Unitarity method for one loop calculations

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Numerical unitarity formalism for evaluating one-loop amplitudes

R. K. Ellis, W. Giele, Z.K, arXiv.0708.2398

W. Giele, Z.K., K. Melnikov,arXiv0801.2237

Production of jets, heavy quarks and gauge bosons at the LHC: multi-leg processes

At LO

- **First estimates: leading order MC's based on Born amplitudes**
- **More quantitative estimates require NLO (QCD and EW) corrections**
- **If we use standard Feynman-diagram approach already LO calculations are problematic:**

  Stronger than factorial growth in number of external particles
  N-gluon scattering: CPU grows as $N^{(N-3)}$ (E-algorithm)

- **Solution: use recursion relations** (Berends, Giele; Britto, Cachazo, Feng) :
  CPU time has polynomial growth in the number of the external legs
  N^{α} (P- algorithm)
- **Tree-level general purpose softwares: ALPGEN, HELAC (P), MADGRAPH (E)**

At NLO, E-algorithms

- **Standard Feynman-graph based method**

  QGRAF , FORM , Tensor Reduction (Passarino,Veltman)

- **Many diagram, many terms from tensor reduction, simplifications in analytic treatments**

- **Semi-analytic, standard methods pushed to their limits**

  K. Ellis, Giele, Zanderighi **6g one-loop amplitudes**

  Denner Dittmaier $e^+e^- \to \mu^+\mu^- \tau^+\tau^-$ **QED NLO**

  Dittmaier, Uwer, Weinzierl **$p+p \to t+t$ jet NLO**

- **Completely numerical methods** avoid tensor reduction

  Nagy, Soper; Lazapoulos, Melnikov, Petriello

P-algorithm for loop amplitudes : Unitarity method

S-matrix theory : **two particle scattering amplitude is given in terms of its imaginary part** (Landau, ...1950's)

Perturbative gauge theories at NLO (Bern, Dixon, Dunbar, Kosower,1994) **:**

i) Decomposition of one-loop amplitudes in terms of finite number of well defined scalar integral functions
*(*Passarino, Veltman*)*

ii) imaginary part of one-loop amplitudes is given in terms of products gauge invariant tree amplitudes

Early applications (BDK, 1994-1998)**:**

i) cut lines are in 4 dimension:

- **--** a number of clever tricks, analytic treatments (magic vs. automation)
- **--** impressive results for SYM but also for QCD

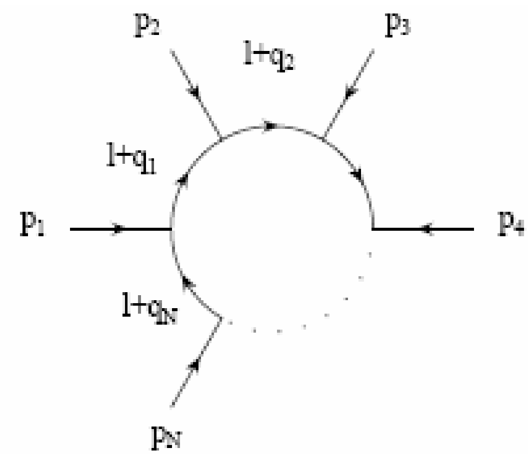
ii) Rational parts and unitarity in D-dimension ?
(van Neerven, Bern Morgan)

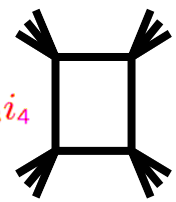
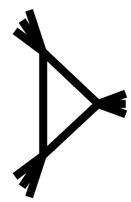
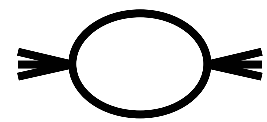
One-loop N-point amplitudes in terms of master integrals

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N) = \int [d\,l]\ \mathcal{A}(p_1,p_2,\ldots,p_N;l)$$

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N;l) = \frac{\mathcal{N}(p_1,p_2,\ldots,p_N;l)}{d_1 d_2 \cdots d_N}$$

$$d_i = (l+q_i)^2 - m_i^2 = (l - q_0 + \sum_{j=1}^{i} p_i)^2 - m_i^2$$



$$\mathcal{A}_N(\{p_i\}) = \sum d_{i_1 i_2 i_3 i_4}$$  $$+ \sum c_{i_1 i_2 i_3}$$  $$+ \sum b_{i_1 i_2}$$ 

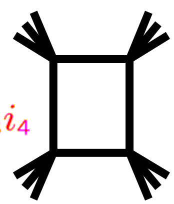
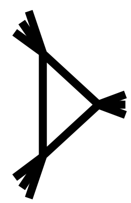
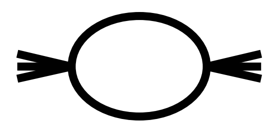
+ Tadpoles + Rational part

- Calculate the discontinuites of the left-hand side using unitarity
- Calculate the discontinuites of the right-hand side and read out the coefficients

Decomposing one-loop N-point amplitudes in terms of master integrals (cont.)

$$\mathcal{A}_N(p_1,p_2,\ldots,p_N) = \sum_{1\le i_1<i_2<i_3<i_4\le N} d_{i_1i_2i_3i_4}(p_1,p_2,\ldots,p_N)I_{i_1i_2i_3i_4}$$
$$+ \sum_{1\le i_1<i_2<i_3\le N} c_{i_1i_2i_3}(p_1,p_2,\ldots,p_N)I_{i_1i_2i_3}$$
$$+ \sum_{1\le i_1<i_2\le N} b_{i_1i_2}(p_1,p_2,\ldots,p_N)I_{i_1i_2}$$
$$+ \sum_{1\le i_1\le N} a_{i_1}(p_1,p_2,\ldots,p_N)I_{i_1}$$

$$I_{i_1\cdots i_M} = \int [d\,l]\,\frac{1}{d_{i_1}\cdots d_{i_M}}$$

$$\mathcal{A}_N(\{p_i\}) = \sum d_{i_1i_2i_3i_4}$$

$$+\sum c_{i_1i_2i_3}$$

$$+\sum b_{i_1i_2}$$


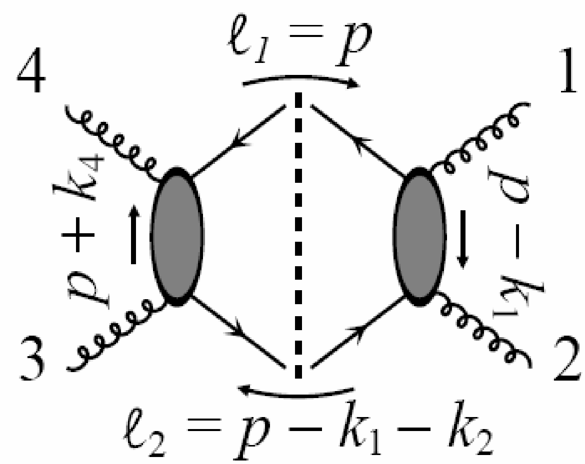
FDHS scheme: coefficients d and c are independent from ε

Rational part is generated by the order ε part of b_{ij}

Unitarity based on-shell method

$$T^\dagger - T = -2iT^\dagger T$$

Im $T^{\text{1-loop}} = \sum c_j$ Im I_j



$$-i\,\mathrm{Disc}\; A_4(1,2,3,4)\Big|_{s-\mathrm{cut}} = \int \frac{d^4p}{(2\pi)^4}\, 2\pi\delta^{(+)}(\ell_1^2 - m^2)\, 2\pi\delta^{(+)}(\ell_2^2 - m^2) \times A_4^{\mathrm{tree}}(-\ell_1, 1, 2, \ell_2)\, A_4^{\mathrm{tree}}(-\ell_2, 3, 4, \ell_1)\,,$$

Most important QCD application: $e^+ + e^-$ annihilation to four jets in NLO

New ideas inspired by twistor formulation

Twistors for string theory and gauge theory: analytic continuation of the amplitude to complex values in the external momenta

Witten (2004); Cachazo, Witten (2004)

i) New on-shell recursion relations for tree amplitudes

Britto, Feng, Cachaco; Britto, Feng, Cachazo, Witten; .

ii) Generalized unitarity Bern Dixon,Kosower, Britto, Cachazo, Feng

quadruple cuts, triple cuts, unitarity conditions are solved by complex on-shell four momenta, suitable for numerical implementation

New ideas for reduction, numerical implementations

- **Reduction of tensor integrals using spinorial integration**
  Britto Cachazo Feng; +Mastrolia ('05)
- **Use recursion relations to calculate the rational part**
  Bern,Dixon, Kosower,+Berger,Forde ('05)
- **Algebraic reduction at the integrand level (parametric integration)**
  Ossala Papadopoulos Pittau ('06)
- **Spinorial integration reduction in D-dimension**
  Anastasiou, Britto, Feng, Z.K., Mastrolia ('06)
- **Automated numerical implementations (all with OPP reduction)**

  -- unitarity cut in 4dim Ossala Papapdopoulos, Pittau; Ellis, Giele, ZK ('07)

  -- unitarity cut in D-dim Giele, ZK, Melnikov ('08)

  -- unitarty cut in 4dim, recursion relation for rational part black hat collaboration ('08)

Parametric integration, disentangling the residues in four dimension

Ossola, Papadopoulos, Pittau: there is a systematic way of algebraic reduction at the integrand level. The numerator can be decomposed as linear combination of 4-,3-,2,-1 denominator factors

EGK follow OPP but use the van Neerven Vermaseren basis and multi-pole expansion of rational functions

$$\mathcal{A}_N(p_1, p_2, \ldots, p_N; l) = \frac{\mathcal{N}(p_1, p_2, \ldots, p_N; l)}{d_1 d_2 \cdots d_N} =$$

$$\sum_{1 \le i_1 < i_2 < i_3 < i_4 \le N} \frac{\overline{d}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} + \sum_{1 \le i_1 < i_2 < i_3 \le N} \frac{\overline{c}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{1 \le i_1 < i_2 \le N} \frac{\overline{b}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{1 \le i_1 \le N} \frac{\overline{a}_{i_1}(l)}{d_{i_1}}$$

Denominator factors : $d_i = (l + q_i)^2 - m_i^2$

The residues can be decomposed into finite number of Lorenz-structure in the loop momenta

Parametrizing the loop momenta

The loop momenta can be decomposed in terms of suitable fixed basis vectors.

We follow OPP but use the van Neerven Vermaseren basis and multi-pole expansion of rational functions .

For box, triangle and bubble integrals split the 4 dimensional space-time to "physical" space and "trivial" space;

physical space is spanned by: **dual momenta v_i** **$p_i v_j = \delta_{ij}$**
trivial space is spanned by: **unit vectors n_i**

Decomposition of the loop-momentum

$$l^\mu = V_R^\mu + \sum_{i=1}^{D_P} \frac{1}{2}(d_i - d_{i-1})\, v_i^\mu + \sum_{i=1}^{D_T} \alpha_i\, n_i^\mu \,,$$

$$V_R^\mu = -\frac{1}{2} \sum_{i=1}^{D_P} \left((q_i^2 - m_i^2) - (q_{i-1}^2 - m_{i-1}^2) \right) v_i^\mu$$

Solving the unitarity conditions

Contributions with four cut propagators $d_i=d_j=d_k=d_l=0$ two solutions

$$l^\mu = V_4^\mu + \alpha_1\, n_1^\mu$$

$$l_\pm^\mu = V_4^\mu \pm i\,\sqrt{V_4^2 - m_l^2} \times n_1^\mu$$

Triangle, infinite # of solutions (on a circle)

$$l^\mu = V_3^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu$$

$$l_{\alpha_1\alpha_2}^\mu = V_3^\mu + \alpha_1\, n_1^\mu + \alpha_2\, n_2^\mu;\;\; \alpha_1^2 + \alpha_2^2 = -(V_3^2 - m_k^2)$$

Bubble, infinite # of solutions (on a “sphere”)

$$l^\mu = V_2^\mu + \alpha_1 n_1^\mu + \alpha_2 n_2^\mu + \alpha_3 n_3^\mu$$

$$l_{\alpha_1\alpha_2\alpha_3}^\mu = V_2^\mu + \alpha_1\, n_1^\mu + \alpha_2\, n_2^\mu + \alpha_3\, n_3^\mu;\;\; \alpha_1^2 + \alpha_2^2 + \alpha_3^2 = -(V_2^2 - m_j^2)\ .$$

l-dependence of the residues in 4 dimension

Box residue, 2 structures:

$$\bar{d}_{ijkl}(l) \equiv \bar{d}_{ijkl}(n_1 \cdot l)\,, \quad (n_1 \cdot l)^2 \sim n_1^2 = 1\,, \quad \bar{d}_{ijkl}(l) = d_{ijkl} + \tilde{d}_{ijkl}\, l \cdot n_1$$

Triangle residues, 7 structures:

$$\bar{c}_{ijk}(l) = c_{ijk}^{(0)} + c_{ijk}^{(1)} s_1 + c_{ijk}^{(2)} s_2 + c_{ijk}^{(3)} (s_1^2 - s_2^2) + s_1 s_2 (c_{ijk}^{(4)} + c_{ijk}^{(5)} s_1 + c_{ijk}^{(6)} s_2)$$

Bubble residues, 9 structures

$$\bar{b}_{ij}(l) = b_{ij}^{(0)} + b_{ij}^{(1)} s_1 + b_{ij}^{(2)} s_2 + b_{ij}^{(3)} s_3 + \ldots$$

Non-vanishing contributions come from d_{ijkl},c_{ijk}^{(0)} and b_{ij}^{(0)}.
The other terms are called spurious.

The residues of the poles can be obtained algebraically.

The residue is taken at special loop momentum defined by the unitarity conditions.

$$\mathrm{Res}_{ij\cdots k}\left[F(l)\right] \equiv \left[d_i(l)d_j(l)\cdots d_k(l)F(l)\right]\Big|_{l=l_{ij\cdots k}} .$$

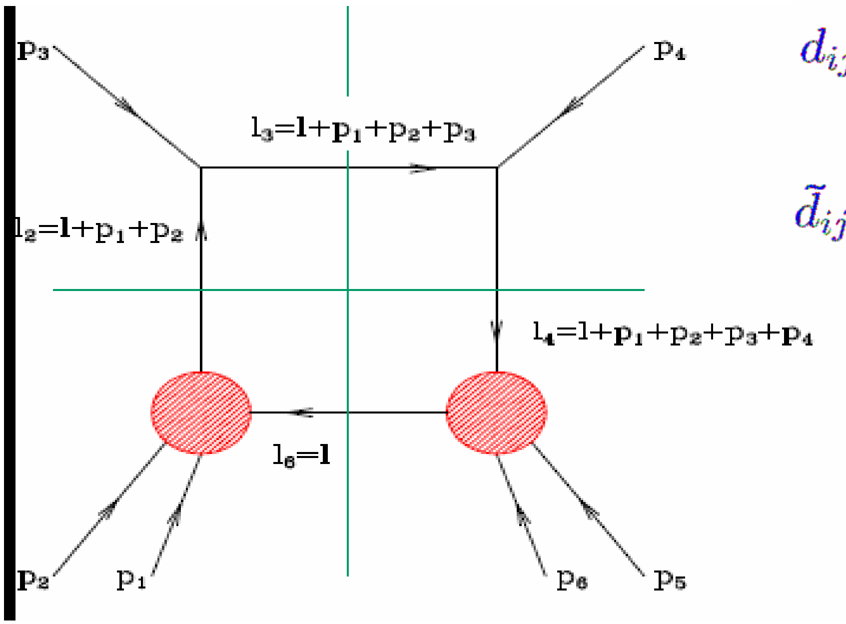
$$\bar{d}_{ijkl}(l) = \mathrm{Res}_{ijkl}\left(\mathcal{A}_N(l)\right)$$ $d_i=d_j=d_k=d_l=0$

$$\bar{c}_{ijk}(l) = \mathrm{Res}_{ijk}\left(\mathcal{A}_N(l) - \sum_{l\neq i,j,k}\frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$ $d_i=d_j=d_k=0$

$$\bar{b}_{ij}(l) = \mathrm{Res}_{ij}\left(\mathcal{A}_N(l) - \sum_{k\neq i,j}\frac{\bar{c}_{ijk}(l)}{d_i d_j d_k} - \frac{1}{2!}\sum_{k,l\neq i,j}\frac{\bar{d}_{ijkl}(l)}{d_i d_j d_k d_l}\right)$$ $d_i=d_j=0$

Calculating the box residue

$$\bar{d}_{ijkl}(l) \equiv \bar{d}_{ijkl}(n_1 \cdot l)\,, \quad (n_1 \cdot l)^2 \sim n_1^2 = 1\,, \quad \bar{d}_{ijkl}(l) = d_{ijkl} + \tilde{d}_{ijkl}\, l \cdot n_1$$



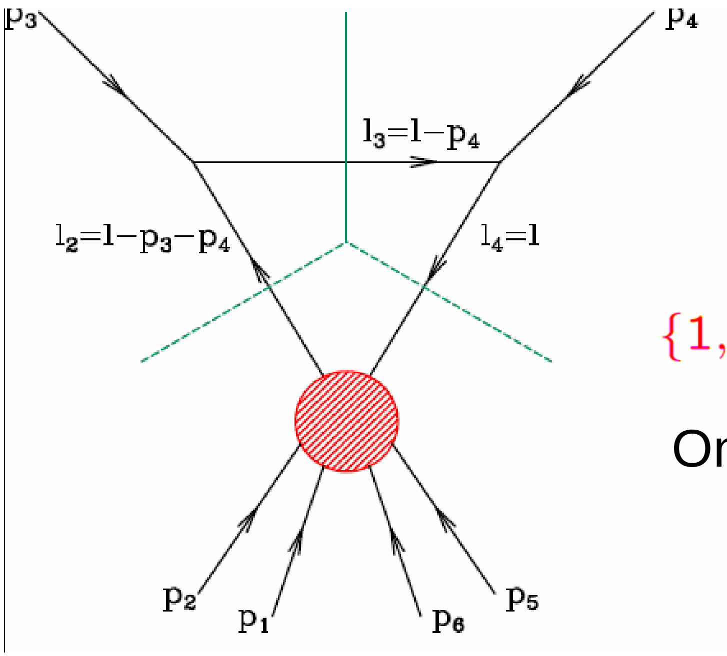
$$d_{ijkl} = \frac{\text{Res}_{ijkl}\big(\mathcal{A}_N(l^+)\big) + \text{Cut}_{ijkl}\big(\mathcal{A}_N(l^-)\big)}{2}$$

$$\tilde{d}_{ijkl} = \frac{\text{Res}_{ijkl}\big(\mathcal{A}_N(l^+)\big) - \text{Res}_{ijkl}\big(\mathcal{A}_N(l^-)\big)}{2i\sqrt{V_4^2 - m_l^2}}$$

the residue of the amplitude factorizes to the product of tree amplitudes

$$\text{Rest}_{2346}\big(\mathcal{A}_6(l^\pm)\big) = \mathcal{M}_4^{(0)}(l_6^\pm; p_1, p_2; -l_2^\pm) \times \mathcal{M}_3^{(0)}(l_2^\pm; p_3; -l_3^\pm)\mathcal{M}_3^{(0)}(l_3^\pm; p_4; -l_4^\pm)$$
$$\times \mathcal{M}_4^{(0)}(l_4^\pm; p_5, p_6; -l_6^\pm)$$

Calculating the triangle residue



l-dependence of the triangle residue

$$\bar{c}_{ijk}(l) \equiv \bar{c}_{ijk}(s_1, s_2);\;\; s_1 = n_1 \cdot l,\, s_2 = n_2 \cdot l$$

Maximum rank: 3 -> 7 possible terms

$$\{1, s_1, s_2, s_1^2, s_1 s_2, s_2^2, s_1^3, s_1^2 s_2, s_1 s_2^2, s_2^3\}$$

One constraint : $s_1^2 + s_2^2 \sim n_1^2 + n_2^2 = 2$

$C^{(0)}$ + six spurious terms

$$\bar{c}_{ijk}(l) = c_{ijk}^{(0)} + c_{ijk}^{(1)} s_1 + c_{ijk}^{(2)} s_2 + c_{ijk}^{(3)}(s_1^2 - s_2^2) + s_1 s_2 (c_{ijk}^{(4)} + c_{ijk}^{(5)} s_1 + c_{ijk}^{(6)} s_2)$$

the residue of the amplitude factorizes to the product of tree amplitudes

$$\mathrm{Res}_{234}\left(\mathcal{A}_6(l^{\alpha_1\alpha_2})\right) = \mathcal{M}_3^{(0)}(l_2^{\alpha_1\alpha_2}; p_3; -l_3^{\alpha_1\alpha_2}) \times \mathcal{M}_3^{(0)}(l_3^{\alpha_1\alpha_2}; p_4; -l_4^{\alpha_1\alpha_2}) \times \mathcal{M}_6^{(0)}(l_4^{\alpha_1\alpha_2}; p_5, p_6, p_1, p_2; -l_2^{\alpha_1\alpha_2})$$

New features in D-dimensional space-time

Virtual particles live in D-dimensional space-time

Two sources of D-dependence

i) spin-polarization states live in D_s .
ii) loop momentum component live in D.

$$\mathcal{A}_{(D,D_s)}(\{p_i\},\{J_i\}) = \int \frac{d^D l}{i(\pi)^{D/2}} \frac{\mathcal{N}^{(D_s)}(\{p_i\},\{J_i\};l)}{d_1 d_2 \cdots d_N}.$$

$$\sum_{i=1}^{D_s-2} e_\mu^{(i)}(l) e_\nu^{(i)}(l) = -g_{\mu\nu}^{(D_s)} + \frac{l_\mu b_\nu + b_\mu l_\nu}{l \cdot b},$$

$$l^2 = \bar{l}^2 - \tilde{l}^2 = l_1^2 - l_2^2 - l_3^2 - l_4^2 - \sum_{i=5}^{D} l_i^2$$

The dependence of amplitude on D_s is linear

Trivial parametrization of D_s dependence :

we need to choose two values $D_s = D_1$ and $D_s = D_2$

$$\mathcal{N}^{(D_s)}(l) = \mathcal{N}_0(l) + (D_s - 4)\mathcal{N}_1(l)$$

$$\mathcal{N}_0(l) = \frac{(D_2 - 4)\mathcal{N}^{(D_1)}(l) - (D_1 - 4)\mathcal{N}^{(D_2)}(l)}{D_2 - D_1}$$

$$\mathcal{N}_1(l) = \frac{\mathcal{N}^{(D_1)}(l) - \mathcal{N}^{(D_2)}(l)}{D_2 - D_1}.$$

We can interpolate D_s either $\rightarrow$ 4 or and $D_s \rightarrow$ 4-2ε

t' Hooft-Veltam scheme four dim.hel. Scheme (FDH)

Numerical implementation is straightforward

Fit the D_s dependence

The amplitudes in the FDH scheme :

$$\mathcal{A}^{\mathrm{FDH}} = \left(\frac{D_2 - 4}{D_2 - D_1}\right) \mathcal{A}_{(D,D_s=D_1)} - \left(\frac{D_1 - 4}{D_2 - D_1}\right) \mathcal{A}_{(D,D_s=D_2)}$$

Each cut imposes one constraint, d_i=0.
In D_s=4 at most four-point integral function is needed.

How many master integrals in D_s>4 dimensions?

$$s_e^2 = -\sum_{i=5}^{D} (l \cdot n_i)^2 = -\sum_{i=5}^{D} (\tilde{l} \cdot n_i)^2,$$

Only one extra momentum component. It enters quadratically.

Reduction in D-dimensions

The parametrization of the N-particle amplitude

$$\frac{\mathcal{N}^{(D_s)}(l)}{d_1 d_2 \cdots d_N} = \sum_{[i_1|i_5]} \frac{\bar{e}^{(D_s)}_{i_1 i_2 i_3 i_4 i_5}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4} d_{i_5}} + \sum_{[i_1|i_4]} \frac{\bar{d}^{(D_s)}_{i_1 i_2 i_3 i_4}(l)}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}}$$

$$+ \sum_{[i_1|i_3]} \frac{\bar{c}^{(D_s)}_{i_1 i_2 i_3}(l)}{d_{i_1} d_{i_2} d_{i_3}} + \sum_{[i_1|i_2]} \frac{\bar{b}^{(D_s)}_{i_1 i_2}(l)}{d_{i_1} d_{i_2}} + \sum_{[i_1|i_1]} \frac{\bar{a}^{(D_s)}_{i_1}(l)}{d_{i_1}}$$

Parametrization of the residues

Pentuple residue: $\bar{e}^{(D_s)}_{ijkmn}(l) = e^{(D_s,(0))}_{ijkmn}$

Box residue: $\bar{d}^{\mathrm{FDH}}_{ijkn}(l) = d^{(0)}_{ijkn} + d^{(1)}_{ijkn} s_1 + (d^{(2)}_{ijkn} + d^{(3)}_{ijkn} s_1) s_e^2 + d^{(4)}_{ijkn} s_e^4$

Three extra structures for triple, three for double and zero for single cuts

Four new master integrals

Four of the s_e^2 dependent master integrals are not spurious

$$\int \frac{\mathrm{d}^D l}{(i\pi)^{D/2}} \frac{s_e^2}{d_{i_1} d_{i_2} d_{i_3} d_{i_4}} = -\frac{D-4}{2} I^{D+2}_{i_1 i_2 i_3 i_4}, \quad \ldots.$$

We obtain the full D-dependence of the amplitude

$$\mathcal{A}_{(D)} = \sum_{[i_1|i_5]} e^{(0)}_{i_1 i_2 i_3 i_4 i_5} \; I^{(D)}_{i_1 i_2 i_3 i_4 i_5}$$

$$+ \sum_{[i_1|i_4]} \left(d^{(0)}_{i_1 i_2 i_3 i_4} \; I^{(D)}_{i_1 i_2 i_3 i_4} - \frac{D-4}{2} d^{(2)}_{i_1 i_2 i_3 i_4} \; I^{(D+2)}_{i_1 i_2 i_3 i_4} + \frac{(D-4)(D-2)}{4} d^{(4)}_{i_1 i_2 i_3 i_4} \; I^{(D+4)}_{i_1 i_2 i_3 i_4} \right)$$

+ similar terms for triangle, bubble and tadpole contributions.

As ε -> 0 the new master integrals can be decomposed in the old basis and generate ε dependent bubble coefficients !

One-loop amplitudes up to terms of order ε

One loop amplitudes as sum of cut-constructible and rational parts:

$$\mathcal{A}_N = \mathcal{A}_N^{CC} + R_N .$$

The cut constructible part is as before (EGK):

$$\mathcal{A}_N^{CC} = \sum_{[i_1|i_4]} \tilde{d}^{(0)}_{i_1 i_2 i_3 i_4} \, I^{(4-2\epsilon)}_{i_1 i_2 i_3 i_4} + \sum_{[i_1|i_3]} c^{(0)}_{i_1 i_2 i_3} \, I^{(4-2\epsilon)}_{i_1 i_2 i_3} + \sum_{[i_1|i_2]} b^{(0)}_{i_1 i_2} \, I^{(4-2\epsilon)}_{i_1 i_2} + \sum_{i_1=1}^{N} a^{(0)}_{i_1} \, I^{(4-2\epsilon)}_{i_1} ,$$

The rational part is new (GKM):

$$R_N = - \sum_{[i_1|i_4]} \frac{d^{(4)}_{i_1 i_2 i_3 i_4}}{3} - \sum_{[i_1|i_3]} \frac{c^{(9)}_{i_1 i_2 i_3}}{2} - \sum_{[i_1|i_2]} \left(\frac{(q_{i_1} - q_{i_2})^2}{6} - \frac{m_{i_1}^2 + m_{i_2}^2}{2} \right) b^{(9)}_{i_1 i_2} .$$

4g,5g,6g scattering amplitudes in QCD

- **One type of color ordered subamplitude for each helicity.**
- **We choose D_1=5 and D_2=6.**

$$\mathcal{A}^{\mathrm{FDH}} = 2\mathcal{A}_{(D,D_s=5)} - \mathcal{A}_{(D,D_s=6)}$$

- **For the computation of the residues we have to consider four and five dimensional loop-momenta on the cuts, embedded into five- and six-dimensional space-time and fulfilling the unitarity constraints;**
- **we do not use supersymmetry;**
- **tree amplitudes are calculated with Berends-Giele recursion relations in D_s=5 and D_s=6 dimensions;**
- **Individual coefficients have been obtained by projection (sum over specially chosen loop momenta on the cut).**

Numerical Implementation

Check the singular parts:

$$m^{(1)}(1,2,\ldots,n) \sim \left(-\frac{n}{\epsilon^2} + \frac{1}{\epsilon} \left(-\frac{11}{3} + \sum_{i=1}^{n} \log\left(\frac{s_{i,i+1}}{\mu^2}\right) \right) \right) \times m^{(0)}(1,2,\ldots,n) + \mathcal{O}(1)$$

$$m^{(1)}(1,2,\ldots,n) \sim -\frac{n}{\epsilon^2} \times m^{(0)}(1,2,\ldots,n) + \mathcal{O}\left(\epsilon^{-1}\right) \ .$$

Compare with known analytic and numerical results.

Compare CPU time with those of the traditional method in case of 6g, 5g ,… amplitudes

Numerical Implementation (cont.)

i) known analytic results (Bern,Kosower, Britto; Feng, Mastrolia)
ii) known semi-numerical results (IBP) (Ellis, Giele, Zanderighi)

$\lambda_1, \lambda_2, \ldots, \lambda_6$	$\Delta^{\rm cut}$	$\Delta^{\rm rat}$	Δ
- - + + + +	-19.481065+78.147162 i	28.508591-74.507275 i	9.027526+3.639887 i
- + - + + +	-241.10930+27.176200 i	250.27357-25.695269 i	9.164272+1.480930 i
- + + - + +	5.4801516-12.433657 i	0.19703574+0.25452928 i	5.677187-12.179127 i
- - - + + +	15.478408-2.7380153 i	2.2486654+1.0766607 i	17.727073-1.661354 i
- - + - + +	-339.15056-328.58047 i	348.65907+336.44983 i	9.508509+7.869351 i
- + - + - +	31.947346+507.44665 i	-17.430910-510.42171 i	14.516436-2.975062 i

TABLE III: Finite parts of singular six-gluon scattering amplitudes for various gluon helicities.

Numerical Implementation (cont.)

Comparison of CPU times

100000 points are generated away from soft and collinear region.
Cuts on transverse momenta, rapidity and separation of the outgoing gluons

EGZ: 9s per ordered amplitude on 2.8GHz Pentium processor
EGK: 0.01s per ordered amplitude on 2.8GHz Pentium processor

	ev.time	# of cuts
4 gluon:	0.0009s	6
5 gluon:	0.0035s	20
6 gluon :	0.0107s	44

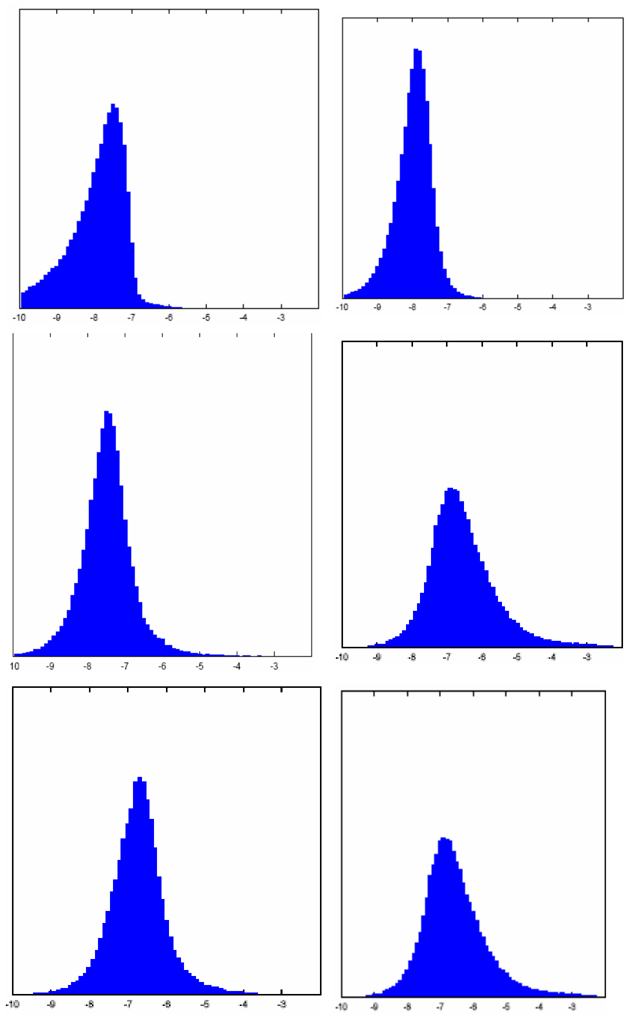
Computer time: scales with $\approx$ n^4 (# of cuts) not as n!

Concluding remarks

- **We have a novel method for calculating the full one-loop scattering amplitudes including the rational parts.**
- **The method is based on unitarity cuts in higher-dimensional space-time.**
- **Similar to four-dimensional unitarity, one loop amplitudes are obtained from tree amplitudes, calculated using efficient recursive algorithm.**
- **The algorithm has polynomial complexity and suitable for efficient numerical evaluation one-loop amplitudes**
- **Important step towards automated computation of NLO cross-section.**
- **Applicable to multiparticle processes that involve virtual particles of spin 0,1/2,1 and arbitrary masses.**
- **One can realistically try the NLO calculations for such complicated process as PP $\rightarrow$ tt + 2,3 jets and PP $\rightarrow$ V + 3,4,5 jets.**

Relative errors for 100000 ordered amplitudes

4-gluon 6-gluon



Horizontal axis: $S = \log_{10}\left(\left|\frac{m^{(1)}_{\text{unitarity}} - m^{(1)}_{\text{analytic}}}{m^{(1)}_{\text{analytic}}}\right|\right)$.

Range of S: (-10, -2)

Vertical axis: number of events

Majority of events agree with rel. precision 10^{-6} or better

Numerical instabilities

1) Matrix inversion needed for the calculation of triangle and bubble spurious contributions has numerical instabilities.

   Possible improvements: χ^2 fit to a large number of points in the space of solutions of the unitarity constraints
   Forde's method?

2) Presence of Gram-determinants in the box, triangle and bubble coefficients.
   In the solution of the unitarity constraints we have maximum power: -2

3) Decomposition to the given integrals basis may become degenerate
   Change the integration basis?

4g,5g,6g scattering amplitudes in QCD

- **We obtain a tirivial derivation of the known result**

$$\mathcal{A}^{\mathrm{HV}} = \mathcal{A}^{\mathrm{FDH}} - \frac{c_\Gamma}{3}\mathcal{A}_{\mathrm{tree}} \; .$$

- **Polarization vectors in five and six dimension for loop momenta embedded in four and five dimensions.**