

SCET goings on in $b \rightarrow s \ell^+ \ell^-$

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SCET Workshop 2008
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- Introduction — measurements and theory
- Third independent observable besides $d\Gamma/dq^2$ and dA_{FB}/dq^2
Exclusive $B \rightarrow K^* \ell^+ \ell^-$ and constraining $\zeta_{\perp}^J/\zeta_{\perp}$
- $B \rightarrow X_s \ell^+ \ell^-$ in the small q^2 region
Cuts on q^2 & $m_X \Rightarrow$ more cleanly shape function region than $B \rightarrow X_u \ell \bar{\nu}$
- $B \rightarrow X_s \ell^+ \ell^-$ in the large q^2 region — normalize to $B \rightarrow X_u \ell \bar{\nu}$
Can eliminate m_b -dependence and dominant $1/m_b^n$ uncertainty

Details: K. Lee, ZL, I. Stewart, F. Tackmann: hep-ph/0612156 & hep-ph/0512191

ZL, F. Tackmann: arXiv:0707.1694, K. Lee, I. Stewart: hep-ph/0511334

Perturbative $b \rightarrow sl^+l^-$ calculations

- Complementary to $B \rightarrow X_s \gamma$
- Subtleties in power counting (as in $K \rightarrow \pi e^+ e^-$)

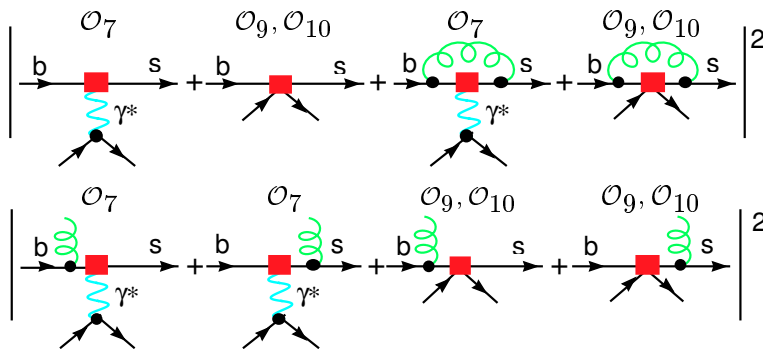
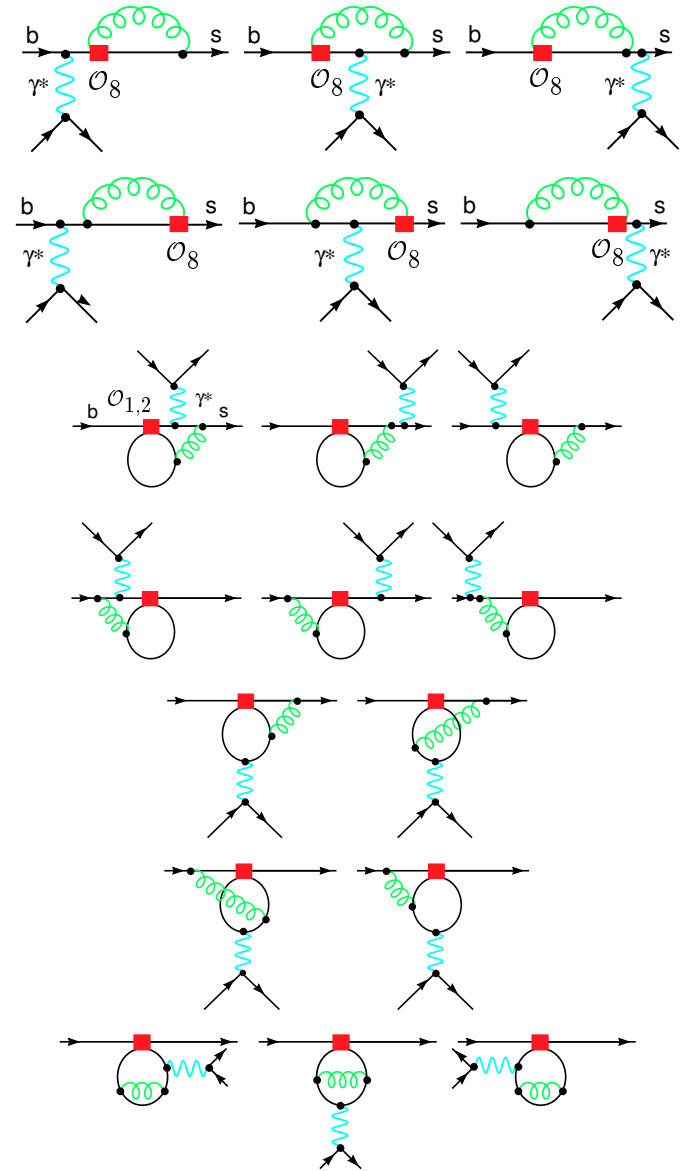
$$C_9(m_b) \sim C_9(m_W) + (\dots) \frac{C_2(m_W)}{\alpha_s(m_W)} \left\{ 1 - \left[\frac{\alpha_s(m_b)}{\alpha_s(m_W)} \right]^{(\dots)} \right\}$$

Scale & scheme dependence cancellation tricky

- NNLL: 2-loop matching, 2- and 3-loop running
2-loop matrix elements

$$\mathcal{B}(B \rightarrow X_s l^+ l^-) \Big|_{1 < q^2 < 6 \text{ GeV}^2} = (1.63 \pm 0.20) \times 10^{-6}$$

[Many authors: Bobeth, Misiak, Urban, Munz, Gambino, Gorbahn, Haisch, Asatryan, Asatrian, Bieri, Hovhannisyian, Greub, Walker, Ghinculov, Hurth, Isidori, Yao, etc.]



The q^2 spectrum in $B \rightarrow X_s \ell^+ \ell^-$

- Rate depends (mostly) on

$$O_7 = \bar{m}_b \bar{s} \sigma_{\mu\nu} e F^{\mu\nu} P_R b,$$

$$O_9 = e^2 (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \ell),$$

$$O_{10} = e^2 (\bar{s} \gamma_\mu P_L b) (\bar{\ell} \gamma^\mu \gamma_5 \ell)$$

Theory most precise for $1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$

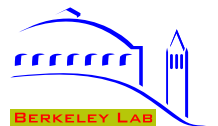
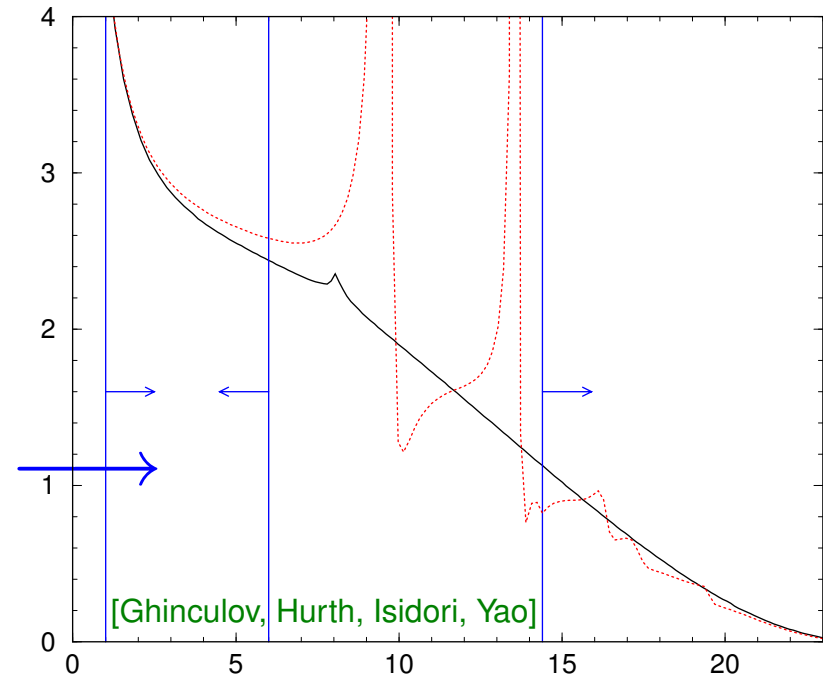
- NNLL $b \rightarrow s \ell^+ \ell^-$ perturbative calculations

Introduce $C_{7,9}^{\text{eff}}$ — complex with usual definition

- Nonperturbative corrections to q^2 spectrum ($1/m_b^2$ and $1/m_c^2$)

[Falk, Luke, Savage; Ali, Hiller, Handoko, Morozumi; Buchalla, Isidori, Rey]

- Small q^2 region has the larger rate \Rightarrow smaller experimental errors



Standard approaches

- Earlier analyses concentrated on two observables: $(s = q^2/m_b^2)$

$$\frac{d\Gamma}{ds} \sim \Gamma_0 (1-s)^2 \left[\left(|C_9|^2 + C_{10}^2 \right) (1+2s) + \frac{4}{s} |C_7|^2 (2+s) + 12 \operatorname{Re}(C_7 C_9^*) \right]$$

$$\frac{dA_{\text{FB}}}{ds} \sim -3\Gamma_0 (1-s)^2 s C_{10} \operatorname{Re} \left(C_9 + \frac{2}{s} C_7 \right) \quad [\text{Ali, Mannel, Morozumi}]$$

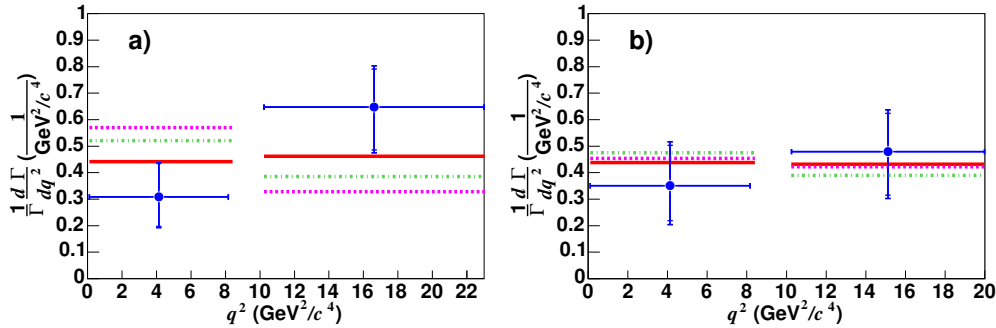
$O_{1-6,8}$ contributions absorbed in $C_{7,9} \rightarrow C_{7,9}^{\text{eff}}(s)$, which are complex

- To look for NP or to extract C_i :
 - Compute rate in SM (or in a NP model) and compare with data
(have to be redone for each model, hard to incorporate improvements in theory)
 - Extract C_i from fits to decay distributions (poor sensitivity, needs lots of data)
(zero of A_{FB} near $-2C_7/C_9$ argued to be model independent in $B \rightarrow K^* \ell^+ \ell^-$)
- Want most effective ways to extract C_i from simple observables integrated over q^2

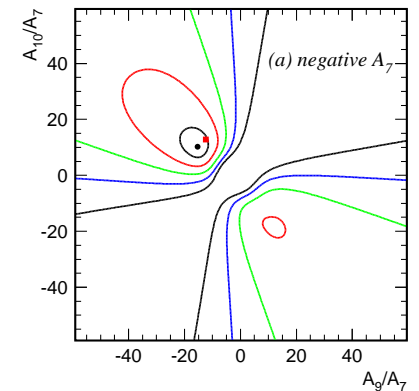
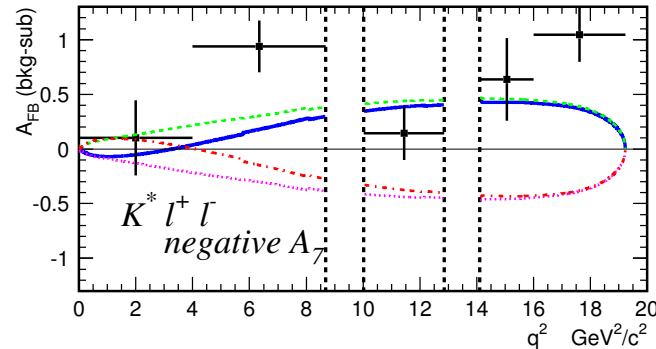
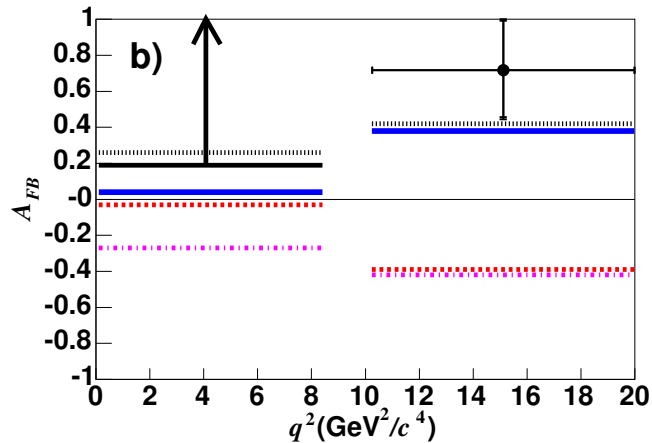
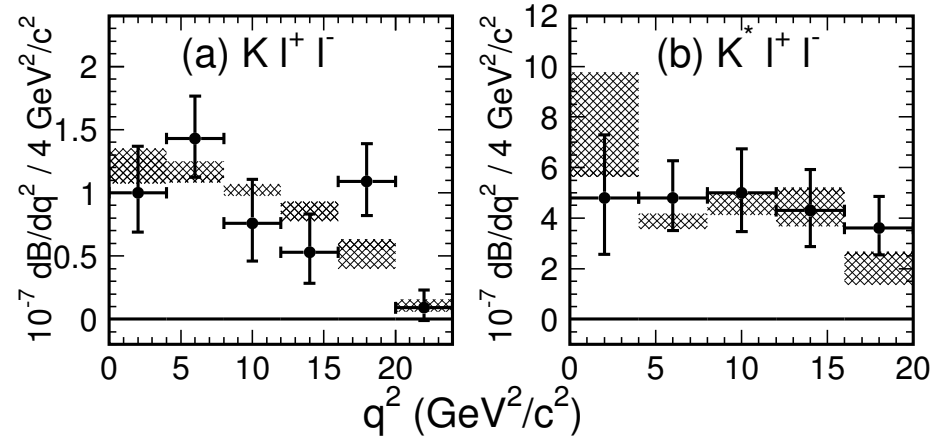


Exclusive $B \rightarrow K^{(*)} \ell^+ \ell^-$ measurements

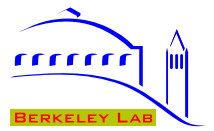
BaBar, 229 m Υ



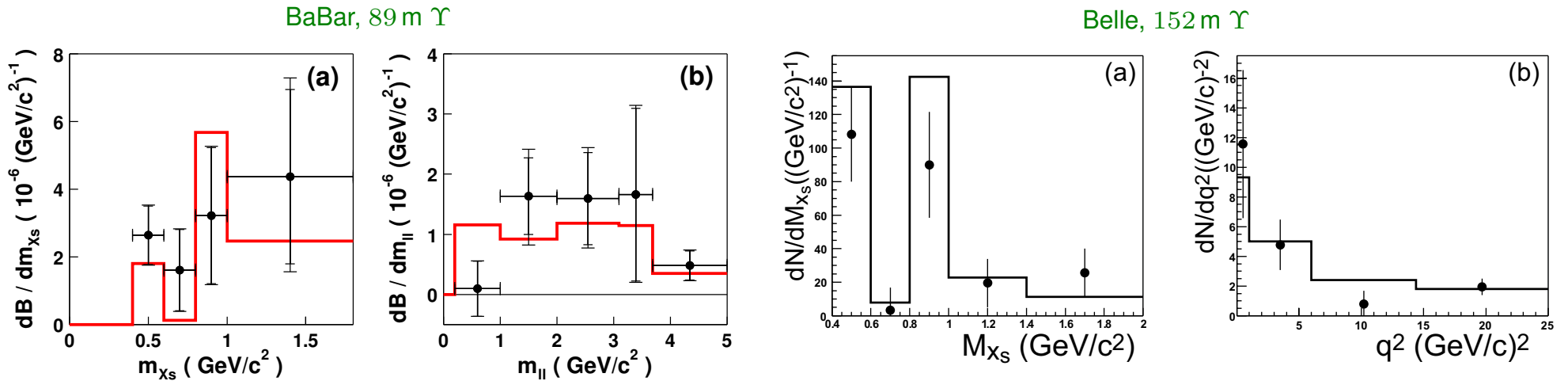
Belle, 253 m and 386 m Υ



- World averages: $\mathcal{B}(B \rightarrow K^{(*)} \ell^+ \ell^-) = (0.44 \pm 0.05) \times 10^{-6}, (1.17 \pm 0.16) \times 10^{-6}$
- LHCb expects $(2, 10 \text{ fb}^{-1})$: $\sigma(q_{A_{FB}=0}^2) \approx 0.46, 0.27 \text{ GeV}^2 \Rightarrow \sigma(C_7^{\text{eff}}/C_9^{\text{eff}}) \sim 12, 7\%$



Inclusive $B \rightarrow X_s l^+ l^-$ measurements



- Cut out J/ψ and ψ' regions, and impose an additional cut $m_X < 1.8 \text{ GeV}$ or 2 GeV to suppress huge $b \rightarrow cl^- \bar{\nu} \rightarrow sl^+ l^- \nu \bar{\nu}$ background

Current measurements not really inclusive — sum $\sim 50\%$ of exclusive modes

- **World average:** $\mathcal{B}(B \rightarrow X_s l^+ l^-) = (4.5 \pm 1.0) \times 10^{-6}$ (with some black magic)
- **Small q^2 region:** $\mathcal{B}(1 < q^2 < 6 \text{ GeV}^2) = (1.60 \pm 0.51) \times 10^{-6}$
- A key measurement that utilizes only a small fraction of the available data



Optimal observables

Details: K. Lee, ZL, I. Stewart, F. Tackmann, [hep-ph/0612156](https://arxiv.org/abs/hep-ph/0612156)

Angular decomposition

- θ : angle between \vec{p}_{ℓ^+} and \vec{p}_{B^0, B^-} [\vec{p}_{ℓ^-} and \vec{p}_{B^0, B^+}] in $\ell^+\ell^-$ center of mass frame

Once θ is measured, there are 3 (not 2) terms with different sensitivities to C_i 's

$$\frac{d^2\Gamma}{dq^2 dz} = \frac{3}{8} \Gamma_0 \left[(1+z^2) H_T(q^2) + 2z H_A(q^2) + 2(1-z^2) H_L(q^2) \right] \quad (s = q^2/m_b^2, z = \cos \theta)$$

$$H_T \sim 2(1-s)^2 s \left[\left(C_9 + \frac{2}{s} C_7 \right)^2 + C_{10}^2 \right] \quad [\Gamma = H_T + H_L]$$

$$H_L \sim (1-s)^2 \left[(C_9 + 2C_7)^2 + C_{10}^2 \right] \quad [\text{no } C_7/s \text{ pole}]$$

$$H_A \sim -4(1-s)^2 s C_{10} \left(C_9 + \frac{2}{s} C_7 \right) \quad [H_A \equiv (4/3) A_{\text{FB}}]$$

- Dependence on C_i : H_L is q^2 independent; $H_{T,A}$'s sensitivity to C_i depends on q^2
- Same structure for $B \rightarrow X_s \ell^+\ell^-$ and $B \rightarrow K^* \ell^+\ell^-$ — different at $\mathcal{O}(\alpha_s, 1/m_{c,b})$
 $B \rightarrow K^* \ell^+\ell^-$: Two further angles (even more if ℓ^\pm polarizations considered)
- Three terms sensitive to different combinations of Wilson coefficients



Higher order corrections

- Introduce a scheme to separate NP-sensitive terms from four-quark operator contributions (for which the SM is assumed)
- Define $C_{7,9}$ as μ - and q^2 -independent constants, **real** in the SM

$$C_{7,9}^{\text{incl}}(q^2) = C_{7,9} + \underbrace{F_{7,9}(q^2)}_{\alpha_s} + \underbrace{G_{7,9}(q^2)}_{1/m_c^2} \quad (F_{7,9} \text{ include NNLL})$$

- Use m_b^{1S} to improve perturbation series; do not normalize to $\Gamma(B \rightarrow X \ell \bar{\nu})$

Keep $\bar{m}_b(\mu)C_7(\mu)$ together and unexpanded — no reason to expand $\bar{m}_b(\mu)$

- Numerically small Λ^2/m_c^2 correction can be simply included:

$$G_9(q^2) = \frac{10}{1-2s} G_7(q^2) = -\frac{5}{6} \frac{\lambda_2}{m_c^2} C_2 \frac{\mathcal{F}[q^2/(4m_c^2)]}{1-q^2/(4m_c^2)}$$

Blows up as $(4m_c^2 - q^2)^{-1/2}$ as $q^2 \rightarrow 4m_c^2$; assume OK for $q^2 \lesssim 3m_c^2 \sim 6 \text{ GeV}^2$

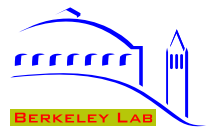
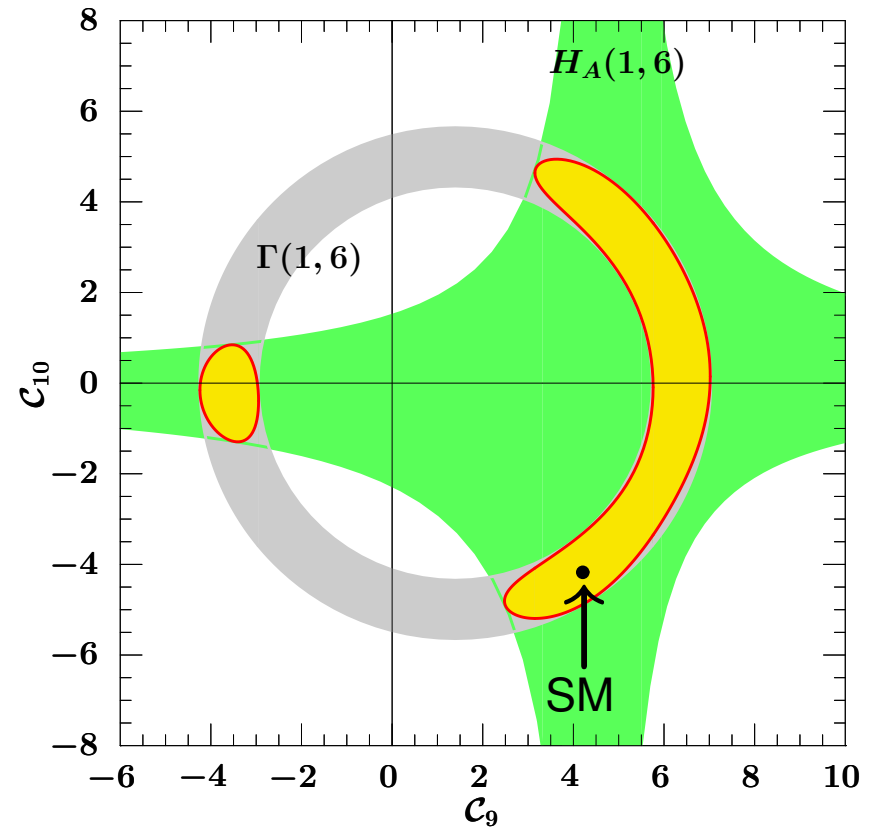


An illustrative toy analysis

- Inclusive, with guesstimated error for 1 ab^{-1}

Define:
$$H_i(q_1^2, q_2^2) = \int_{q_1^2}^{q_2^2} dq^2 H_i(q^2)$$

- Small q^2 -dependence

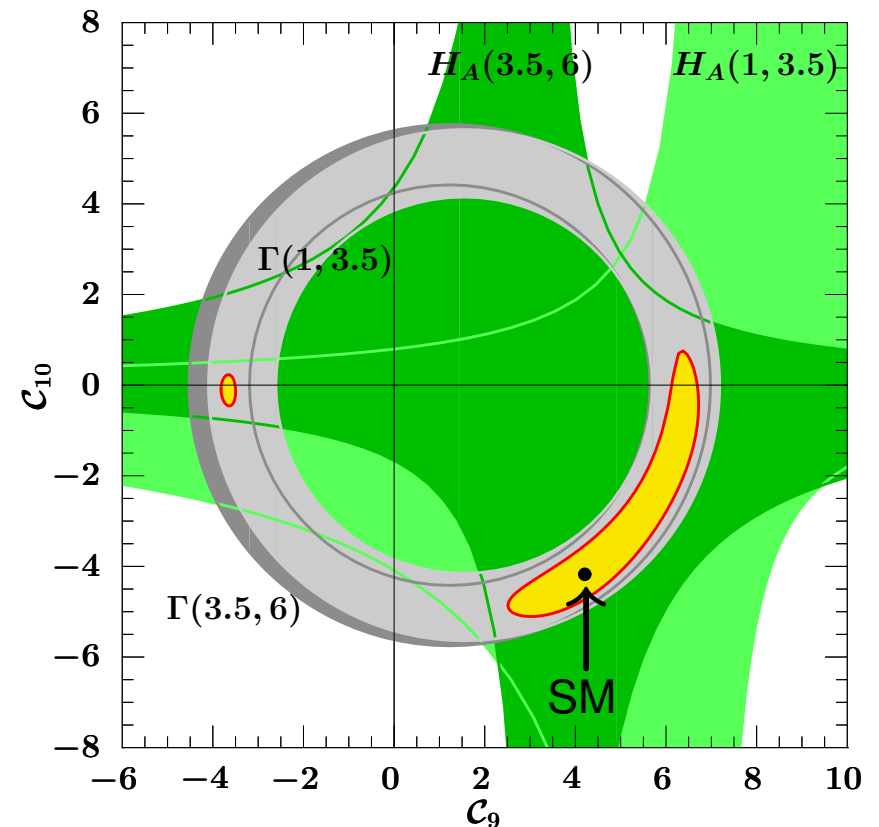


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- Small q^2 -dependence \Rightarrow splitting Γ in two regions not useful (splitting $H_A \equiv A_{\text{FB}}$ is!)

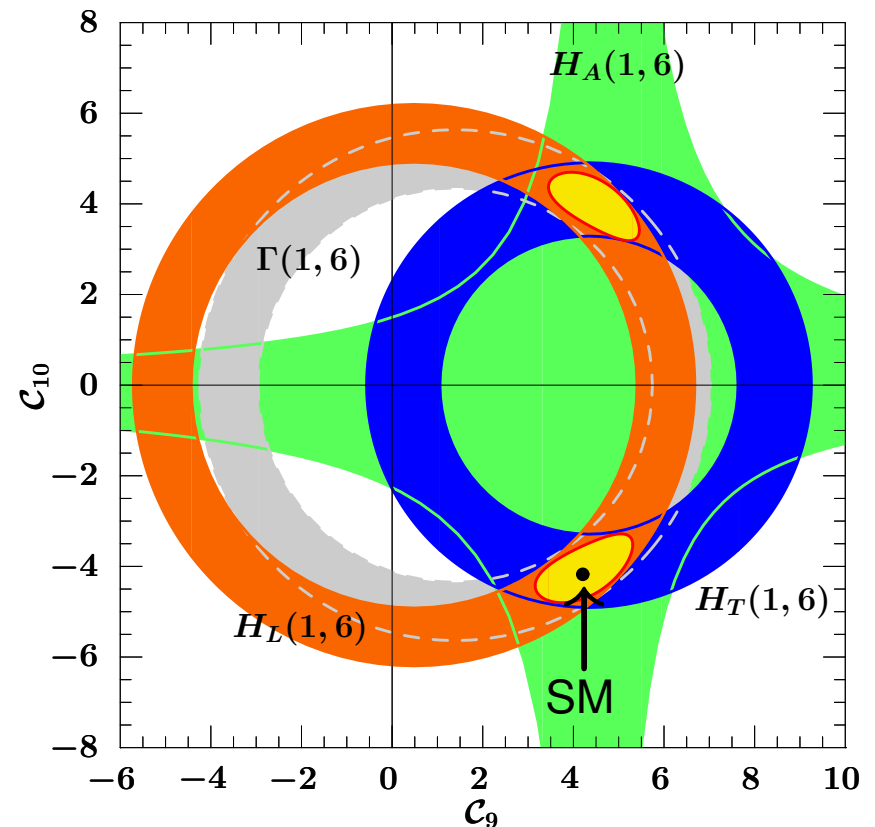


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- $H_L \propto q^2$ -independent combination of C_i 's
 \Rightarrow integrate over as large region as possible



- Separating H_T and H_L is very powerful

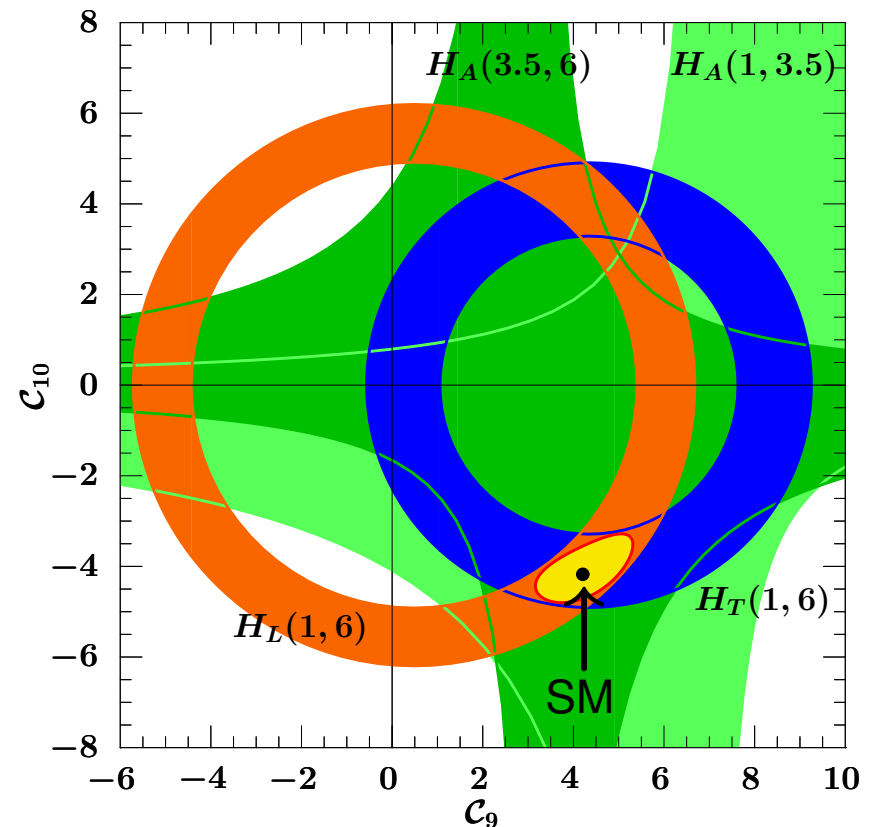


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- $H_L \propto q^2$ -independent combination of \mathcal{C}_i 's
 \Rightarrow integrate over as large region as possible
- H_T and H_A : different q^2 regions sensitive to different combinations of \mathcal{C}_i 's
 Separating $H_A(1, 3.5)$ vs $H_A(3.5, 6)$ and/or $H_T(1, 3.5)$ vs $H_T(3.5, 6)$ appears promising
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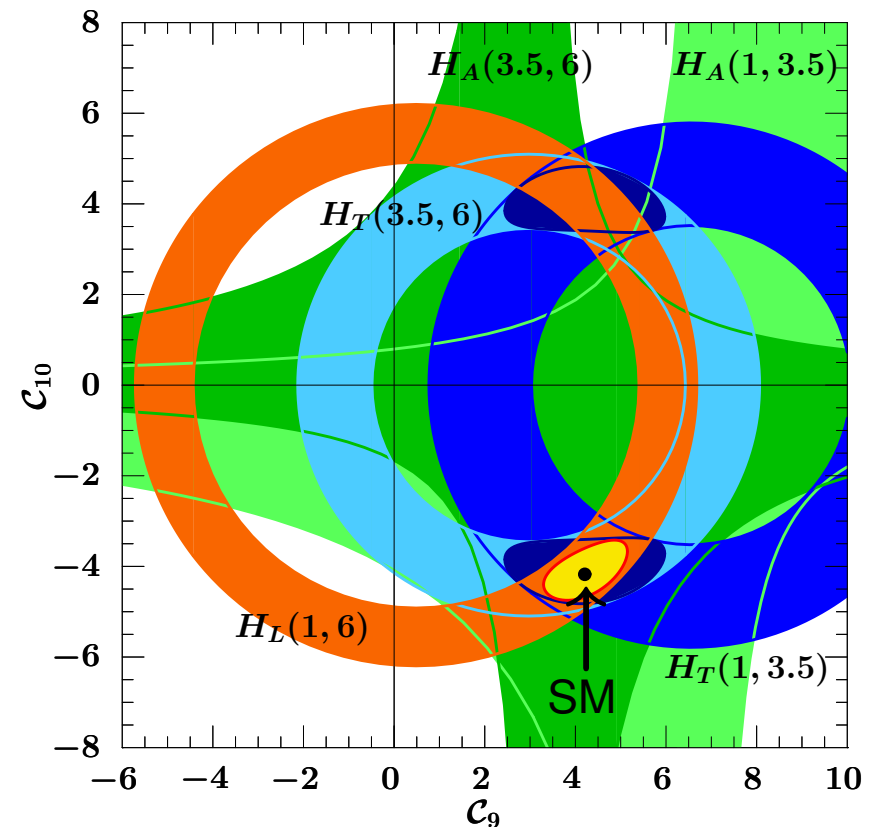
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- Separating H_T and H_L is very powerful
- Can extract all information from a few integrated rates

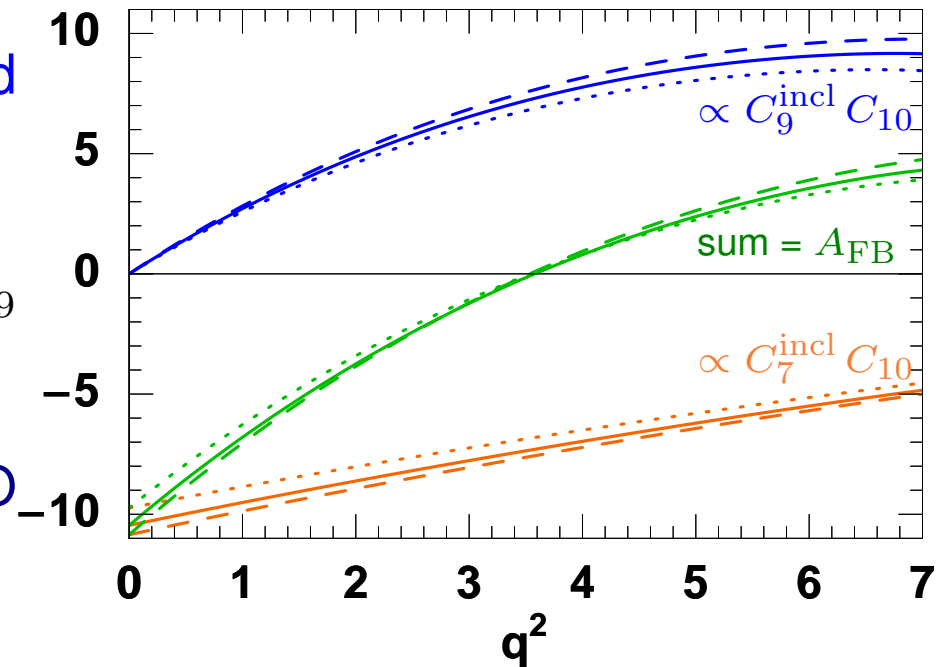


The μ dependence of A_{FB}

- Zero of A_{FB} , $A_{\text{FB}}(q_0^2) = 0$ sometimes said to be particularly clean in inclusive as well
- μ -dep. smaller than for rate, linear in C_7, C_9 (C_{10} is μ independent, rate is quadratic)

Cancellations reduce μ -dep of zero @NLO

Some terms tend to cancel even at NNLO



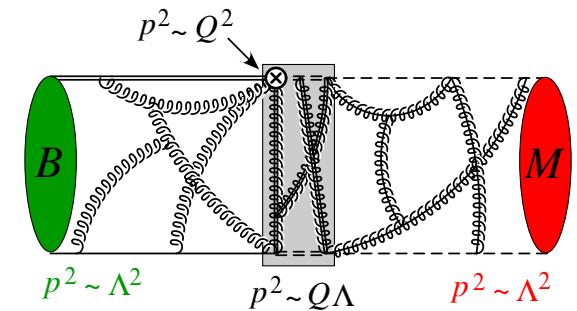
- Uncertainty of q_0^2 not relevant; only physical question is sensitivity to C_7/C_9 , for which it's not clear that the zero has an advantage
- Whether uncertainty from q_0^2 is parametrically reduced in $B \rightarrow K^* \ell^+ \ell^-$ depends on relative size of factorizable / nonfactorizable contributions to form factors



Recall: heavy-to-light form factors

- At leading order in Λ/Q and to all orders in α_s , form factors for $q^2 \ll m_B^2$ factorize ($Q = E, m_b$; omit μ -dep's)

[Beneke & Feldmann; Bauer, Pirjol, Stewart; Becher, Hill, Lange, Neubert]



$$f(E) = C(E) \zeta(E) + \int dz T(z, E) \zeta^J(z, E), \quad \zeta^J = \int dx dk_+ J(z, x, k_+, E) \phi_M(x) \phi_B(k_+)$$

Matrix elements of distinct $\int d^4x T[J^{(n)}(0) \mathcal{L}_{\xi q}^{(m)}(x)]$ terms (turn spectator $q_{us} \rightarrow \xi$)

- Symmetries \Rightarrow nonfactorizable (1st) term obey form factor relations [Charles et al.]

$3 B \rightarrow P$ and $7 B \rightarrow V$ form factors related to just 3 fn's: $\zeta_{P, \parallel, \perp}$

Issues: perturbation theory at E and $\sqrt{E} \Lambda_{\text{QCD}}$; zero-bin (endpoint singularity)

- Applications: pert. calculate T ; SCET: fit both ζ & ζ^J ; QCDF: fit ζ , calculate ζ^J ; PQCD: k_{\perp} factorization, dependence on $\phi(x, k_{\perp})$



Exclusive $B \rightarrow K^* \ell^+ \ell^-$ with SCET

- Angular decomposition involves: $\zeta_{\parallel, \perp}(s)$ and $\zeta_{\parallel, \perp}^J(s) \sim$ (non-)factorizable parts

$$H_T \sim 2s\lambda^3 \left\{ C_{10}^2 [\zeta_{\perp}(s)]^2 + \left| C_9 \zeta_{\perp}(s) + \frac{2C_7}{s} \frac{m_b}{m_B} [\zeta_{\perp}(s) + (1-s)\zeta_{\perp}^J(s)] \right|^2 \right\}$$

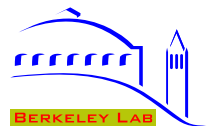
$$H_A \sim -4s\lambda^3 C_{10} \zeta_{\perp}(s) \operatorname{Re} \left\{ C_9 \zeta_{\perp}(s) + \frac{2C_7}{s} \frac{m_b}{m_B} [\zeta_{\perp}(s) + (1-s)\zeta_{\perp}^J(s)] \right\}$$

$$H_L \sim \frac{1}{2} \lambda^3 \left(C_{10}^2 + \left| C_9 + 2C_7 \frac{m_b}{m_B} \right|^2 \right) [\zeta_{\parallel}(s) - \zeta_{\parallel}^J(s)]^2 \quad (\lambda = \sqrt{(1-s)^2 - 2\rho(1+s) + \rho^2})$$

- Form factors: reduce to a few numbers using asymptotic dependence

$$\zeta_{\perp}^{(J)}(s) = \frac{\zeta_{\perp}^{(J)}(0)}{(1-s)^2} \left[1 + \mathcal{O}\left(\alpha_s, \frac{\Lambda}{E}\right) \right] \quad (1.9 < E < 2.7 \text{ GeV})$$

- Without nonperturbative input [or $SU(3)$], cannot use $H_L^{(B \rightarrow K^* \ell^+ \ell^-)}$ & $B \rightarrow K \ell^+ \ell^-$



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$$\Gamma(B \rightarrow K^* \gamma) = \frac{G_F^2}{8\pi^3} \frac{\alpha_{\text{em}}}{4\pi} |V_{tb} V_{ts}^*|^2 m_B^3 (m_b^{1S})^2 (1-\rho)^3 |C_7(0)|^2 [\zeta_{\perp}(0) + \zeta_{\perp}^J(0)]^2$$



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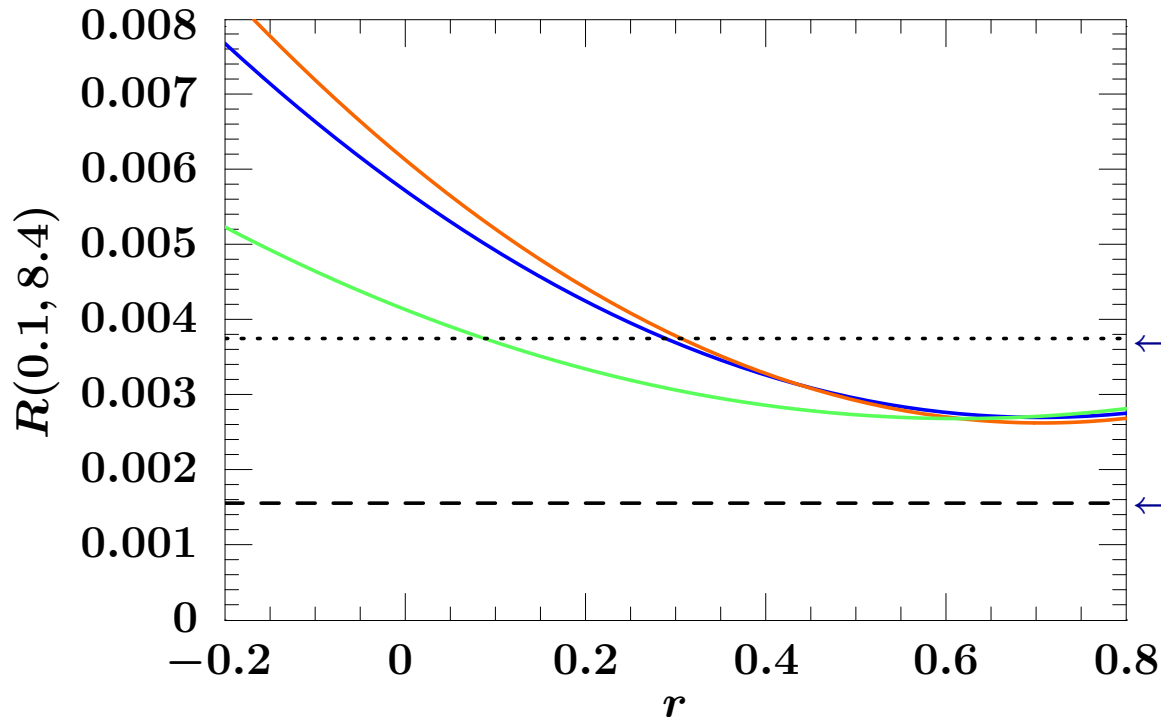
- Three ratios of: $\Gamma(B \rightarrow K^* \gamma)$, $H_T(0, 8)$, $H_A(0, 4)$, $H_A(4, 8)$

Determine: C_{10}/C_7 , C_9/C_7 , and hadronic parameter $\zeta_{\perp}^J(0)/[\zeta_{\perp}(0) + \zeta_{\perp}^J(0)]$



Constraining hadronic physics

$$R(q_1^2, q_2^2) \equiv \frac{H_T(q_1^2, q_2^2)}{\Gamma(B \rightarrow K^* \gamma)} = \frac{\alpha_{\text{em}} m_B^2}{12\pi m_b^2} \int_{q_1^2/m_B^2}^{q_2^2/m_B^2} ds \frac{\lambda^3 s}{(1-\rho)^3 (1-s)^4} \times \left\{ \frac{C_{10}^2}{C_7^2} (1-r)^2 + \left[\frac{C_9}{C_7} (1-r) + \frac{2 m_b}{s m_B} (1-sr) \right]^2 \right\}$$



Plot $R(0.1, 8.41)$ and BaBar data

$$r \equiv \frac{\zeta_{\perp}^J(0)}{\zeta_{\perp}(0) + \zeta_{\perp}^J(0)}$$

← 1σ upper bound

← central value

Too early to tell...

- BaBar & Belle have already a lot more data (expect / predict H_T to increase)



Small q^2 region

Details: K. Lee, ZL, I. Stewart, F. Tackmann, hep-ph/0512191; K. Lee, I. Stewart: hep-ph/0511334

Recall: NNLL for $B \rightarrow X_s \ell^+ \ell^-$

- Rate depends (mostly) on

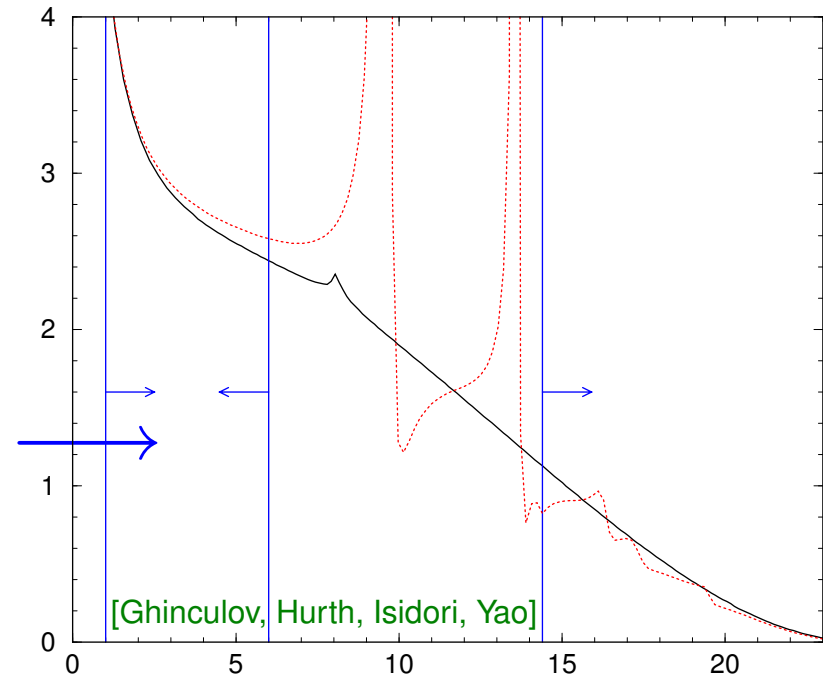
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Theory most precise for $1 \text{ GeV}^2 < q^2 < 6 \text{ GeV}^2$

- NNLL $b \rightarrow s \ell^+ \ell^-$ perturbative calculations
- Nonperturbative corrections to q^2 spectrum



- In small q^2 region experiments need additional $m_{X_s} \lesssim 2 \text{ GeV}$ cut to suppress $b \rightarrow c(\rightarrow s \ell^+ \nu) \ell^- \bar{\nu} \Rightarrow$ nonperturbative effects
- Theory similar to that for phase space cuts in inclusive $|V_{ub}|$ measurements



$B \rightarrow X_s \ell^+ \ell^-$ kinematics at small q^2

- Only two kinematic variables are symmetric in p_{ℓ^+} and p_{ℓ^-}

$$2m_B E_X = m_B^2 + m_X^2 - q^2$$

q^2 not large and $m_X^2 \ll m_B^2 \Rightarrow E_X = \mathcal{O}(m_B) \Rightarrow E_X^2 \gg m_X^2$, so p_X near light-cone

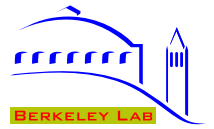
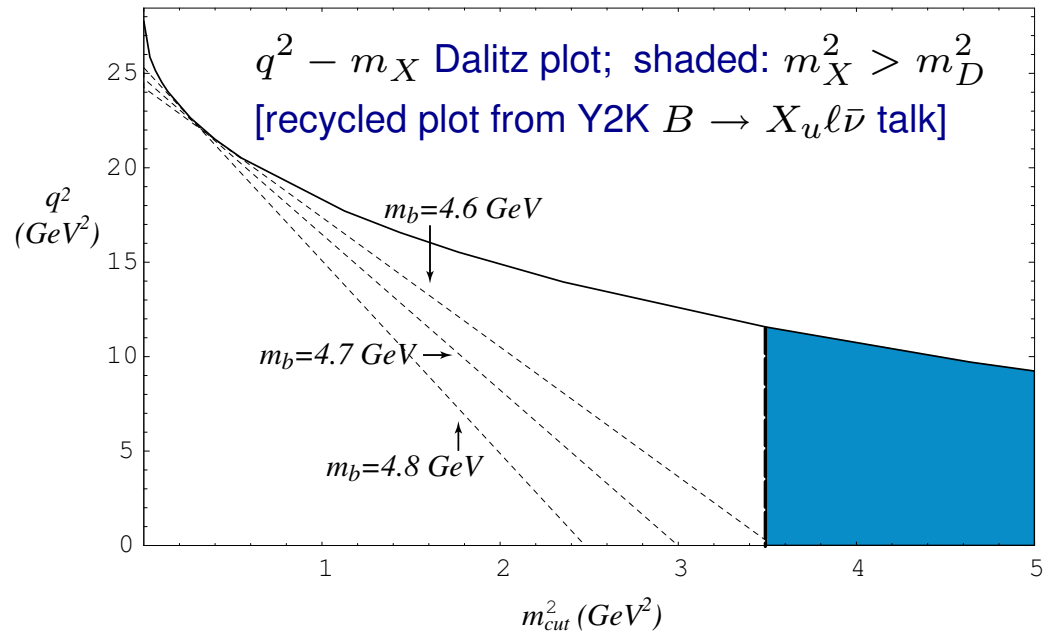
$$p_X^+ = n \cdot p_X = \mathcal{O}(\Lambda_{\text{QCD}}) \quad p_X^- = \bar{n} \cdot p_X = \mathcal{O}(m_B) \quad n, \bar{n} = (1, \pm \vec{p}_X / |\vec{p}_X|)$$

- $p_X^+ \ll p_X^-$: jet-like hadronic final state

- Parton level: $\Gamma \propto f(q^2) \delta[(m_b v - q)^2]$
 $m_X^2 \geq \bar{\Lambda}(m_B - q^2/m_b)$

rate vanishes left of the dashed lines

- Nonperturbative physics is important
 Same shape fn as in $B \rightarrow X_s \gamma, X_u \ell \bar{\nu}$



$B \rightarrow X_s \ell^+ \ell^-$ kinematics at small q^2

- Only two kinematic variables are symmetric in p_{ℓ^+} and p_{ℓ^-}

$$2m_B E_X = m_B^2 + m_X^2 - q^2$$

q^2 not large and $m_X^2 \ll m_B^2 \Rightarrow E_X = \mathcal{O}(m_B) \Rightarrow E_X^2 \gg m_X^2$, so p_X near light-cone

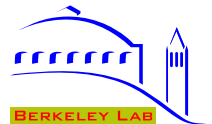
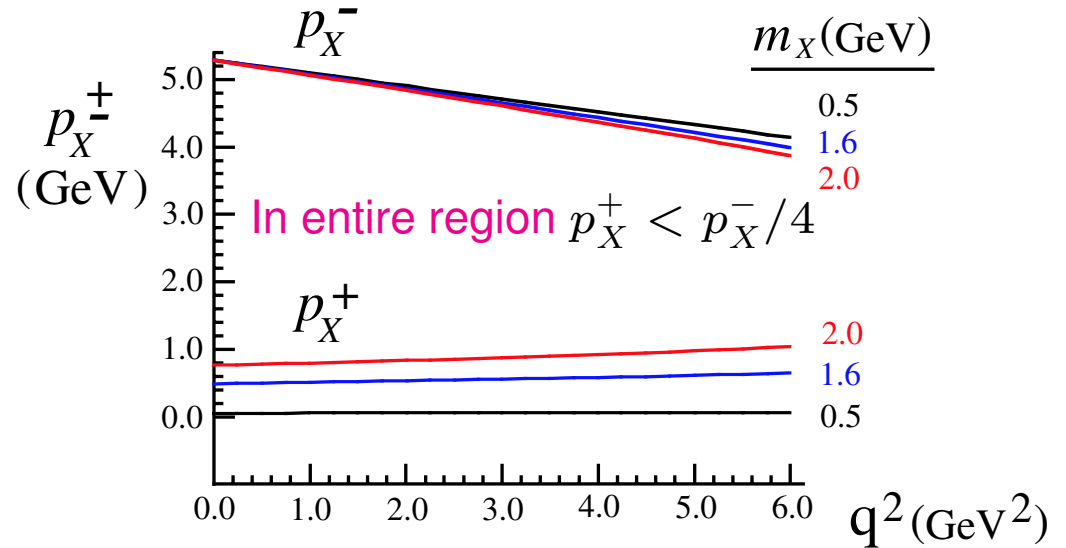
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Effects of m_X cut at lowest order

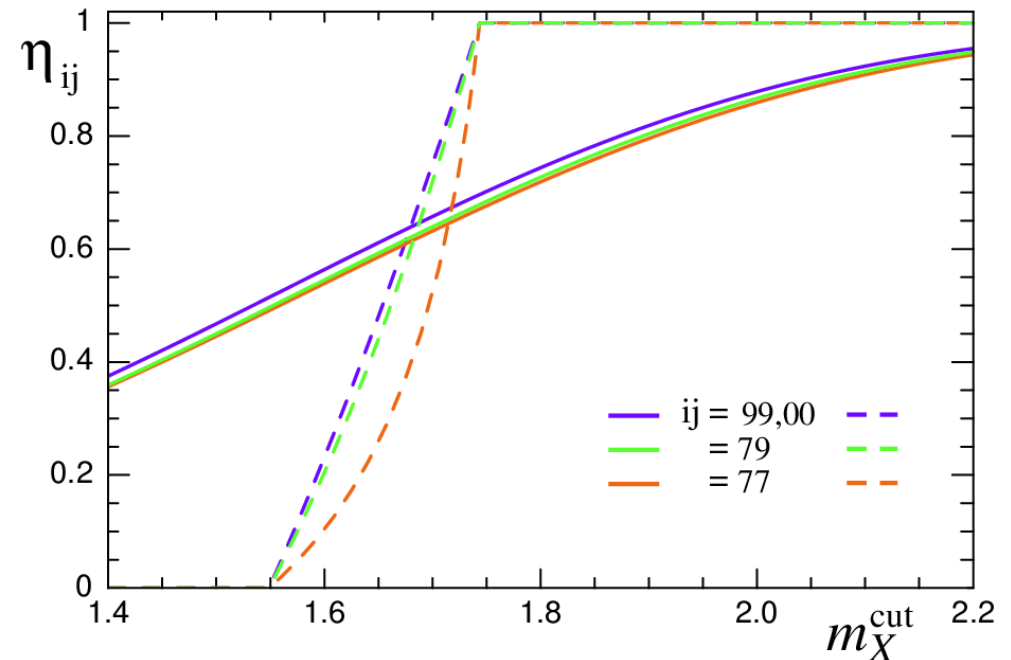
- Define:

$$\eta_{ij} = \frac{\int_{1 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 \int_0^{m_X^{\text{cut}}} dm_X^2 \frac{d\Gamma_{ij}}{dq^2 dm_X^2}}{\int_{1 \text{ GeV}^2}^{6 \text{ GeV}^2} dq^2 \frac{d\Gamma_{ij}}{dq^2}}$$

ij : C_9^2 and C_{10}^2 , $C_7 C_9$, C_7^2 — different functionally for each contribution

Dashed: tree level in local OPE [wrong]

Solid: with a fixed shape function model



- η_{ij} determine fraction of rate that is measured in presence of m_X cut

I.e., a 30% deviation at $m_X^{\text{cut}} = 1.8 \text{ GeV}$ may be hadronic physics, not new physics

Experiments use Fermi-motion model to incorporate m_X^{cut} effect [Earlier work: Ali & Hiller, '98]



Effects of m_X cut at lowest order

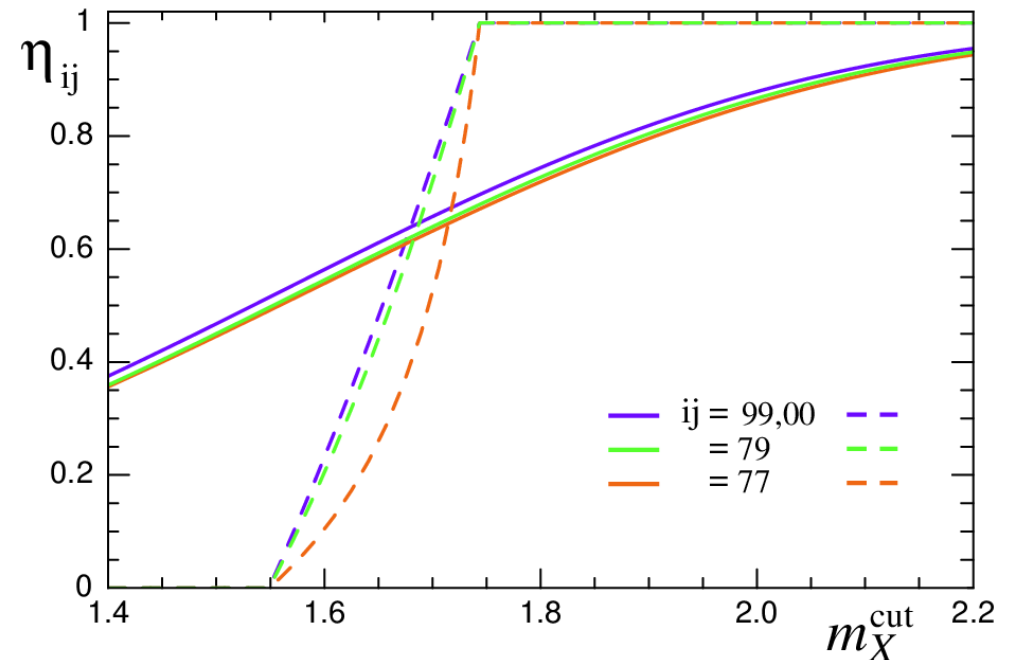
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ij : C_9^2 and C_{10}^2 , $C_7 C_9$, C_7^2 — different functionally for each contribution

Dashed: tree level in local OPE [wrong]

Solid: with a fixed shape function model



- Strong m_X^{cut} dependence: Raising it (if possible) would reduce uncertainty

If $1 - \eta$ is sizable, so is its uncertainty

- Approximate universality of η_{ij} : since shape function varies on scale $p_X^+/\Lambda_{\text{QCD}}$, while $\Gamma_{ij}^{\text{parton}}$ varies on scale $p_X^+/m_b \Rightarrow \eta \approx \eta_{ij}$



Perturbation theory for amplitude or rate?

- Usual power counting: expand $\langle s\ell^+\ell^-|\mathcal{H}|b\rangle$ in α_s , treating $\alpha_s \ln(m_W/m_b) = \mathcal{O}(1)$
OK in local OPE region: include small nonpert. corrections ($\lambda_{1,2}$, etc.) at the end
- Shape function region: only the rate is calculable, $\Gamma \sim \text{Im} \langle B|T\{O_i^\dagger(x)O_j(0)\}|B\rangle$
 $C_9(m_b) \sim \ln(m_W/m_b) \sim 1/\alpha_s$ “enhancement”, but $|C_9(m_b)| \sim C_{10}$
 - Need to take it seriously to cancel scheme- and scale-dependence in running
 - Don’t want power counting: $\langle B|O_9^\dagger O_9|B\rangle$ at $\mathcal{O}(\alpha_s^2) \sim \langle B|O_{10}^\dagger O_{10}|B\rangle$ at tree level

- “Split matching” in SCET: separate μ -dependence in matrix element which cancels that in $m_{\text{weak}} \rightarrow m_b$ running from dependencies on scales $\mu_i \sim \sqrt{m_b \Lambda_{\text{QCD}}}$ and $\mu_\Lambda \sim 1 \text{ GeV}$ — can work to different orders



Shape function modelling

- Decay rates: $d\Gamma \sim H(p_X^-, p_X^+, \mu_i) \int d\omega J(p_X^+ - \omega, \mu_i) S(\omega, \mu_i)$

Shape function: $S(\omega, \mu) = \langle B | \bar{b} \delta(i\mathcal{D}_+ + \omega) b | B \rangle \quad (i\mathcal{D} = iD - m_B v)$

- Generate perturbative tail of $S(\omega, \mu)$ from a model using partonic matrix element:

$$S(\omega, \mu) = \int_0^\infty dk_+ F_{\text{mod}}(k_+) S_{\text{part}}(\omega - k_+, \mu) \quad S_{\text{part}}(\omega, \mu) = \langle b | \bar{b} \delta(i\mathcal{D}_+ + \omega) b | b \rangle$$

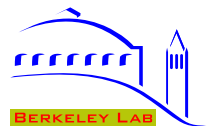
Optimal: large ω determined by perturbative tail, small ω by (unavoidable) model

– Modelling SF at μ_i or at a lower scale and running it up makes little difference

– Straightforward to include moment constraints: $\int_0^\infty dk_+ k_+^n F_{\text{mod}}(k_+) = \langle B | O_n | B \rangle$

Other applications: Extraction of $|V_{ub}|$ + see Andre's talk for top physics + etc...

Details to appear + see Frank's talk at Heidelberg V_{xb} workshop



Including NLL corrections

- Universality maintained; estimate shape function uncertainties using $B \rightarrow X_s \gamma$

- Find for $\mathcal{B}(1 < q^2 < 6 \text{ GeV}^2)/10^{-6}$

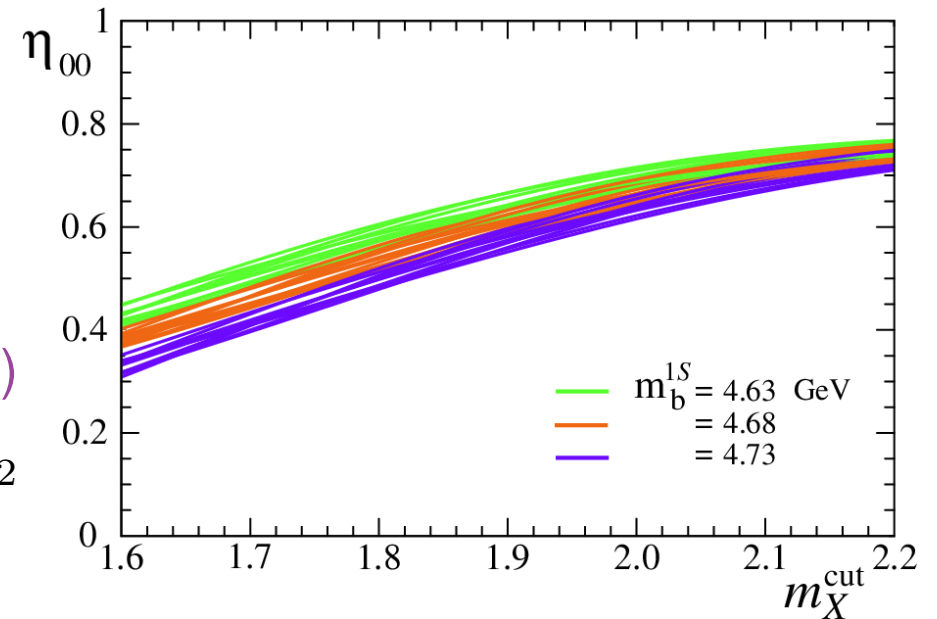
$$m_X^{\text{cut}} = 1.8 \text{ GeV}: 1.20 \pm 0.15$$

$$m_X^{\text{cut}} = 2.0 \text{ GeV}: 1.48 \pm 0.14$$

$$\text{NNLL, no } m_X \text{ cut}: 1.57 \pm 0.11$$

- A_{FB} only slightly affected (a-priori nontrivial)

- NNLL reduces μ dependence, effect on q^2 spectrum small \Rightarrow expect $\eta^{(\text{NLL})} \approx \eta^{(\text{NNLL})}$



- If increasing m_X^{cut} above 2 GeV hard \Rightarrow keep $m_X^{\text{cut}} < m_D$, normalize to $B \rightarrow X_u \ell \bar{\nu}$ with same cuts:

$$R = \Gamma^{\text{cut}}(B \rightarrow X_s \ell^+ \ell^-) / \Gamma^{\text{cut}}(B \rightarrow X_u \ell \bar{\nu})$$

Both shape function (m_X^{cut}) and m_b dependence drastically reduced



Is $C_7(m_b) = -C_7^{\text{SM}}(m_b)$ excluded?

- Inclusive:** rate in small q^2 region, in units of 10^{-6} (world average: 1.60 ± 0.51)

[Gambino, Haisch, Misiak, hep-ph/0410155]

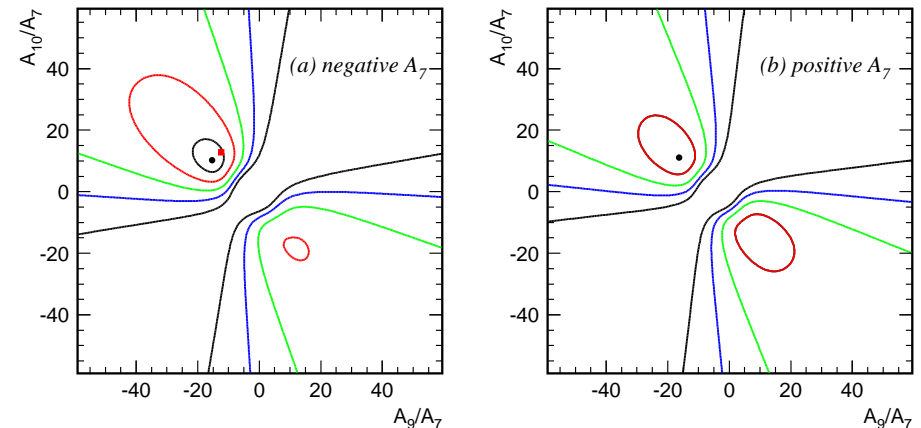
TABLE II: Predictions for $B(\bar{B} \rightarrow X_s l^+ l^-)$ [10^{-6}] in the Standard Model and with reversed sign of \tilde{C}_7^{eff} for the same ranges of q^2 as in Tab. I.

Range	SM	$\tilde{C}_7^{\text{eff}} \rightarrow -\tilde{C}_7^{\text{eff}}$
(a)	4.4 ± 0.7	8.8 ± 1.0
(b)	1.57 ± 0.16	3.30 ± 0.25

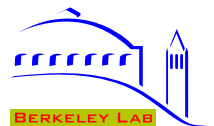
$\tilde{C}_7^{\text{eff}} \rightarrow -\tilde{C}_7^{\text{eff}}$ is not the best way to proceed

“Preliminary”	m_X^{cut}	rate (C_7^{SM})	rate ($C_7^{\text{non-SM}}$)
NNLL “GHM”	—	1.57	3.18
NNLL “us”	—	1.57	2.99
NLL	—	1.74	3.61
NLL	2.0 GeV	1.35	3.09
NLL	1.8 GeV	1.10	2.49

- Exclusive:** with some model dependence, Belle’s A_{FB} measurement fixes sign of C_9/C_{10} , but not sign of C_7 relative to $C_{9,10}$



- I also think $C_7 > 0$ is unlikely, but probably disfavored only about the 2σ level



Large q^2 region

ZL, F. Tackmann, arXiv:0707.1694

Large q^2 region: complementary with small q^2

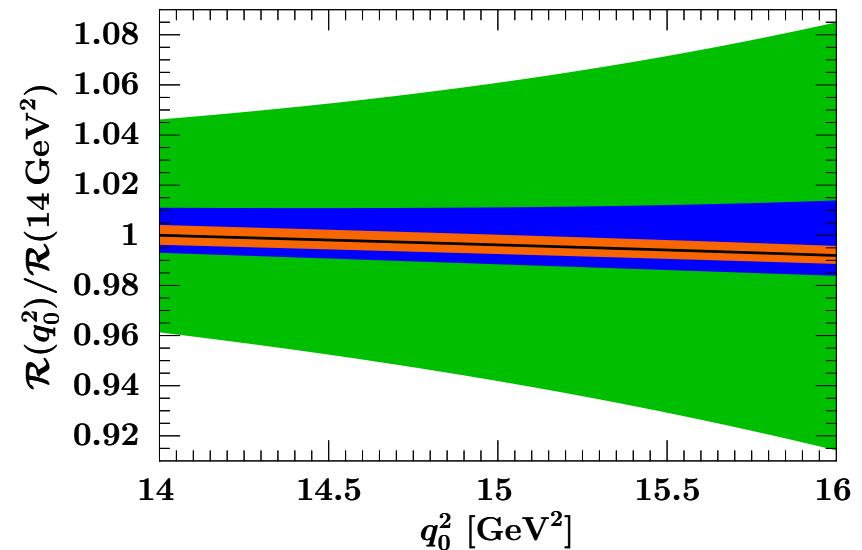
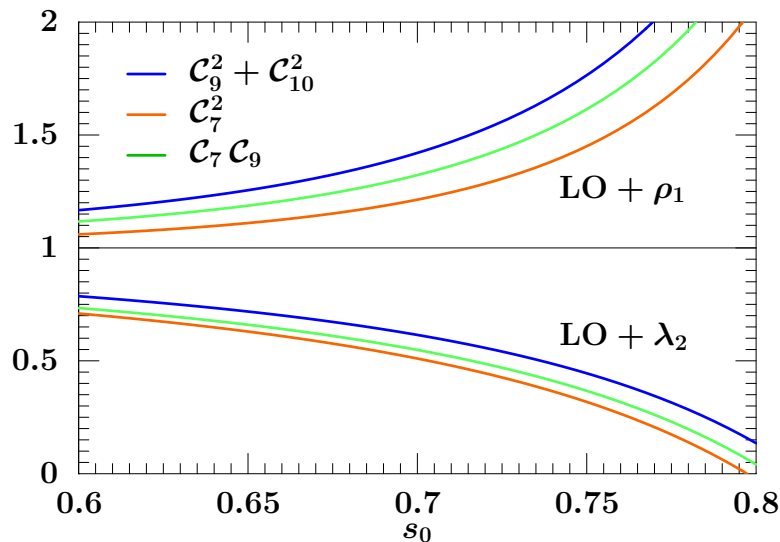
- Theory: largest errors (i) expansion in $\Lambda_{\text{QCD}}/(m_b - \sqrt{q^2})$; (ii) huge m_b dependence
Experiment: smaller rate, but higher efficiency

- Both can be reduced / eliminated \Rightarrow uncertainty $\sim 5\%$ (missing NNLL at large q^2)

$$\frac{\int_{q_0^2}^{m_B^2} \frac{d\Gamma(B \rightarrow X_s \ell^+ \ell^-)}{dq^2} dq^2}{\int_{q_0^2}^{m_B^2} \frac{d\Gamma(B^0 \rightarrow X_u \ell \bar{\nu})}{dq^2} dq^2} = \frac{|V_{tb}V_{ts}^*|^2}{|V_{ub}|^2} \frac{\alpha_{\text{em}}^2}{8\pi^2} \mathcal{R}(q_0^2)$$

uncertainties suppressed by:

$$1 - \frac{(\mathcal{C}_9 + 2\mathcal{C}_7)^2 + \mathcal{C}_{10}^2}{\mathcal{C}_9^2 + \mathcal{C}_{10}^2} \simeq 0.12$$



Large q^2 region measured in $B \rightarrow X_u \ell \bar{\nu}$

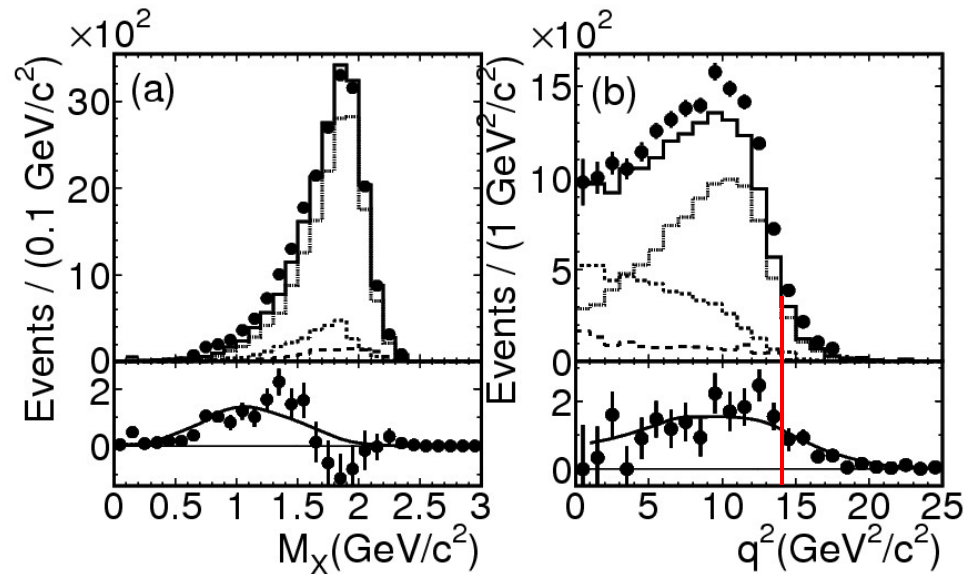
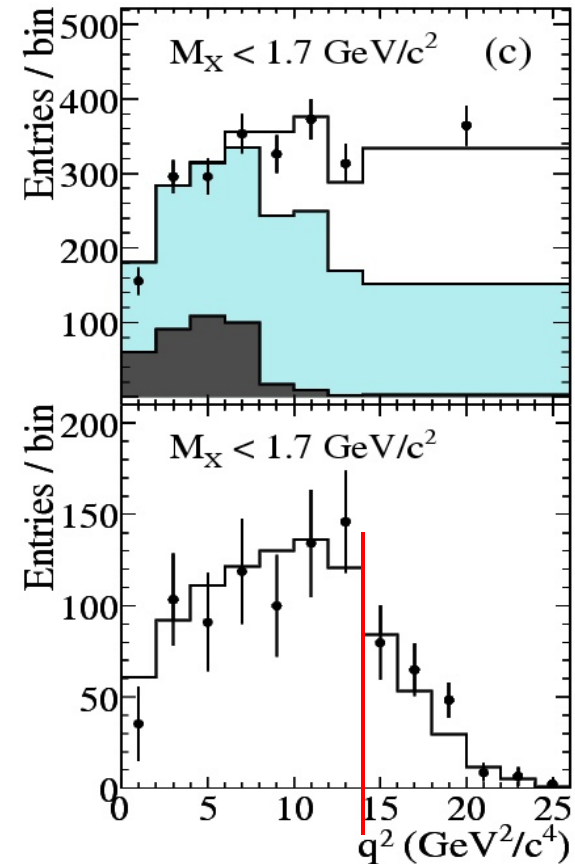


FIG. 4: (a) M_X distribution for $q^2 > 8.0 \text{ GeV}^2/c^2$. (b) q^2 distribution for $M_X < 1.7 \text{ GeV}/c^2$. Points are the data and histograms are backgrounds from $D^* \ell \nu$ (dotted), $D \ell \nu$ (short dashed), others (long dashed), and total background contribution (solid). Lower plots show the data after background subtraction. Solid curves show the inclusive MC predictions for $B \rightarrow X_u \ell \bar{\nu}$.

Belle, 87 fb^{-1} , PRL **92** (2004) 101801 [hep-ex/0311048]



BaBar, $383 \text{ m } \Upsilon$, arXiv:0708.3702

- The $m_X > 1.7 \text{ GeV}$ cut is irrelevant for $q^2 > 12.8 \text{ GeV}^2$ (up to resolution effects)
- Separating B^0 from B^\pm is important to control 4-quark operator contribution (WA)



Summary and conclusions

- To achieve optimal sensitivity to NP, separate terms: $1 + \cos^2 \theta$, $1 - \cos^2 \theta$, $\cos \theta$
Both in inclusive & exclusive; latter can be analyzed without form factor models
- Few integrated rates may give as good info as fit to 2-d distribution & zero of A_{FB}
- Small q^2 : measured rate depends on shape function (local OPE is not sufficient)
SF region: expansion for rate, not the amplitude, reorganize perturbation theory
- Model independent calculation of $q^2 < 6\text{GeV}^2$ rate with m_X^{cut} using new SF model
- Theoretical uncertainty at large q^2 can approach 5% by combining measurements
BaBar & Belle should quote $\mathcal{B}(B \rightarrow X_u \ell \bar{\nu})|_{q^2 > 14, 15\text{GeV}^2}$, try to separate B^0 / B^\pm
- SCET needed to take hadronic effects into account using $B \rightarrow X_s \gamma$ & $B \rightarrow X_u \ell \bar{\nu}$
Sensitivity to NP survive (can be improved) both in small and large q^2 regions





Backup slides

Aside: long distance effects

- A worry (at least, for me) that will be ignored in this talk:

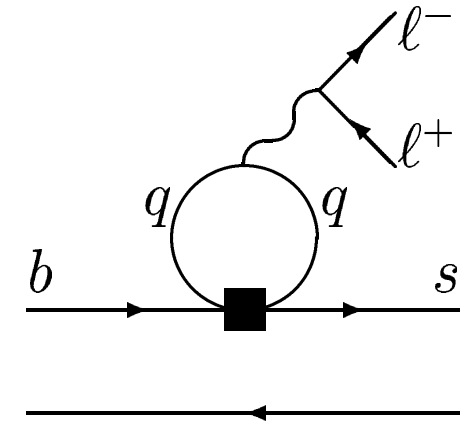
$$\mathcal{B}(B \rightarrow \psi X_s) \sim 4 \times 10^{-3}$$

↓

$$\mathcal{B}(\psi \rightarrow \ell^+ \ell^-) \sim 6 \times 10^{-2}$$

Combined rate: $\mathcal{B}(B \rightarrow X_s \ell^+ \ell^-) \sim 2 \times 10^{-4}$

This is ~ 30 times the short distance contribution!



- Averaged over a large region of q^2 , the $c\bar{c}$ loop expected to be dual to $\psi + \psi' + \dots$. This is what happens in $e^+e^- \rightarrow$ hadrons, in τ decay, etc., but NOT here
- Is it consistent to “cut out” the ψ and ψ' regions and then compare data with the short distance calculation? (Maybe..., but understanding is unsatisfactory)



“Split matching” and running below m_b

- Match H_w at $\mu_0 \sim m_b$ onto scale invariant operators

[Lee & Stewart]

$$C_9^{\text{mix}}(\mu_0) (\bar{s}_L \gamma^\mu b_L) (\bar{\ell} \gamma_\mu \ell) + C_{10} (\bar{s}_L \gamma^\mu b_L) (\bar{\ell} \gamma_\mu \gamma_5 \ell) - C_7^{\text{mix}}(\mu_0) \frac{2\bar{m}_b q_\nu}{q^2} (\bar{s}_L \sigma^{\mu\nu} b_R)_{\mu=m_b} (\bar{\ell} \gamma_\mu \ell)$$

- Match onto SCET at $\mu_b \sim m_b$ [$\mu_b = \mu_0$ in practice; common anom. dim. below m_b]
- Run down to $\mu_i \sim \sqrt{m_b \Lambda_{\text{QCD}}}$, calculate

$$d^3\Gamma^{(0)} = H \int dk J(k) f^{(0)}(k)$$

H and J perturbative, $f^{(0)}$ nonperturbative

- Take $f^{(0)}(k)$ from $B \rightarrow X_s \gamma$, or run model from μ_Λ to μ_i

[Bosch, Lange, Neubert, Paz]

(recall: Λ_{QCD}/m_b suppressed shape functions are non-universal)

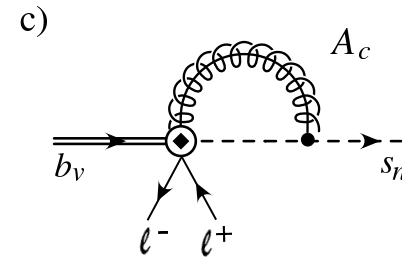
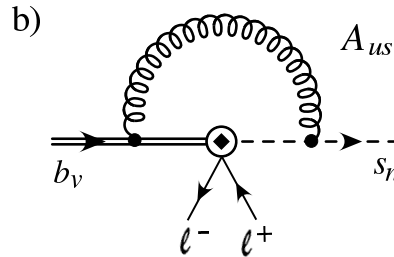
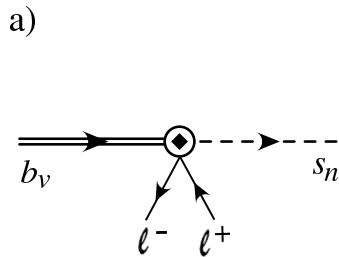
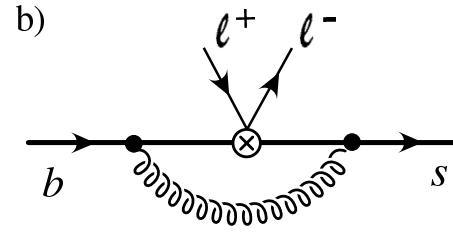
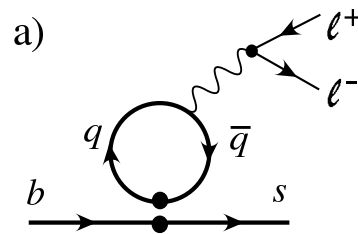
$$f^{(0)}(\hat{\omega}, \mu_i) = \frac{e^{V_S(\mu_i, \mu_\Lambda)}}{\Gamma(1 + \eta)} \left(\frac{\hat{\omega}}{\mu_\Lambda} \right)^\eta \int_0^1 dt f^{(0)} \left[\hat{\omega} (1 - t^{1/\eta}), \mu_\Lambda \right] \quad \eta = \frac{16}{25} \ln \frac{\alpha_s(\mu_\Lambda)}{\alpha_s(\mu_i)}$$



Matching onto SCET

- SCET operators: $J_{\ell\ell}^{(0)} = \sum_{i=a,b,c} C_{9i}(s) \left(\bar{\chi}_{n,p} \Gamma_i^\mu \mathcal{H}_v \right) (\bar{\ell} \gamma_\mu \ell) + \text{similar } C_{10,7} \text{ terms}$

$$\mathcal{H}_v = Y^\dagger h_v, \quad \chi_n = W^\dagger \xi_n, \quad \Gamma_{a-c}^\mu = P_R \left\{ \gamma^\mu, v^\mu, \frac{n^\mu}{n \cdot v} \right\}$$



SCET Wilson coefficients: $C_{9a} = \tilde{C}_9^{\text{eff}} [1 + \mathcal{O}(\alpha_s)]$ $C_{9b,c} = \mathcal{O}(\alpha_s)$

- Some parts of the “usual” NLL $\mathcal{O}(\alpha_s)$ corrections included in \tilde{C}_9^{eff} [Misiak; Buras, Munz] now contribute to the jet function, J , some others to the shape function, $f^{(0)}(k)$

