Electroweak Corrections to High Energy Processes using Effective Field Theory

Aneesh Manohar

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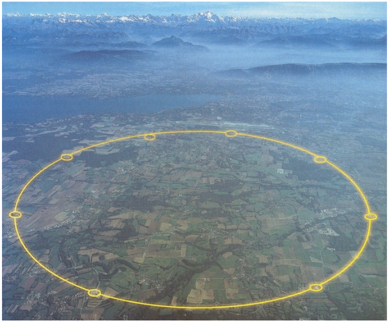
Outline

1. Introduction
2. Sudakov Double Logarithms
3. Effective Field Theory
4. Standard Model Results and Plots
5. Conclusions

Finding the Higgs, New Physics, Black Holes, ?

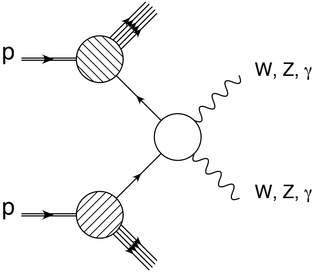
Large Hadron Collider (LHC)

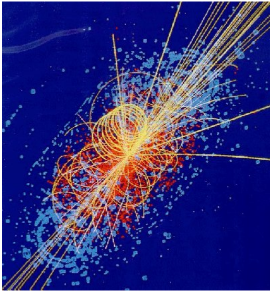
- proton - proton collider
- $E_{cm} \sim 14$ TeV
- increased luminosity



proton-proton collisions

- strong interaction dynamics complicates computation
- asymptotic freedom allows for perturbative calculation of parton-parton collisions.
- look at parton-parton subprocesses, and turn into cross-sections using parton distribution functions





The standard model

- $SU(3) \times SU(2)_L \times U(1)_Y$ gauge theory
- Describes all interactions between elementary particles except gravity, which can be treated as an EFT.

Gauge Fields

- $g, W^{1,2,3}, B$
- $SU(2) \times U(1)$ mixing leads to $W^{\pm}$, Z and γ.

Higgs Field (scalar)

- H only untested part. Responsible for symmetry breaking.

Matter fields (fermion)

- quark doublets ($i = u, c, t$)

$$Q^{(i)} = \begin{pmatrix} u \\ d' \end{pmatrix}_L, \begin{pmatrix} c \\ s' \end{pmatrix}_L, \begin{pmatrix} t \\ b' \end{pmatrix}_L$$

- quark singlets $U_R^{(i)}$, $D_R^{(i)}$ (or t_R, etc.)
- lepton doublets:

$$L^{(i)} = \begin{pmatrix} \nu_e \\ e \end{pmatrix}, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}$$

- lepton singlets $E_R^{(i)}$, (or e_R, etc.)

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Parton Processes

Typical LHC processes being studied such as jet production, t-quark pair production, squark pair production proceed via energetic partonic processes

$$qq \to qq, \quad q\bar{q} \to q\bar{q}, \quad q\bar{q} \to t\bar{t}, \quad q\bar{q} \to \tilde{q}\tilde{q}^*$$

with $Q \sim \sqrt{s}$ of order (few) TeV.

Final state invariant masses are much smaller than Q.

Describe these using SCET. Work in the regime

$$s \sim -t \sim -u \sim Q^2$$

(Hard Scattering)

Sudakov Double Logarithms

There are no electroweak singlet targets or beams, so all processes behave like the exclusive case and have double logs.

M. Ciafaloni, P. Ciafaloni and D. Comelli, PRL 84 (2000) 4810

Typical form of the radiative corrections:

$$\frac{\alpha}{4\pi \sin^2\theta_W} \log^2 \frac{s}{M_{W,Z}^2} \sim 0.15$$

for $\sqrt{s} \sim 4$ TeV.

Can be much larger. Need to be combined with the QCD radiative corrections, which are 5 times larger.

Sudakov form factor: *the purely electroweak* corrections reduce the amplitude by $\sim 10\%$.

For scattering, the cross-section is reduced by about a factor of two (? see Chiu's talk).

These (QED and weak) corrections are not small. They are important for the LHC, and need to be included.

How many papers would be written if a LHC cross-section differed from the standard model value by 15%?

Previous Work

M. Ciafaloni, P. Ciafaloni and D. Comelli

V. S. Fadin, L. N. Lipatov, A. D. Martin and M. Melles

B. Jantzen=Feucht, J. H. Kuhn, A. A. Penin and V. A. Smirnov

M. Beccaria, F. M. Renard and C. Verzegnassi

A. Denner and S. Pozzorini

M. Hori, H. Kawamura and J. Kodaira

W. Beenakker and A. Werthenbach

This talk based on

J. Chiu, F. Golf, R. Kelley, A.M, PRL 100 (2008) 021802

J. Chiu, F. Golf, R. Kelley, A.M, PRD 77 (2008) 053004

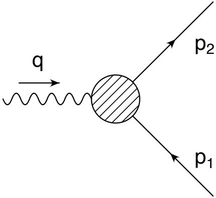
J. Chiu, R. Kelley, A.M, in preparation

Thanks to G. Bell

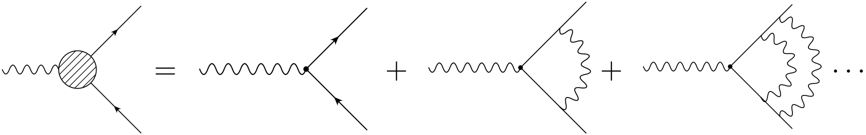
Sudakov Form Factor

- $(Q^2 \equiv -q^2 = -(p_2 - p_1)^2)$,

$$F_E(Q)\left[\bar{u}(p_2)\gamma^\mu u(p_1)\right] = \langle p_2|\, J^\mu_{EM}(q)\, |p_1\rangle =$$



- If coupling strength is small we calculate $F_E(Q^2)$ perturbatively in powers of $\alpha = \frac{e^2}{4\pi}$.



- $F_E(Q)$ for $Q^2 \gg m_e^2 \sim 0$ is called the Sudakov Form Factor
- We will work with the on-shell form factor, i.e. an S-matrix element for scattering.
- The off-shell form-factor with $p_i^2 \neq m_i^2$ is also considered in the literature. There is a factor of 2 in the double-logarithm between the two cases.
- pair production: analytically continue $Q^2 \to -q^2 - i0^+$,

$$\log \frac{Q^2}{\mu^2} \to \log \frac{q^2}{\mu^2} - i\pi$$

- In QED, look at $\log Q^2/m_e^2$ terms. Here we study $\log Q^2/M_{W,Z}^2$ corrections.

Got interested in the problem due to a talk by Pozzorini at LMU

IREE (InfraRed Evolution Equation) inspired approach — a well-motivated guess as to the structure of the corrections.

Previous computations done with all gauge bosons having a common mass M. Conceptual problems with symmetry breaking and $SU(2) \times U(1)$ mixing which lead to $M_W \neq M_Z$.

$$\log \frac{Q^2}{M^2} \to \log \frac{Q^2}{M_W^2} + \log \frac{Q^2}{M_Z^2} + \log \frac{Q^2}{M_\gamma^2} \qquad \text{but } M_\gamma = 0$$

Can address the issues using effective field theory methods.

Can include m_t — multiscale problem with m_t and M.

General perturbative structure of $F_E(Q)$

$$\mathsf{L} = \log \mathsf{Q}^2/\mathsf{M}^2$$

(Each term has a coefficient)

$$\begin{aligned} F_E(Q) = \Big[1 \quad &+ \quad \alpha^1\left(\mathsf{L}^2+\mathsf{L}^1+\mathsf{L}^0\right) && \mathsf{LO}+\mathsf{NLO} \\ &+ \quad \alpha^2\left(\mathsf{L}^4+\mathsf{L}^3+\mathsf{L}^2+\mathsf{L}^1+\mathsf{L}^0\right) && \mathsf{N^2LO} \\ &+ \quad \alpha^3\left(\mathsf{L}^6+\mathsf{L}^5+\mathsf{L}^4+\mathsf{L}^3+\mathsf{L}^2+\mathsf{L}^1+\mathsf{L}^0\right)\Big] && \mathsf{N^3LO} \\ &+ \quad \alpha^4\left(\mathsf{L}^8+\mathsf{L}^7+\mathsf{L}^6+\mathsf{L}^5+\cdots+\mathsf{L}^0\right)\Big] && \mathsf{N^4LO} \end{aligned}$$

The α^n term has powers of L up to L^{2n}.

Structure of series

- The $\alpha \mathsf{L}^2$, $\alpha^2 \mathsf{L}^4$, $\alpha^3 \mathsf{L}^6$ series is called $\mathsf{LL}_{\mathsf{FO}}$.
- The $\alpha \mathsf{L}$, $\alpha^2 \mathsf{L}^3$, $\alpha^3 \mathsf{L}^5$ series is called $\mathsf{NLL}_{\mathsf{FO}}$.

The series for $\log F_E(Q^2)$ takes a simpler form

$$\begin{aligned}\log F_E &= \alpha\left(\mathsf{L}^2+\mathsf{L}+\mathsf{L}^0\right)\\ &\quad+\alpha^2\left(\mathsf{L}^3+\mathsf{L}^2+\mathsf{L}+\mathsf{L}^0\right)\\ &\quad+\alpha^3\left(\mathsf{L}^4+\ldots+\mathsf{L}^0\right)+\ldots\end{aligned}$$

with the α^n term having power of L upto L^{n+1}.

$$\log F_E = \mathsf{L}\, f_0(\alpha \mathsf{L}) + f_1(\alpha \mathsf{L}) + \alpha\, f_2(\alpha \mathsf{L}) + \ldots$$

Counting of Logs

Only get L^{n+1} at order α^n, so there are far fewer terms.

$$\log F_E \quad = \quad \mathsf{L}\, f_0(\mathsf{L}) + f_1(\alpha \mathsf{L}) + \alpha\, f_2(\alpha \mathsf{L}) + \ldots$$

RGE counting: f_0 is LL, f_1 is NLL, etc.

If we then expand to get F_E and look (for example) at order α^2:

- $\alpha^2 \mathsf{L}^4$ is LL ($\mathsf{LL}_{\mathsf{FO}}$),
- $\alpha^2 \mathsf{L}^3$ is NLL ($\mathsf{NLL}_{\mathsf{FO}}$),
- $\alpha^2 \mathsf{L}^2$ is NNLL ($\mathsf{NNLL}_{\mathsf{FO}}$),
- $\alpha^2 \mathsf{L}$ is NNLL ($\mathsf{N^3LL}_{\mathsf{FO}}$)

mismatch in number of N's increases at higher orders in α.

Infrared Evolution Equation

Collins

$$\log F_E(Q^2) = \log F_0(a(M))$$

$$+ \int_{M^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2} \left[\zeta(a(\mu)) + \xi(a(M)) + \int_{M^2}^{\mu^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \Gamma(a(\mu')) \right]$$

ξ integral can be done.
F_0, ζ, ξ and Γ have the expansions

$$\begin{aligned} F_0(a) &= 1 + F_0^{(1)} a + F_0^{(2)} a^2 + \ldots \\ \Gamma(a) &= \Gamma^{(1)} a + \Gamma^{(2)} a^2 + \ldots \\ \zeta(a) &= \zeta^{(1)} a + \zeta^{(2)} a^2 + \ldots \\ \xi(a) &= \xi^{(1)} a + \xi^{(2)} a^2 + \ldots . \end{aligned}$$

$$\frac{1}{2}\int_{y^2}^{z^2}\frac{\mathrm{d}\mu^2}{\mu^2}\frac{\partial G(a(\mu))}{\partial a(\mu)}\beta_a(a(\mu)) \quad = \quad G(a(z)) - G(a(y))$$

can move terms between scales μ and M.

Invariant under:

$$\begin{aligned}
\Gamma(a(\mu)) &\rightarrow \Gamma(a(\mu)) + \frac{\partial G(a(\mu))}{\partial a}\beta_a(a(\mu)) \\
\zeta(a(\mu)) &\rightarrow \zeta(a(\mu)) - 2G(a(\mu)) \\
\xi(a(M)) &\rightarrow \xi(a(M)) + 2G(a(M))
\end{aligned}$$

SCET Form

$$\log F_E(Q^2) = C(a(Q)) + D_0(a(M)) + D_1(a(M)) \log \frac{Q^2}{M^2}$$

$$+ \int_Q^M \frac{\mathrm{d}\mu}{\mu} \left[A(a(\mu)) \log \frac{\mu^2}{Q^2} + B(a(\mu)) \right]$$

- C: matching at Q
- $A \log \mu^2/Q^2 + B$: SCET anomalous dimension
- $D_0 + D_1 \log Q^2/M^2$: matching at M
- There is a log Q in the matching at M

Bell and Feldmann — a single log in NR system

$$\int_{M^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2} \int_{M^2}^{\mu^2} \frac{\mathrm{d}\mu'^2}{\mu'^2} \Gamma(a(\mu')) = \int_{M^2}^{Q^2} \frac{\mathrm{d}\mu^2}{\mu^2} \Gamma(a(\mu)) \log \frac{Q^2}{\mu^2}$$

Invariance:

$$\begin{aligned}
A(a(\mu)) &\to A(a(\mu)) + \frac{\partial \tilde{G}(a(\mu))}{\partial a} \beta_a(a(\mu)) \\
B(a(\mu)) &\to B(a(\mu)) + \frac{\partial \tilde{H}(a(\mu))}{\partial a} \beta_a(a(\mu)) + 2\tilde{G}(a(\mu)) \\
C(a(Q)) &\to C(a(Q)) + \tilde{H}(a(Q)) \\
D_0(a(M)) &\to D_0(a(M)) - \tilde{H}(a(M)) \\
D_1(a(M)) &\to D_1(a(M)) + \tilde{G}(a(M)).
\end{aligned}$$

Mapping between SCET and IREE

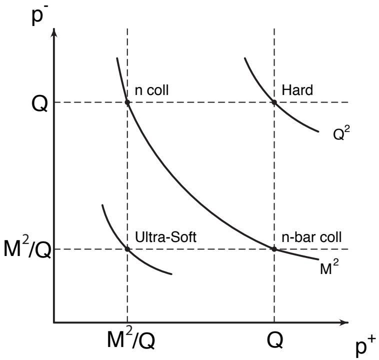
The SCET and cusp anomalous dimension computations are the same $\Rightarrow$:

$$\begin{aligned}
\frac{1}{2}A(a) &= \Gamma(a) \\
D_1(a) &= \xi(a) \\
-\frac{1}{2}B(a) + \frac{1}{2}\frac{\partial C(a)}{\partial a}\beta_a(a) &= \zeta(a) \\
C(a) + D_0(a) &= \log F_0(a).
\end{aligned}$$

The log in the low scale matching, D_1, is ξ.

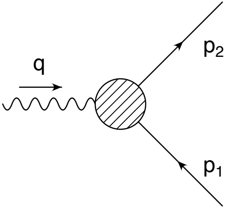
SCET degrees of freedom (modes)

- Light Cone Coordinates:
- Hard Modes: $p^2 \sim Q^2$
  integrated out
- Collinear modes: $p^2 \sim M^2$
- Ultra-Soft modes: $p^2 \sim M^4/Q^2$
  do not contribute



Toy Theory

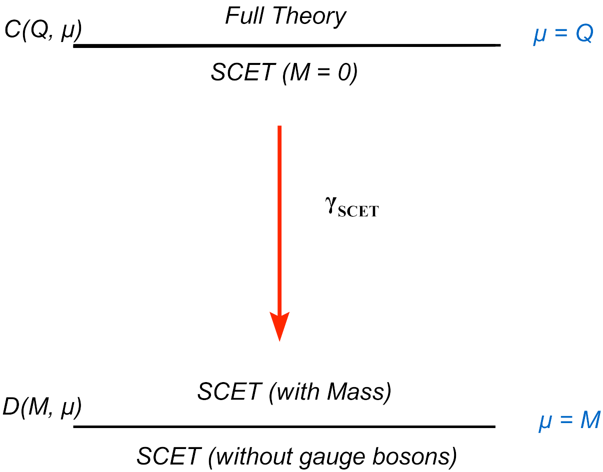
- Calculate on-shell scattering amplitude $\langle p_2 | \bar{\psi}\gamma^\mu\psi | p_1 \rangle$



- SU(2) gauge theory with massive boson
- vector like fermions
- Higgs in the fundamental (doublet) representation of SU(2)
- use generic group theory factors:

$$C_F, \qquad T_F, \qquad C_A$$

Outline of Calculation



Full Theory

$C(Q, \mu)$

$\mu = Q$

SCET (M = 0)

γ_{SCET}

SCET (with Mass)

$D(M, \mu)$

$\mu = M$

SCET (without gauge bosons)

Outline of Calculation

1. match at Q onto SCET with gauge bosons

$$\langle p_2 | \hat{\mathcal{O}}_{full} | p_1 \rangle = \exp[C(Q)] \langle p_2 | \hat{\mathcal{O}}_{SCET} | p_1 \rangle$$

2. run from $Q \to M$:

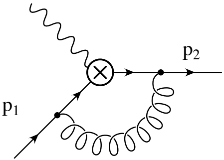
$$\exp[C(M)] = \exp[C(Q)] \exp\left[\int_Q^M \frac{d\mu}{\mu} \gamma_{SCET}(\mu)\right]$$

3. match at M onto SCET without gauge bosons

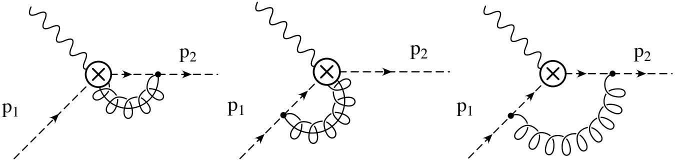
$$\begin{aligned} \langle p_2 | \hat{\mathcal{O}}_{SCET} | p_1 \rangle &= \exp[D(M)] \langle p_2 | \hat{\mathcal{O}}_{SCET\ w/o\ W's} | p_1 \rangle \\ D(M) &= D_0(\alpha(M)) + D_1(\alpha(M)) \log \frac{Q^2}{M^2} \end{aligned}$$

High scale matching: $\mu \sim Q$

- full theory:



- EFT:



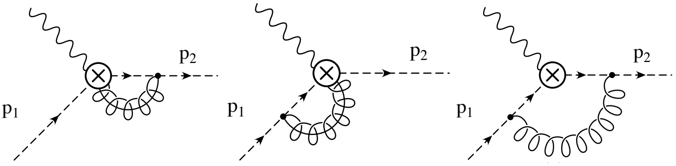
Same as for DIS, since small scales such as M can be neglected.

- Matching at Q:

$$\begin{aligned} \mathrm{L}_Q &= \log \frac{Q^2}{\mu^2} \\ c(\mu) &= \exp C(\mu) \\ C(\mu) &= a(\mu)\, C_F \left(-\mathrm{L}_Q^2 + 3\mathrm{L}_Q + \frac{\pi^2}{6} - 8 \right) \end{aligned}$$

No large logs if μ is of order Q, e.g. if $\mu = \eta Q$, then

$$\mathrm{L}_Q = \log \frac{1}{\eta^2}$$



- Compute running between Q and M using SCET anomalous dimension.
- From UV divergences, so independent of IR scales such as M,
- same as DIS

$$\begin{aligned} \mu \frac{\mathrm{d}c(\mu)}{\mathrm{d}\mu} &= \gamma(\mu) c(\mu) \\ \gamma(\mu) &= a(\mu)\, C_F \left[4\mathrm{L}_Q - 6\right] \end{aligned}$$

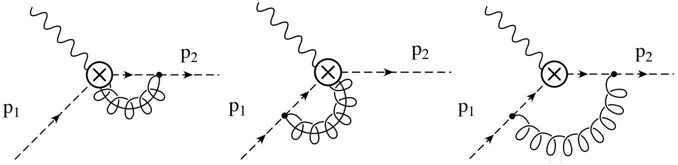
$$\log c(M) - \log c(Q) \; = \; \int_Q^M \frac{\mathrm{d}\mu}{\mu} \, \gamma(\mu)$$

An additive shift in $C(\mu)$, i.e. in $\log F_E$.

The last step is to integrate out the massive gauge bosons at $\mu = M$.

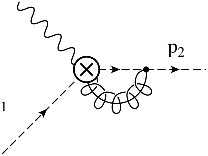
Low scale matching: $\mu \sim M$

- EFT (SCET with gauge bosons):



- EFT (SCET without gauge bosons):

NONE



$$I_n = -2ig^2\mu^{2\epsilon}C_F\int\frac{\mathrm{d}^dk}{(2\pi)^d}\,\frac{\bar{n}\cdot(p_2-k)}{(p_2-k)^2}\,\gamma^\mu\,\frac{1}{-\bar{n}\cdot k}\,\frac{1}{k^2-M^2}$$

Graph is IR divergent even in dimensional regularization with an off-shellness.

Analytic Regulator

Use an analytic regulator Beneke and Feldmann, Smirnov

$$\frac{1}{(p_i-k)^2} \quad\to\quad \frac{(-\nu_i^2)^{\delta_i}}{\left[(p_i-k)^2\right]^{1+\delta_i}}\,.$$

Use different δ_i for the two particles.

$(p_1-k)^2 \to (p_1^+ - k^+)(p_1^- - k^-) - k_\perp^2 \to p_1^+(-k^-)$.

$$\frac{1}{(p_1-k)^2} \quad\to\quad \frac{(-\nu_1^2)^{\delta_1}}{[(n\cdot p_1)(-\bar{n}\cdot k)]^{1+\delta_1}}\,.$$

n-Collinear Graph

We will therefore analytically continue the $-\bar{n} \cdot k$ propagator from the W_n Wilson line in $\mathcal{O}$ using

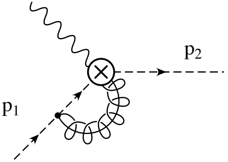
$$\frac{1}{-\bar{n} \cdot k} \quad \to \quad \frac{(-\nu_1^-)^{\delta_1}}{(-\bar{n} \cdot k)^{1+\delta_1}}$$

where $\nu_1^- \equiv \nu_1^2/p_1^+$

The p_2 propagator is

$$\frac{1}{(p_2 - k)^2} \quad \to \quad \frac{(-\nu_2^2)^{\delta_2}}{\left[(p_2 - k)^2\right]^{1+\delta_2}} \,.$$

$\bar{n}$-Collinear Graph



$$\frac{1}{-n\cdot k} \quad \to \quad \frac{(-\nu_2^+)^{\delta_2}}{(-n\cdot k)^{1+\delta_2}}, \qquad \nu_2^+ \equiv \nu_2^2/p_2^-$$

The p_1 propagator is

$$\frac{1}{(p_1-k)^2} \quad \to \quad \frac{(-\nu_1^2)^{\delta_1}}{\left[(p_1-k)^2\right]^{1+\delta_1}} \,.$$

Integral with Analytic Regulator

$$
\begin{aligned}
I_n &= -2\frac{\alpha}{4\pi} C_F c(\mu)\gamma^\mu \left(\frac{\mu^2}{M^2}\right)^{\epsilon} \left(\frac{\nu_2^2}{M^2}\right)^{\delta_2} \left(\frac{\nu_1^-}{p_2^-}\right)^{\delta_1} \\
&\quad \times \frac{\Gamma(\epsilon+\delta_2)}{\Gamma(1+\delta_2)} \frac{\Gamma(2-\epsilon-\delta_2)\Gamma(\delta_2-\delta_1)}{\Gamma(2-\epsilon-\delta_1)} \,.
\end{aligned}
$$

The regulated value of I_n is given by setting $\delta_i = r_i\delta$ and taking the limit $\delta \to 0$ first, followed by $\epsilon \to 0$.

Limit exists except at the symmetric point $\delta_1 = \delta_2$.

I_n

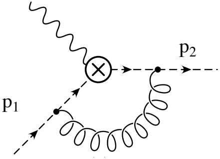
$$\begin{aligned}
I_n \;=\;& \frac{\alpha}{4\pi} C_F c(\mu) \gamma^\mu \left[\frac{2}{r_1 - r_2} \frac{1}{\delta \epsilon} + \frac{2}{r_1 - r_2} \frac{1}{\delta} \log \frac{\mu^2}{M^2} - \frac{2 r_2}{r_1 - r_2} \frac{1}{\epsilon^2} \right. \\
& \frac{1}{\epsilon} \left(2 + \frac{2 r_1}{r_1 - r_2} \log \frac{\nu_1^-}{p_2^-} + \frac{2 r_2}{r_1 - r_2} \log \frac{\nu_2^2}{\mu^2} \right) \\
& + 2 + 2 \log \frac{\mu^2}{M^2} + \frac{2 r_2}{r_1 - r_2} \log \frac{\mu^2}{M^2} \log \frac{\nu_2^2}{\mu^2} \\
& + \frac{2 r_1}{r_1 - r_2} \log \frac{\mu^2}{M^2} \log \frac{\nu_1^-}{p_2^-} + \frac{r_2}{r_1 - r_2} \log^2 \frac{\mu^2}{M^2} \\
& \left. + \frac{r_2 \pi^2}{2(r_1 - r_2)} - \frac{r_1 \pi^2}{3(r_1 - r_2)} \right],
\end{aligned}$$

Other graphs

Add $I_{\bar{n}}$, the other collinear graph.

Equal to I_n with $r_1 \leftrightarrow r_2$, $\nu_2^2 \to \nu_1^2$, $p_2^- \to p_1^+$, $\nu_1^- \to \nu_2^+$.

There is also an ultrasoft graph, which vanishes on-shell.



and wavefunction renormalization.

One-loop on-shell amplitude

$$\frac{\alpha}{4\pi} C_F c(\mu) \gamma^\mu \left[\frac{2}{\epsilon^2} + \frac{1}{\epsilon}(3 - 2\mathsf{L}_Q) + 2\mathsf{L}_M \mathsf{L}_Q - \mathsf{L}_M^2 - 4\mathsf{L}_M + \frac{9}{2} - \frac{5\pi^2}{6} \right]$$

$1/\epsilon$ terms give anomalous dimension:

$$\gamma = a\, C_F \left[4\mathsf{L}_Q - 6 \right]$$

agrees with the $M = 0$ DIS calculation.

The break-up into I_n, $I_{\bar{n}}$ and I_{us} depends on the regulator, and possibly on the gauge. We only need the sum, not each term separately.

Finite part gives the matching correction:

$$\exp D = a\, C_F \left[2\mathsf{L}_M \mathsf{L}_Q - \mathsf{L}_M^2 - 4\mathsf{L}_M + \frac{9}{2} - \frac{5\pi^2}{6} \right]$$

There is a $\log Q^2/M^2$ term in the matching at $\mu \sim M$.

Take $\mu = M$, then $\mathsf{L}_M = 0$ and the $\log Q$ term vanishes. But this is a fake. If $\mu = \eta M$, then

$$\mathsf{L}_Q \mathsf{L}_M \to \left(\log \frac{Q^2}{M^2} + \log \frac{1}{\eta^2} \right) \log \frac{1}{\eta^2}$$

Single log in low scale matching

There is a single log in the matching.

At two-loop order, it does not vanish even if $\mu = M$.

It cannot be moved to the anomalous dimension, because it depends on particle masses. There is a piece of the form

$$\mathrm{Cl}_2(\pi/3), \qquad \mathrm{Cl}_2(x) = \sum_{1}^{\infty} \frac{\sin nx}{n^2}$$

for massive particles at two loops, which is absent for massless particles.

Consistency Condition for Anomalous Dimension

Theory above M has anomalous dimension γ_h, below M has γ_ℓ.

$$\mu \frac{\mathrm{d}D}{\mathrm{d}\mu} = \gamma_\ell - \gamma_h$$

Here $\gamma_h \propto 4\mathsf{L}_Q - 6$ contains L_Q, $\gamma_\ell = 0$.

In DIS, $\gamma_\ell \propto \log N$, and

$$\gamma_\ell - \gamma_h \propto \log \frac{Q^2}{N\mu^2} = \log \frac{M_J^2}{\mu^2}$$

and $M_J^2 = Q^2/N$ is the (jet) matching scale. DIS has no large log in the matching, since γ does not have a big jump.

The L_Q would ruin the whole program of summing logarithms using RG evolution.

One can prove that there is at most a single logarithm in D, the logarithm of the matching condition

$$D = D_0(M,\mu) + D_1(M,\mu)\,\mathsf{L}_Q$$

and this holds to all orders of perturbation theory.

True at two-loop order by explicit computation.

Log summation by RGE

Normal RGE:

$$\alpha^n \left(L^n, L^{n-1}, \ldots L^2, L \right)$$

summed using RGE equations — n terms at order α^n.

Sudakov (SCET) RGE — wanted to sum:

$$\alpha^n \left(L^{2n}, L^{2n-1}, \ldots L^2, L \right)$$

but can only sum

$$\alpha^n \left(L^{2n}, L^{2n-1}, \ldots L^2, \cancel{L} \right)$$

which sums $2n - 1$ terms instead of $2n$. Not so bad.

Agrees with the known results. Can reproduce the known two-loop fixed order computations using a one-loop computation plus RGE: $\alpha^2 \times \mathrm{L}^4, \mathrm{L}^3, \mathrm{L}^2$.

Required Terms

High scale matching C, Low scale matching D. A is the cusp anomalous dimension.

$$\begin{aligned} \gamma &= A\,\mathsf{L}_Q + B \\ D &= D_0 + D_1\,\mathsf{L}_Q \end{aligned}$$

In the RGE counting:

LL requires $A^{(1)}$

NLL requires $A^{(2)}$, $B^{(1)}$, $D_1^{(1)}$

NNLL requires $A^{(3)}$, $B^{(2)}$, $D_1^{(2)}$, $D_0^{(1)}$, $C^{(1)}$

NNNLL requires $A^{(4)}$, $B^{(3)}$, $D_1^{(3)}$, $D_0^{(2)}$, $C^{(2)}$

It is now simple to compute other results, which have not been done:

Other operators, e.g. $\bar{\psi}\psi$, $\phi^\dagger\phi$, $\phi^\dagger\psi$ etc. (SCET for scalars)

Can include mass effects — two fermions have masses m_1 and m_2, and sum logs of m_i.

Can include Higgs fields and Yukawa couplings, and compute for the standard model, including the top-quark Yukawa coupling.

Do the case M_H, M_W, and M_Z all different, and treat electroweak mixing.

$\mathcal{O}$	$C(\mu)/C_F$	$\gamma_{EFT}(\mu)/C_F$	$D(\mu)/C_F$
$\bar\psi\psi$	$-\mathsf{L}_Q^2+\frac{\pi^2}{6}-2$	$4\mathsf{L}_Q-6$	$-\mathsf{L}_M^2+2\mathsf{L}_M\mathsf{L}_Q-3\mathsf{L}_M+\frac{9}{2}-\frac{5\pi^2}{6}$
$\bar\psi\gamma^\mu\psi$	$-\mathsf{L}_Q^2+3\mathsf{L}_Q+\frac{\pi^2}{6}-8$	$4\mathsf{L}_Q-6$	$-\mathsf{L}_M^2+2\mathsf{L}_M\mathsf{L}_Q-3\mathsf{L}_M+\frac{9}{2}-\frac{5\pi^2}{6}$
$\bar\psi\sigma^{\mu\nu}\psi$	$-\mathsf{L}_Q^2+4\mathsf{L}_Q+\frac{\pi^2}{6}-8$	$4\mathsf{L}_Q-6$	$-\mathsf{L}_M^2+2\mathsf{L}_M\mathsf{L}_Q-3\mathsf{L}_M+\frac{9}{2}-\frac{5\pi^2}{6}$
$\phi^\dagger\phi$	$-\mathsf{L}_Q^2+\mathsf{L}_Q+\frac{\pi^2}{6}-2$	$4\mathsf{L}_Q-8$	$-\mathsf{L}_M^2+2\mathsf{L}_M\mathsf{L}_Q-4\mathsf{L}_M+\frac{7}{2}-\frac{5\pi^2}{6}$
$i(\phi^\dagger D^\mu\phi-D^\mu\phi^\dagger\phi)$	$-\mathsf{L}_Q^2+4\mathsf{L}_Q+\frac{\pi^2}{6}-8$	$4\mathsf{L}_Q-8$	$-\mathsf{L}_M^2+2\mathsf{L}_M\mathsf{L}_Q-4\mathsf{L}_M+\frac{7}{2}-\frac{5\pi^2}{6}$
$\bar\psi\phi$	$-\mathsf{L}_Q^2+2\mathsf{L}_Q+\frac{\pi^2}{6}-4$	$4\mathsf{L}_Q-7$	$-\mathsf{L}_M^2+2\mathsf{L}_M\mathsf{L}_Q-\frac{7}{2}\mathsf{L}_M+4-\frac{5\pi^2}{6}$

Note there is a factorization structure in the EFT

All $\bar\psi\psi$ are equal, all $\phi^\dagger\phi$ are equal, $\bar\psi\phi$ is the average in the EFT.

Factorization Structure

For massive fermions also have a factorization form:

$$\Gamma_{12}(Q^2, 1, 2) \quad = \quad \Gamma_{12}(2p_1 \cdot p_2) + f_1(m_1) + f_2(m_2)$$

where $\Gamma(Q)$ is independent of particle type, and f_i only depends on the properties of particle i (including whether it is a scalar or fermion).

$$\Gamma_{12}(2p_1 \cdot p_2) \quad = \quad a \log(2p_1 \cdot p_2) + b$$

Factorization up to a calculable single log term.

top quark with Higgs corrections:

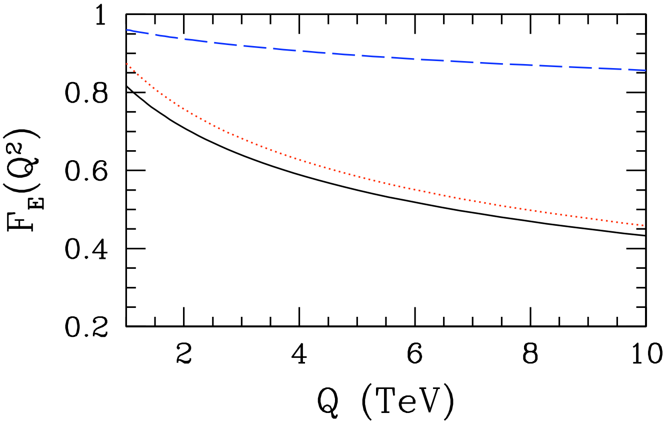
$m_t - m_b$ is large, and breaks $SU(2) \times U(1)$ symmetry. If one uses a sequence of theories with $M_H > m_t > M_Z > M_W$, then a mess.

But instead, can integrate them all out at a common μ.

Go from $SU(3) \times SU(2) \times U(1)$ directly to $SU(3) \times U(1)$, and a theory with SCET $Q^{(t)}$, t_R and b_R fields to one with SCET b_L, b_R fields and HQET t_L, t_R fields.

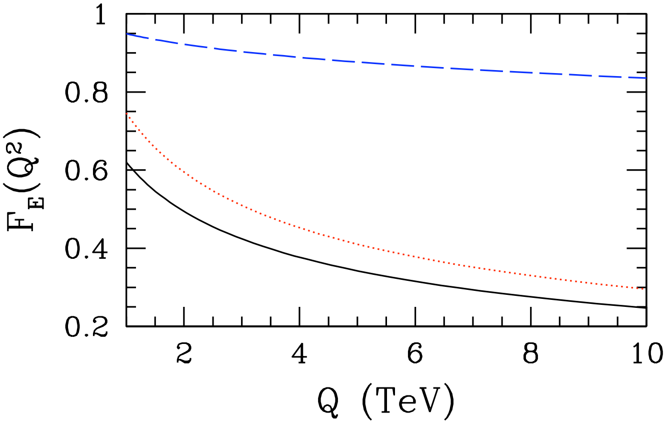
Then compute observables as before. For cross-sections, the SCET field matrix elements in the proton are the Collins-Soper parton distribution functions. For final states, construct jet observables, or t-observables, etc.

$\mu = M_Z$



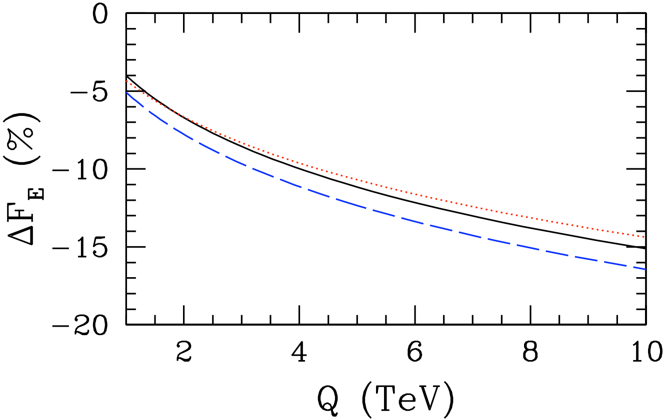
black: *u*, red: *t*, blue: *e*

$\mu = 30$ GeV



black: *u*, red: *t*, blue: *e*

Electroweak contribution ($\mu = 30$ GeV)



black: *u*, red: *t*, blue: *e*

Scattering

This is still the toy Sudakov form-factor problem, which gives the rate for a gauge-singlet particle or current to produce two particles.

Can easily extend this to arbitary LHC processes, e.g. $q\bar{q} \to t\bar{t}$ by combining the calculations with the appropriate group theory factors. Basically, sum the Sudakov result over all pairs.

The anomalous dimensions are larger.

(see talk by J. Chiu)

Conclusions

- Include electroweak corrections in a systematic way.
- Include dependence on M_W, M_Z and m_t in a spontaneously broken gauge theory including gauge mixing.
- Include Higgs corrections due to m_t.
- Can be extended to other electroweak processes such as squark production (see Chiu's talk)
- Purely electroweak corrections are important for LHC cross-sections.