perp) \longrightarrow p_c = (r^+, p^-, r_\perp)$
- usoft $\longrightarrow$ soft.
- diagrams with only collinear particles: $\lambda^\delta \rightarrow \eta^\delta$
- diagrams with usoft and collinear particles:
  internal lines with offshellness $\sim Q\Lambda_{QCD}$ integrated out



λ^2

SCET$_I$

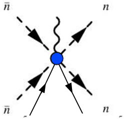
$\longrightarrow$



η

SCET$_{II}$

Matching Result



$$= \int d\omega_i d\bar\omega_i \int d\sigma_1 d\bar\sigma_2\, C^{(j)}(\omega_i,\bar\omega_i,\sigma_1,\bar\sigma_2)\, \mathcal{O}_j(\omega_i,\bar\omega_i,\sigma_1,\bar\sigma_2)$$

with $\sigma_1 \to n\cdot\partial$, $\bar\sigma_2 \to \bar n\cdot\partial$, label operators on soft fields.

- One Example:

$$\begin{aligned}\mathcal{O}_a = {} & (n^\mu + \bar n^\mu)\, \bar\chi_{n\omega_1}\frac{\bar{\not{n}}}{2}\gamma^5\tau^+\chi_{n\omega_4}\, \bar\chi_{\bar n\bar\omega_3}\frac{\not{n}}{2}\gamma^5\tau^-\chi_{\bar n\bar\omega_2}\, \bar Q_{n\sigma_1} S_n^\dagger\, \mathrm{Q}_u \frac{1-\tau^3}{2} S_{\bar n}\, i\not{\partial}_\perp Q_{\bar n\bar\sigma_2} \\ & + (n^\mu + \bar n^\mu)\, \bar\chi_{n\omega_1}\frac{\bar{\not{n}}}{2}\gamma^5\tau^-\chi_{n\omega_4}\, \bar\chi_{\bar n\bar\omega_3}\frac{\not{n}}{2}\gamma^5\tau^+\chi_{\bar n\bar\omega_2}\, \bar Q_{n\sigma_1} S_n^\dagger\, \mathrm{Q}_d \frac{1+\tau^3}{2} S_{\bar n}\, i\not{\partial}_\perp Q_{\bar n\bar\sigma_2} + \text{h.c.}\end{aligned}$$

- Soft operators & matching coefficients:

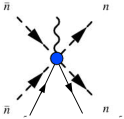
$$\mathcal{O}_a \sim \bar Q_{n\sigma_1} S_n^\dagger\, S_{\bar n}\, i\not{\partial}_\perp Q_{\bar n\bar\sigma_2}$$

$$\mathcal{O}_c \sim \bar Q_{n\sigma_1} S_n^\dagger\, S_{\bar n}\frac{\not{n}}{2} i\bar n\cdot\partial\, Q_{\bar n\bar\sigma_2}$$

$$C_a = -8\pi^2\alpha_s^2\left(\frac{C_F}{N_c}\right)^2 \frac{1}{\sigma_1\bar\sigma_2^2}\left[\frac{1}{\bar\omega_2-\bar\omega_3}\,\frac{1}{\bar\omega_3\omega_4} + \frac{1}{\omega_4\bar\omega_3^2}\right]$$

$$C_c = -32\pi^2\alpha_s^2\left(\frac{C_F}{N_c}\right)^2 \frac{1}{\sigma_1\bar\sigma_2^2}\left[\frac{1}{\omega_4\bar\omega_3^2}\right]$$

Matching Result



$$= \int d\omega_i d\bar{\omega}_i \int d\sigma_1 d\bar{\sigma}_2 C^{(j)}(\omega_i, \bar{\omega}_i, \sigma_1, \bar{\sigma}_2)\, \mathcal{O}_j(\omega_i, \bar{\omega}_i, \sigma_1, \bar{\sigma}_2)$$

with $\sigma_1 \to n \cdot \partial$, $\bar{\sigma}_2 \to \bar{n} \cdot \partial$, label operators on soft fields.

- One Example:

$$\mathcal{O}_a = (n^\mu + \bar{n}^\mu)\, \bar{\chi}_{n\omega_1} \frac{\bar{n}\!\!\!/}{2} \gamma^5 \tau^+ \chi_{n\omega_4}\, \bar{\chi}_{\bar{n}\bar{\omega}_3} \frac{n\!\!\!/}{2} \gamma^5 \tau^- \chi_{\bar{n}\bar{\omega}_2}\, \bar{Q}_{n\sigma_1} S_n^\dagger\, \mathrm{Q}_u \frac{1-\tau^3}{2} S_{\bar{n}} i\partial\!\!\!/_\perp Q_{\bar{n}\bar{\sigma}_2}$$
$$+ (n^\mu + \bar{n}^\mu)\, \bar{\chi}_{n\omega_1} \frac{\bar{n}\!\!\!/}{2} \gamma^5 \tau^- \chi_{n\omega_4}\, \bar{\chi}_{\bar{n}\bar{\omega}_3} \frac{n\!\!\!/}{2} \gamma^5 \tau^+ \chi_{\bar{n}\bar{\omega}_2}\, \bar{Q}_{n\sigma_1} S_n^\dagger\, \mathrm{Q}_d \frac{1+\tau^3}{2} S_{\bar{n}} i\partial\!\!\!/_\perp Q_{\bar{n}\bar{\sigma}_2} + \text{h.c.}$$

- Soft operators & matching coefficients:

$$\mathcal{O}_b \sim \bar{Q}_{n\sigma_1} (i\partial\!\!\!/_\perp)^\dagger\, S_n^\dagger\, S_{\bar{n}} Q_{\bar{n}\bar{\sigma}_2}$$

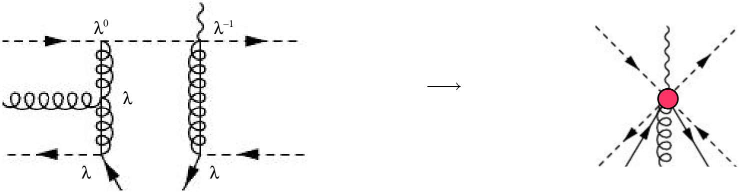
$$\mathcal{O}_d \sim \bar{Q}_{n\sigma_1} (in \cdot \partial)^\dagger\, S_n^\dagger\, S_{\bar{n}} \frac{\bar{n}\!\!\!/}{2} Q_{\bar{n}\bar{\sigma}_2}$$

$$C_b = -8\pi^2 \alpha_s^2 \left(\frac{C_F}{N_c}\right)^2 \frac{1}{\sigma_1^2 \bar{\sigma}_2} \left[\frac{1}{\omega_1 - \omega_4} \frac{1}{\bar{\omega}_3 \omega_4} + \frac{1}{\omega_4^2 \bar{\omega}_3}\right]$$

$$C_d = -32\pi^2 \alpha_s^2 \left(\frac{C_F}{N_c}\right)^2 \frac{1}{\sigma_1^2 \bar{\sigma}_2} \left[\frac{1}{\omega_4^2 \bar{\omega}_3}\right]$$

Matching Results

- Matching at one soft gluon



$$\mathcal{O}_e \sim \bar{Q}_{n\sigma_1} S_n^\dagger \, \mathrm{Q}_u \frac{1-\tau^3}{2} S_{\bar{n}} i g \not{\mathcal{B}}_{\perp \bar{n}\bar{\sigma}_3} Q_{\bar{n}\bar{\sigma}_2} \qquad C_e = 8\pi^2 \alpha_s^2 \frac{1}{\sigma_1 \bar{\sigma}_2 \left(\bar{\sigma}_2 + \bar{\sigma}_3\right)} \left[\left(\frac{C_F}{N_c}\right)^2 \frac{1}{\bar{\omega}_2 - \bar{\omega}_3} \frac{1}{\bar{\omega}_3 \omega_4} + \frac{C_F}{2N_c} \frac{1}{\omega_4 \bar{\omega}_3^2} \right]$$

$$\mathcal{O}_f \sim \bar{Q}_{n\sigma_1} i g \not{\mathcal{B}}_{\perp n\sigma_3} S_n^\dagger \, \mathrm{Q}_u \frac{1-\tau^3}{2} S_{\bar{n}} Q_{\bar{n}\bar{\sigma}_2} \qquad C_f = 8\pi^2 \alpha_s^2 \frac{1}{\sigma_1 \left(\sigma_1 - \sigma_3\right) \bar{\sigma}_2} \left[\left(\frac{C_F}{N_c}\right)^2 \frac{1}{\omega_1 - \omega_4} \frac{1}{\bar{\omega}_3 \omega_4} + \frac{C_F}{2N_c} \frac{1}{\omega_4^2 \bar{\omega}_3} \right]$$

RPI and Soft

Is this set of the operators RPI invariant?
naively: no, to ensure RPI, the soft part of the operator should depend only on full Lorentz scalar, like $\slashed{\partial}$, $\slashed{B}$, . . .

Introduce two distinct collinear directions n_1 and n_2

- Four vector basis in the n_1, n_2 sector n_1^μ, n_2^μ, $v_\perp^\mu$, $w_\perp^\mu = \frac{1}{n_1\cdot n_2}\varepsilon^{\mu\nu\alpha\beta} v^\nu_\perp n_1^\alpha n_2^\beta$.

$$V^\mu = \frac{n_1\cdot V}{n_1\cdot n_2} n_2^\mu + \frac{n_2\cdot V}{n_1\cdot n_2} n_1^\mu + V_\perp^\mu$$

- but for power counting: bases n_1^μ, $\bar{n}_1^\mu$, $e^\mu_{\perp,\,1}$ and n_2^μ, $\bar{n}_2^\mu$, $e^\mu_{\perp,\,2}$.

$$\begin{aligned}
n_1\cdot V &= n_1\cdot V\\
n_2\cdot V &= \frac{n_1\cdot n_2}{2}\bar{n}_1\cdot V + \frac{n_1\cdot V}{2} n_2\cdot\bar{n}_1 + n_2\cdot V_{\perp,\,1}\\
V_\perp^\mu &= V^\mu_{\perp,\,1} - \frac{n_1^\mu}{n_1\cdot n_2} n_2\cdot V_{\perp,\,1} - n_1\cdot V\left(\frac{n_2\cdot\bar{n}_1}{n_2\cdot n_1} n_1^\mu + \frac{1}{n_1\cdot n_2} g^{\mu\sigma}_{\perp,\,1} n_2^\sigma\right)
\end{aligned}$$

- Dirac matrices

$$\bar{\chi}_{n_1}\,\Gamma\,\chi_{n_2}:\quad \Gamma = \left\{1,\ \gamma^5,\ \gamma_\perp^\mu\right\} \qquad\qquad \bar{\chi}_{n_1}\,\Gamma\,\chi_{n_1}:\quad \Gamma = \left\{\bar{\slashed{n}}_1,\ \bar{\slashed{n}}_1\gamma^5,\ \bar{\slashed{n}}_1\gamma^\mu_{\perp,1}\right\}$$

RPI transformations

$$\textbf{RPI}_I : \begin{cases} n_2^\mu \longrightarrow n_2^\mu + \Delta_{\perp,2}^\mu \\ \bar{n}_2^\mu \longrightarrow \bar{n}_2^\mu \end{cases} \quad \text{and} \quad \begin{cases} n_1^\mu \longrightarrow n_1^\mu \\ \bar{n}_1^\mu \longrightarrow \bar{n}_1^\mu \end{cases}$$

vector with soft scaling

- $\Delta_{\perp,2} \sim \lambda$
- the components in the $n_2, \bar{n}_2$ basis:

$$\begin{aligned} n_2 \cdot V &\longrightarrow n_2 \cdot V \\ \bar{n}_2 \cdot V &\longrightarrow \bar{n}_2 \cdot V \\ V_{\perp,2}^\mu &\longrightarrow V_{\perp,2}^\mu . \end{aligned}$$

- components in the n_1, n_2 basis change too:

$$\begin{aligned} n_2 \cdot V &\longrightarrow n_2 \cdot V \\ n_1 \cdot V &\longrightarrow n_1 \cdot V \\ V_\perp^\mu &\longrightarrow V_\perp^\mu \end{aligned}$$

$$\textbf{RPI}_{II} : \begin{cases} n_2^\mu \longrightarrow n_2^\mu \\ \bar{n}_2^\mu \longrightarrow \bar{n}_2^\mu + \varepsilon_{\perp,2}^\mu \end{cases} \quad \text{and} \quad \begin{cases} n_1^\mu \longrightarrow n_1^\mu \\ \bar{n}_1^\mu \longrightarrow \bar{n}_1^\mu \end{cases}$$

vector with soft scaling

- $\varepsilon_{\perp,2} \sim \lambda^0$
- components in the $n_2, \bar{n}_2$:

$$\begin{aligned} n_2 \cdot V &\longrightarrow n_2 \cdot V \\ \bar{n}_2 \cdot V &\longrightarrow \bar{n}_2 \cdot V + \varepsilon_{\perp,2} \cdot V_{\perp,2} \\ V_{\perp,2}^\mu &\longrightarrow V_{\perp,2}^\mu - \frac{n_2 \cdot V}{2} \varepsilon_{\perp,2}^\mu - \frac{n_2^\mu}{2} \varepsilon_{\perp,2} \cdot V_{\perp,2} \end{aligned}$$

- components in the n_1, n_2 basis are unchanged

RPI invariance of the electromagnetic current

$$
\begin{aligned}
J_a &= \bar{\chi}_{n_1\omega_1}\gamma_\perp^\mu \chi_{n_2\omega_2} \\
J_{b,2} &= \left(\frac{2}{n_1\cdot n_2} n_1^\mu + \frac{1}{2} g_\perp^{\mu\lambda}\bar{n}_{2\,\lambda}\right)\bar{\chi}_{n_1\omega_1}\gamma_\perp\cdot\mathcal{P}_{\perp,2}\chi_{n_2\omega_2} \\
J_{c,2} &= \bar{\chi}_{n_1\omega_1}\frac{\bar{n}_2\cdot\gamma_\perp}{2} g_\perp^{\mu\lambda}\mathcal{P}_{\perp,\,\lambda}\chi_{n_2\omega_2} \\
J_{d,2} &= \frac{1}{n_1\cdot n_2}\bar{\chi}_{n_1\omega_1}\gamma_\perp^\mu n_1\cdot\mathcal{P}_{\perp,2}\chi_{n_2\omega_2}
\end{aligned}
$$

$$
\begin{aligned}
J^{(g)}_{a,2} &= \frac{2}{n_1\cdot n_2} n_2^\mu\,\bar{\chi}_{n_1\omega_1}\gamma_\perp\cdot\mathcal{B}_{\perp,2\,\omega_3}\chi_{n_2\omega_2} \\
J^{(g)}_{b,2} &= \left(\frac{2}{n_1\cdot n_2} n_1^\mu + \frac{1}{2} g_\perp^{\mu\lambda}\bar{n}_{2\,\lambda}\right)\bar{\chi}_{n_1\omega_1}\gamma_\perp\cdot\mathcal{B}_{\perp,2}\chi_{n_2} \\
J^{(g)}_{c,2} &= \bar{\chi}_{n_1\omega_1}\frac{\bar{n}_2\cdot\gamma_\perp}{2} g_\perp^{\mu\lambda}\mathcal{B}_{\perp,2\,\omega_3,\,\lambda}\chi_{n_2\omega_2} \\
J^{(g)}_{d,2} &= \frac{1}{n_1\cdot n_2}\bar{\chi}_{n_1\omega_1}\gamma_\perp^\mu n_1\cdot\mathcal{B}_{\perp,2\,\omega_3}\chi_{n_2\omega_2}
\end{aligned}
$$

$$\Downarrow$$

RPI$_I$

$$C_a = -\omega_2\, C_{b,2} = \omega_2\, C_{c,2} = \omega_2\, C_{d,2}.$$

- true (at tree level)
- agrees with C. Marcantonini and I. Stewart, talk given at SCET Workshop 2007

RPI$_{II}$

$$
\begin{aligned}
C_a &= -\,\omega_2\, C_{b,2} = \omega_2\, C_{c,2} = \omega_2\, C_{d,2} \\
C_a &= -\,(\omega_2+\omega_3)C^{(g)}_{b,2} = (\omega_2+\omega_3)C^{(g)}_{c,2} \\
C_a &= +\left[\frac{1}{\omega_2+\omega_3} + \frac{2}{\omega_3}\right]^{-1} C^{(g)}_{d,2}.
\end{aligned}
$$

RPI invariance of the subleading contributions F_π

$$\mathcal{O}_{a,2} = \frac{2}{n_1 \cdot n_2} \bar{\chi}_{n_1\omega_1} \frac{\bar{\slashed{n}}_1}{2} \gamma^5 \chi_{n_1\omega_4} \bar{\chi}_{n_2\omega_3} \frac{\bar{\slashed{n}}_2}{2} \gamma^5 \chi_{n_2\omega_2} \bar{Q}_{n_1,\sigma_1} S^\dagger_{n_1} S_{n_2} \left(\partial_{\perp,2} + \frac{\bar{n}_2}{2} n_2 \cdot \partial \right) \cdot \gamma_\perp Q_{n_2,\sigma_2}$$

$$\mathcal{O}_{c,2} = \frac{2}{n_1 \cdot n_2} \bar{\chi}_{n_1\omega_1} \frac{\bar{\slashed{n}}_1}{2} \gamma^5 \chi_{n_1\omega_4} \bar{\chi}_{n_2\omega_3} \frac{\bar{\slashed{n}}_2}{2} \gamma^5 \chi_{n_2\omega_2} \bar{Q}_{n_1,\sigma_1} S^\dagger_{n_1} S_{n_2} \frac{\slashed{n}_1}{n_1 \cdot n_2} n_2 \cdot \partial Q_{n_2,\sigma_2}$$

- RPI$_I$: trivially realized in the soft sector.
- RPI$_{II}$: each operator is invariant.

RPI & Soft

Naive expectation is wrong because:

- RPI$_I$ does not impose constraint in the soft sector
- two collinear directions n_1 and n_2

$\Rightarrow$

RPI invariant object as

$$\frac{n_2 \cdot \partial}{n_1 \cdot n_2} \slashed{n}_1, \qquad \frac{n_1 \cdot \partial}{n_1 \cdot n_2} \slashed{n}_2$$

can appear in the operators

RPI invariance of the subleading contributions F_π

$$\mathcal{O}_{a,2} = \frac{2}{n_1 \cdot n_2} \bar{\chi}_{n_1\omega_1} \frac{\not{n}_1}{2} \gamma^5 \chi_{n_1\omega_4} \bar{\chi}_{n_2\omega_3} \frac{\not{n}_2}{2} \gamma^5 \chi_{n_2\omega_2} \bar{Q}_{n_1,\sigma_1} S^\dagger_{n_1} S_{n_2} \left(\not{\partial} - \frac{n_1 \cdot \partial}{n_1 \cdot n_2} \not{n}_2 - \frac{n_2 \cdot \partial}{n_1 \cdot n_2} \not{n}_1 \right) Q_{n_2,\sigma_2}$$
$$\mathcal{O}_{c,2} = \frac{2}{n_1 \cdot n_2} \bar{\chi}_{n_1\omega_1} \frac{\not{n}_1}{2} \gamma^5 \chi_{n_1\omega_4} \bar{\chi}_{n_2\omega_3} \frac{\not{n}_2}{2} \gamma^5 \chi_{n_2\omega_2} \bar{Q}_{n_1,\sigma_1} S^\dagger_{n_1} S_{n_2} \frac{\not{n}_1}{n_1 \cdot n_2} n_2 \cdot \partial Q_{n_2,\sigma_2}$$

- RPI$_I$: trivially realized in the soft sector.
- RPI$_{II}$: each operator is invariant.

RPI & Soft

Naive expectation is wrong because:

- RPI$_I$ does not impose constraint in the soft sector
- two collinear directions n_1 and n_2

⇒

RPI invariant object as

$$\frac{n_2 \cdot \partial}{n_1 \cdot n_2} \not{n}_1, \qquad \frac{n_1 \cdot \partial}{n_1 \cdot n_2} \not{n}_2$$

can appear in the operators

Estimate of the correction

- matrix element between ${}_{n}\langle\pi^{\pm}|\,{}_{\bar n}\langle 0|\,{}_{s}\langle 0|$ and $|0\rangle_n\,|\pi^{\pm}\rangle_{\bar n}\,|0\rangle_s$
- charge conjugation simplifies the matrix elements
- one example (operators $\mathcal{O}_c$ and $\mathcal{O}_d$):

$$\begin{aligned}
F_{\pi}^{(c)\pm}(Q^2) &= \pm\frac{f_\pi^2}{Q^2}\left(\frac{8}{9}\right)^2 8\pi^2\alpha_s^2(\Lambda_{QCD}Q)\,(\mathrm{Q}_u-\mathrm{Q}_d)\int_0^1 dz_1\,\frac{\phi_\pi(z_1)}{z_1}\;\int_0^1 dz_2\,\frac{\phi_\pi(z_2)}{z_2^2}\\
&\quad\frac{1}{Q}\int d\sigma_1 d\bar\sigma_2\,\frac{1}{\sigma_1\bar\sigma_2}\,\langle 0|\bar Q_{n\sigma_1}S_n^\dagger\,S_{\bar n}\,\frac{\not{n}}{2}Q_{\bar n\bar\sigma_2}|0\rangle\,,\\
F_{\pi}^{(d)\pm}(Q^2) &= \mp\frac{f_\pi^2}{Q^2}\left(\frac{8}{9}\right)^2 8\pi^2\alpha_s^2(\Lambda_{QCD}Q)\,(\mathrm{Q}_u-\mathrm{Q}_d)\int_0^1 dz_1\,\frac{\phi_\pi(z_1)}{z_1^2}\;\int_0^1 dz_2\,\frac{\phi_\pi(z_2)}{z_2}\\
&\quad\frac{1}{Q}\int d\sigma_1 d\bar\sigma_2\,\frac{1}{\sigma_1\bar\sigma_2}\,\langle 0|\bar Q_{\bar n\bar\sigma_2}S_{\bar n}^\dagger\,S_n\,\frac{\not{\bar n}}{2}Q_{n\sigma_1}|0\rangle
\end{aligned}\;.$$

- but parity $\langle 0|\bar Q_{n\sigma_1}S_n^\dagger\,S_{\bar n}\,\frac{\not{\bar n}}{2}Q_{\bar n\bar\sigma_2}|0\rangle = \langle 0|\bar Q_{\bar n\sigma_1}S_{\bar n}^\dagger\,S_n\,\frac{\not{n}}{2}Q_{n\bar\sigma_2}|0\rangle$.
- simmetry of the coefficient $\sigma_1 \longrightarrow \bar\sigma_2$, rename $z_1 \longrightarrow z_2$

$$\boxed{F_\pi^{(c)\pm}(Q^2) + F_\pi^{(d)\pm}(Q^2) = 0}$$

- same argument holds for the other four operators.

Conclusion (?)

At this moment

- no order $\frac{\Lambda_{QCD}}{Q}$ corrections to $F_\pi(Q^2)$

however

- check the matching & all the signs
- add pion/quark mass effects
- construct the most general SCET$_{II}$ operator contributing to F_π.

Backup Slides

Definition of the fields

- collinear fields

$$
\begin{aligned}
\chi_{n\omega} &= \delta(\omega - \bar{n}\cdot\mathcal{P})W_n^\dagger \xi_n \\
\chi_{\bar{n}\bar{\omega}} &= \delta(\bar{\omega} - n\cdot\mathcal{P})W_{\bar{n}}^\dagger \xi_{\bar{n}} \\
ig\mathcal{B}^\mu_{\perp n\omega} &= \delta(\omega - \bar{n}\cdot\mathcal{P})\frac{1}{\bar{n}\cdot\mathcal{P}}W_n\left[i\bar{n}\cdot D_c, iD^\mu_{c\perp}\right]W_n^\dagger \\
ig\mathcal{B}^\mu_{\perp\bar{n}\bar{\omega}} &= \delta(\bar{\omega} - n\cdot\mathcal{P})\frac{1}{n\cdot\mathcal{P}}W_{\bar{n}}\left[in\cdot D_c, iD^\mu_{c\perp}\right]W_{\bar{n}}^\dagger,
\end{aligned}
$$

- soft fields

$$
\begin{aligned}
Q_{n\sigma} &= \delta(\sigma - n\cdot\mathcal{P})S_n^\dagger q_s \\
Q_{\bar{n}\bar{\sigma}} &= \delta(\bar{\sigma} - \bar{n}\cdot\mathcal{P})S_{\bar{n}}^\dagger q_s \\
ig\mathcal{B}^\mu_{\perp s\sigma} &= \delta(\sigma - n\cdot\mathcal{P})\frac{1}{n\cdot\mathcal{P}}S_n\left[in\cdot D_s, iD^\mu_{s\perp}\right]S_n^\dagger \\
ig\mathcal{B}^\mu_{\perp s\bar{\sigma}} &= \delta(\bar{\sigma} - \bar{n}\cdot\mathcal{P})\frac{1}{\bar{n}\cdot\mathcal{P}}S_{\bar{n}}\left[i\bar{n}\cdot D_c, iD^\mu_{c\perp}\right]S_{\bar{n}}^\dagger,
\end{aligned}
$$

Contributions to F_π

$$
\begin{aligned}
F_\pi^{(a)} =& \frac{f_\pi^2}{Q^3}\left(\frac{C_F}{N_c}\right)^2 8\pi^2\alpha_s^2\,(\mathrm{Q}_u-\mathrm{Q}_d)\int dz_1\,\frac{\phi_\pi(z_1)}{z_1}\ \int dz_2\,\frac{(1-z_2)\,\phi_\pi(z_2)}{z_2^2}\\
&\int d\sigma_1 d\bar{\sigma}_2\,\frac{1}{\sigma_1\bar{\sigma}_2^2}\,\langle 0|\bar{Q}_{n\sigma_1}\,Y_n^\dagger\ Y_{\bar{n}} i\bar{\partial}\!\!\!/_\perp\, Q_{\bar{n}\bar{\sigma}_2}\,|0\rangle\,,\\
F_\pi^{(b)} =& -\frac{f_\pi^2}{Q^3}\left(\frac{C_F}{N_c}\right)^2 8\pi^2\alpha_s^2\,(\mathrm{Q}_u-\mathrm{Q}_d)\int dz_1\,\frac{(1-z_1)\phi_\pi(z_1)}{z_1^2}\ \int dz_2\,\frac{\phi_\pi(z_2)}{z_2}\,.\\
&\int d\sigma_1 d\bar{\sigma}_2\,\frac{1}{\sigma_1^2\bar{\sigma}_2}\,\langle 0|\bar{Q}_{n\sigma_1}\,(i\bar{\partial}\!\!\!/_\perp)^\dagger\ Y_n^\dagger\ Y_{\bar{n}} Q_{\bar{n}\bar{\sigma}_2}\,|0\rangle\\
F_\pi^{(d)} =& -\frac{f_\pi^2}{Q^3}\left(\frac{C_F}{N_c}\right)^2 32\pi^2\alpha_s^2\,(\mathrm{Q}_u-\mathrm{Q}_d)\int dz_1\,\frac{\phi_\pi(z_1)}{z_1^2}\ \int dz_2\,\frac{\phi_\pi(z_2)}{z_2}\\
&\int d\sigma_1 d\bar{\sigma}_2\,\frac{1}{\sigma_1\bar{\sigma}_2}\,\langle 0|\bar{Q}_{n\sigma_1}\,Y_n^\dagger\ Y_{\bar{n}}\,\frac{\bar{n}\!\!\!/}{2}\,Q_{\bar{n}\bar{\sigma}_2}\,|0\rangle
\end{aligned}
$$

Charge conjugation and parity

- charge conjugation

$$\mathrm{C}^{-1} q_s \mathrm{C} = -\left[\bar{q}_s \mathcal{C}\right]^T \qquad \mathrm{C}^{-1} A_s^{\mu} \mathrm{C} = -\left[A_s^{\mu}\right]^T \qquad \mathrm{C}^{-1} Y_{\bar{n}} \mathrm{C} = \left[Y_{\bar{n}}^{\dagger}\right]^T ,$$

implying

$$\mathrm{C}^{-1} Q_{\bar{n}\,\bar{\sigma}_2} \mathrm{C} = -\left[\bar{Q}_{\bar{n},-\bar{\sigma}_2} \mathcal{C}\right]^T \quad \text{and} \quad \mathrm{C}^{-1} \bar{Q}_{n\,\sigma_1} \mathrm{C} = \left[\mathcal{C}^{-1} Q_{n,-\sigma_1}\right]^T .$$

The operator $\mathcal{C}$ is such that $\mathcal{C}\gamma_\mu^T \mathcal{C}^{-1} = -\gamma^\mu$. Using this info,

$$\mathrm{C}^{-1} \bar{Q}_{n\,\sigma_1} Y_n^{\dagger} \frac{\not{n}}{2} Y_{\bar{n}} Q_{\bar{n},\bar{\sigma}_2} \mathrm{C} = -\bar{Q}_{\bar{n}\,-\bar{\sigma}_2} Y_{\bar{n}}^{\dagger} \frac{\not{n}}{2} Y_n Q_{n,-\sigma_1}$$

- parity

$$\mathrm{P}^{-1} q_s(x^+, x^-, x_\perp)\mathrm{P} = \gamma_0 q_s(x^-, x^+, -x_\perp) \qquad \mathrm{P}^{-1} A_s^{\mu}(x^+, x^-, x_\perp)\mathrm{P} = g_{\mu\nu} A_s^{\nu}$$

$$\mathrm{P}^{-1} \bar{n}\cdot A_s \mathrm{P} = n \cdot A_s \qquad \mathrm{P}\bar{n}\cdot\partial q_s \mathrm{P}^{-1} = n\cdot\partial\gamma^0 q_s$$

$$\mathrm{P}^{-1} Q_{\bar{n},\bar{\sigma}_2}(x^+, x^-, x_\perp)\mathrm{P} = \gamma^0 Q_{n,\bar{\sigma}_2}(x^-, x^+, -x_\perp)$$

$$\mathrm{P}^{-1} Q_{n,\sigma_1}(x^+, x^-, x_\perp)\mathrm{P} = \gamma^0 Q_{\bar{n},\sigma_1}(x^-, x^+, -x_\perp)$$

So

$$\mathrm{P}^{-1} \bar{Q}_{n\sigma_1} S_n^{\dagger}\, S_{\bar{n}} \frac{\bar{\not{n}}}{2} Q_{\bar{n}\bar{\sigma}_2} \mathrm{P} = \bar{Q}_{\bar{n}\sigma_1} S_{\bar{n}}^{\dagger}\, S_n \gamma^0 \frac{\bar{\not{n}}}{2} \gamma^0 Q_{n\bar{\sigma}_2} = \bar{Q}_{\bar{n}\sigma_1} S_{\bar{n}}^{\dagger}\, S_n \frac{\not{n}}{2} Q_{n\bar{\sigma}_2}$$

RPI transformations

$$\mathbf{RPI}_I : \begin{cases} n_2^\mu \to n_2^\mu + \Delta_{\perp,2}^\mu \\ \bar{n}_2^\mu \to \bar{n}_2^\mu \end{cases} \quad \text{and} \quad \begin{cases} n_1^\mu \to n_1^\mu \\ \bar{n}_1^\mu \to \bar{n}_1^\mu \end{cases}$$

vector with collinear scaling

- $\Delta_{\perp,2} \sim \lambda$
- the components in the $n_2, \bar{n}_2$ basis:

$$\begin{aligned} n_2 \cdot V &\to n_2 \cdot V + \Delta_{\perp,2} \cdot V_{\perp,2} \\ \bar{n}_2 \cdot V &\to \bar{n}_2 \cdot V \\ V_{\perp,2}^\mu &\to V_{\perp,2}^\mu - \frac{\bar{n}_2 \cdot V}{2} \Delta_{\perp,2}^\mu \end{aligned}$$

- components in the n_1, n_2 basis change too:

$$\begin{aligned} n_2 \cdot V &\to n_2 \cdot V + \Delta_{\perp,2} \cdot V_\perp \\ n_1 \cdot V &\to n_1 \cdot V \\ V_\perp^\mu &\to V_\perp^\mu - \frac{n_1 \cdot V}{n_1 \cdot n_2} \Delta_{\perp,2}^\mu \end{aligned}$$

$$\mathbf{RPI}_{II} : \begin{cases} n_2^\mu \to n_2^\mu \\ \bar{n}_2^\mu \to \bar{n}_2^\mu + \varepsilon_{\perp,2}^\mu \end{cases} \quad \text{and} \quad \begin{cases} n_1^\mu \to n_1^\mu \\ \bar{n}_1^\mu \to \bar{n}_1^\mu \end{cases}$$

vector with collinear scaling

- $\varepsilon_{\perp,2} \sim \lambda^0$
- components in the $n_2, \bar{n}_2$:

$$\begin{aligned} n_2 \cdot V &\to n_2 \cdot V \\ \bar{n}_2 \cdot V &\to \bar{n}_2 \cdot V \\ V_{\perp,2}^\mu &\to V_{\perp,2}^\mu - \frac{n_2^\mu}{2} \varepsilon_{\perp,2} \cdot V_{\perp,2} \end{aligned}$$

- components in the n_1, n_2 basis are unchanged

RPI transformations

$$\mathbf{RPI}_I : \begin{cases} n_2^\mu \to n_2^\mu + \Delta^\mu_{\perp,2} \\ \bar{n}_2^\mu \to \bar{n}_2^\mu \end{cases} \quad \text{and} \quad \begin{cases} n_1^\mu \to n_1^\mu \\ \bar{n}_1^\mu \to \bar{n}_1^\mu \end{cases}$$

vector with soft scaling

- $\Delta_{\perp,2} \sim \lambda$
- the components in the $n_2, \bar{n}_2$ basis:

$$\begin{aligned} n_2 \cdot V &\to n_2 \cdot V \\ \bar{n}_2 \cdot V &\to \bar{n}_2 \cdot V \\ V^\mu_{\perp,2} &\to V^\mu_{\perp,2}. \end{aligned}$$

- components in the n_1, n_2 basis change too:

$$\begin{aligned} n_2 \cdot V &\to n_2 \cdot V \\ n_1 \cdot V &\to n_1 \cdot V \\ V^\mu_\perp &\to V^\mu_\perp \end{aligned}$$

$$\mathbf{RPI}_{II} : \begin{cases} n_2^\mu \to n_2^\mu \\ \bar{n}_2^\mu \to \bar{n}_2^\mu + \varepsilon^\mu_{\perp,2} \end{cases} \quad \text{and} \quad \begin{cases} n_1^\mu \to n_1^\mu \\ \bar{n}_1^\mu \to \bar{n}_1^\mu \end{cases}$$

vector with collinear scaling

- $\varepsilon_{\perp,2} \sim \lambda^0$
- components in the $n_2, \bar{n}_2$:

$$\begin{aligned} n_2 \cdot V &\to n_2 \cdot V \\ \bar{n}_2 \cdot V &\to \bar{n}_2 \cdot V \\ V^\mu_{\perp,2} &\to V^\mu_{\perp,2} - \frac{n_2^\mu}{2} \varepsilon_{\perp,2} \cdot V_{\perp,2} \end{aligned}$$

- components in the n_1, n_2 basis are unchanged

RPI transformations

$$\mathbf{RPI}_I : \begin{cases} n_2^\mu \to n_2^\mu + \Delta^\mu_{\perp,2} \\ \bar{n}_2^\mu \to \bar{n}_2^\mu \end{cases} \quad \text{and} \quad \begin{cases} n_1^\mu \to n_1^\mu \\ \bar{n}_1^\mu \to \bar{n}_1^\mu \end{cases}$$

vector with soft scaling

- $\Delta_{\perp,2} \sim \lambda$
- the components in the $n_2, \bar{n}_2$ basis:

$$\begin{aligned} n_2 \cdot V &\to n_2 \cdot V \\ \bar{n}_2 \cdot V &\to \bar{n}_2 \cdot V \\ V^\mu_{\perp,2} &\to V^\mu_{\perp,2}. \end{aligned}$$

- components in the n_1, n_2 basis change too:

$$\begin{aligned} n_2 \cdot V &\to n_2 \cdot V \\ n_1 \cdot V &\to n_1 \cdot V \\ V^\mu_\perp &\to V^\mu_\perp \end{aligned}$$

$$\mathbf{RPI}_{II} : \begin{cases} n_2^\mu \to n_2^\mu \\ \bar{n}_2^\mu \to \bar{n}_2^\mu + \varepsilon^\mu_{\perp,2} \end{cases} \quad \text{and} \quad \begin{cases} n_1^\mu \to n_1^\mu \\ \bar{n}_1^\mu \to \bar{n}_1^\mu \end{cases}$$

vector with soft scaling

- $\varepsilon_{\perp,2} \sim \lambda^0$
- components in the $n_2, \bar{n}_2$:

$$\begin{aligned} n_2 \cdot V &\to n_2 \cdot V \\ \bar{n}_2 \cdot V &\to \bar{n}_2 \cdot V + \varepsilon_{\perp,2} \cdot V_{\perp,2} \\ V^\mu_{\perp,2} &\to V^\mu_{\perp,2} - \frac{n_2 \cdot V}{2} \varepsilon^\mu_{\perp,2} - \frac{n_2^\mu}{2} \varepsilon_{\perp,2} \cdot V_{\perp,2} \end{aligned}$$

- components in the n_1, n_2 basis are unchanged