Electroweak Sudakov Corrections to Scattering Processes

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Based on:

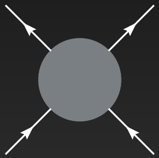
J.C., F. Golf, R. Kelley, A. Manohar, PRL 100 (2008) 021802

J.C., F. Golf, R. Kelley, A. Manohar, PRD 77 (2008) 053004

J.C., R. Kelley, A. Manohar in preparation

- $2 \to 2$ scattering – one of the dominant high-energy processes at LHC.
- The EFT operators become 4 particle operators.
- Shown that we can use previous results, by summing over pairs of particles, with appropriate group theory factors.

- Can be used for:
  - quark production: $q\bar{q} \to q\bar{q}$ or $q\bar{q} \to t\bar{t}$
  - squark production: $q\bar{q} \to \tilde{t}\tilde{t}^*$
  - gauge boson production: $q\bar{q} \to ZZ, WW, \cdots$



SU(2) Toy Theory

$$\alpha = \alpha_2 = \frac{\alpha_{em}}{\sin^2 \theta_W}$$

Full Theory

$C(Q, \mu)$ ——————— $\mu = Q$

SCET w/ M=0

↓ γ_{SCET}

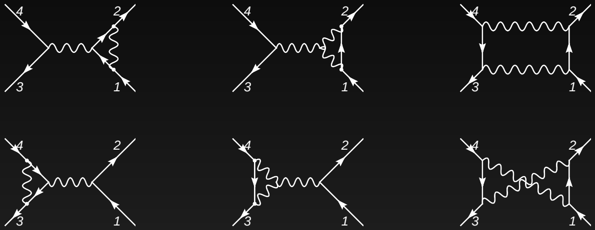
SCET w/ mass M

$D(M, \mu)$ ——————— $\mu = M$

SCET w/o gauge boson

4-particle full theory

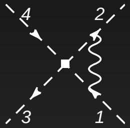
- Full Theory: $\mathcal{O}_{full} = \left[\bar{\psi}\gamma^\mu t^a \psi\right]\left[\bar{\psi}\gamma^\mu t^a \psi\right]$
- One-loop diagrams:



$$\begin{aligned} \mathbf{t^a t^a} &= C_F\,\mathbf{1}; \quad \mathbf{t^a t^b t^a} = \left(C_F - \frac{1}{2}C_A\right)\mathbf{t^b} \\ \mathbf{t^a t^b} \otimes \mathbf{t^a t^b} &= C_1\,\mathbf{1}\otimes\mathbf{1} + \frac{1}{4}\left(C_d - \frac{1}{2}C_A\right)\mathbf{t^a}\otimes\mathbf{t^a} \\ \mathbf{t^a t^b} \otimes \mathbf{t^b t^a} &= C_1\,\mathbf{1}\otimes\mathbf{1} + \frac{1}{4}\left(C_d + \frac{1}{2}C_A\right)\mathbf{t^a}\otimes\mathbf{t^a} \end{aligned}$$

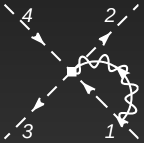
4-particle SCET

- $$\mathcal{O}_{SCET} = C_1 \left[[\bar{\xi}_{n,p_2} W_n] \gamma^\mu t^a [W_{\bar{n}}^\dagger \xi_{\bar{n},p_1}] \right] \left[[\bar{\xi}_{n,p_4} W_n] \gamma^\mu t^a [W_{\bar{n}}^\dagger \xi_{\bar{n},p_3}] \right]$$
  $$+ C_2 \left[[\bar{\xi}_{n,p_2} W_n] \gamma^\mu [W_{\bar{n}}^\dagger \xi_{\bar{n},p_1}] \right] \left[[\bar{\xi}_{n,p_4} W_n] \gamma^\mu [W_{\bar{n}}^\dagger \xi_{\bar{n},p_3}] \right]$$
  $$= C_1 \, [\mathbf{t^a} \otimes \mathbf{t^a}]_{SCET} + C_2 \, [\mathbf{1} \otimes \mathbf{1}]_{SCET}$$
- One Loop : 4 Collinear diagrams + 6 Ultra-Soft diagrams + wavefunction corrections
- Ultra-soft Graphs



$\times$ 6

- Collinear Graphs



$\times$ 4

Matrix Element in Matrix Form (in C_1, C_2 space)

$$\Gamma_{ij} = \mathbf{I}_{n_i} + \mathbf{I}_{n_j} + \mathbf{I}_{s_{ij}}$$

$$\begin{aligned}
R &= \tilde{R}\,\mathbf{1} + R_S \\
\tilde{R} &= C_F[\Gamma_{12} - \frac{1}{2}\delta Z_1^{-1} - \frac{1}{2}\delta Z_2^{-1}] + C_F[\Gamma_{34} - \frac{1}{2}\delta Z_3^{-1} - \frac{1}{2}\delta Z_4^{-1}] \\
R_S &= \begin{bmatrix} \frac{1}{4}C_d r_1 + \frac{1}{4}C_A r_2 & r_1 \\ C_1 r_1 & 0 \end{bmatrix} \\
r_1 &= \Gamma_{14} + \Gamma_{23} - \Gamma_{13} - \Gamma_{24} \\
r_2 &= \Gamma_{14} + \Gamma_{23} + \Gamma_{13} + \Gamma_{24} - \Gamma_{12} - \Gamma_{34}
\end{aligned}$$

$$\Gamma_{ij} \to S_{ij} \equiv \Gamma_{ij} - \frac{1}{2}\delta Z_i^{-1} - \frac{1}{2}\delta Z_j^{-1}$$

$$\Longrightarrow \quad \begin{aligned}
\tilde{R} &= C_F S_{12} + C_F S_{34} \\
r_1 &= S_{14} + S_{23} - S_{13} - S_{24} \\
r_2 &= S_{14} + S_{23} + S_{13} + S_{24} - S_{12} - S_{34}
\end{aligned}$$

(also true for arbitrary number of external particles)

Matrix Element in Matrix Form (in C_1, C_2 space)

$$\begin{aligned} R &= \tilde{R}\,\mathbf{1} + R_S \\ \tilde{R} &= C_F[\Gamma_{12} - \frac{1}{2}\delta Z_1^{-1} - \frac{1}{2}\delta Z_2^{-1}] + C_F[\Gamma_{34} - \frac{1}{2}\delta Z_3^{-1} - \frac{1}{2}\delta Z_4^{-1}] \\ R_S &= \begin{bmatrix} \frac{1}{4}C_d r_1 + \frac{1}{4}C_A r_2 & r_1 \\ C_1 r_1 & 0 \end{bmatrix} \\ r_1 &= \Gamma_{14} + \Gamma_{23} - \Gamma_{13} - \Gamma_{24} \\ r_2 &= \Gamma_{14} + \Gamma_{23} + \Gamma_{13} + \Gamma_{24} - \Gamma_{12} - \Gamma_{34} \end{aligned}$$

$$\Gamma_{ij} = \mathbf{I}_{n_i} + \mathbf{I}_{n_j} + \mathbf{I}_{s_{ij}}$$

$$\Gamma_{ij} \to S_{ij} \equiv \Gamma_{ij} - \frac{1}{2}\delta Z_i^{-1} - \frac{1}{2}\delta Z_j^{-1}$$

$$\Longrightarrow \quad \begin{aligned} \tilde{R} &= C_F S_{12} + C_F S_{34} \\ r_1 &= S_{14} + S_{23} - S_{13} - S_{24} \\ r_2 &= S_{14} + S_{23} + S_{13} + S_{24} - S_{12} - S_{34} \end{aligned}$$

(also true for arbitrary number of external particles)

SU(2) Toy Theory

anomalous dimension:

$$\begin{aligned}\gamma^{(1)} &= \tilde{\gamma}^{(1)}\,\mathbf{1} + \gamma_S^{(1)} \\ \tilde{\gamma}^{(1)} &= 2C_F\left(4\log\frac{-s}{\mu^2} - 6\right) \\ \gamma_S^{(1)} &= \begin{bmatrix} 2C_d\log\frac{t}{u} + 2C_A\log\frac{ut}{s^2} & 8\log\frac{t}{u} \\ 8C_1\log\frac{t}{u} & 0 \end{bmatrix}\end{aligned}$$

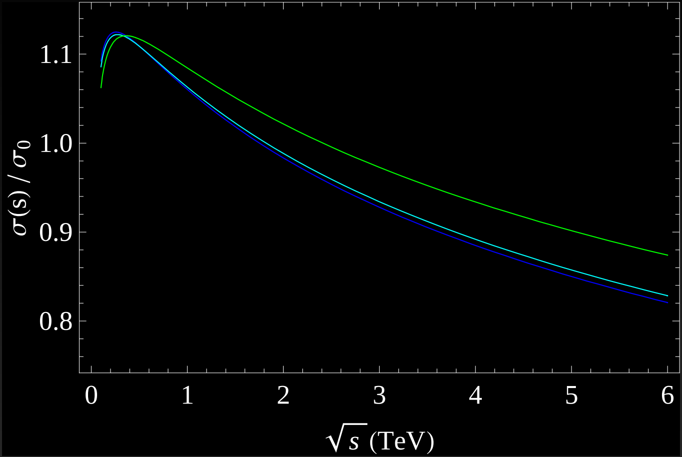
$\tilde{\gamma}^{(1)}$ is twice the Sudakov γ, has a L_Q
$\gamma_S^{(1)}$ has no large logarithm since s, t, and u are all at the order of Q.

Matching Matrix at low scale:
(same structure as anomalous dimension)

$$\begin{aligned} R^{(1)} &= \tilde{R^{(1)}}\,\mathbf{1} + R_S^{(1)} \\ \tilde{R}^{(1)} &= 2C_F\left(-\mathsf{L}_M^2 + 2\mathsf{L}_M \log\frac{-s}{\mu^2} - 2\mathsf{L}_M + \frac{9}{2} - \frac{5\pi^2}{6}\right) \\ R_S^{(1)} &= \mathsf{L}_M \begin{bmatrix} C_d \log\frac{t}{u} + C_A \log\frac{ut}{s^2} & 4\log\frac{t}{u} \\ 4C_1 \log\frac{t}{u} & 0 \end{bmatrix} \end{aligned}$$

cross-section in toy theory

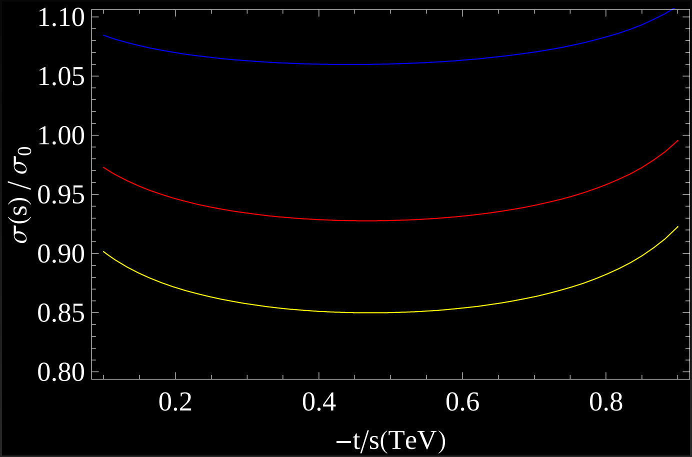
All numerics preliminary.



t=-0.5s t=-0.3s t=-0.1s

$\alpha \frac{M^2}{Q^2}$ term neglected.

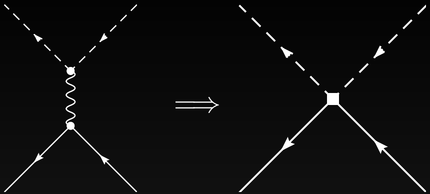
cross-section in toy theory

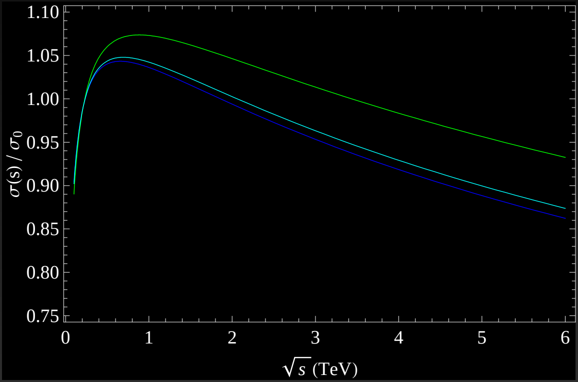


s= 1TeV s=3TeV s=5TeV

squarks production

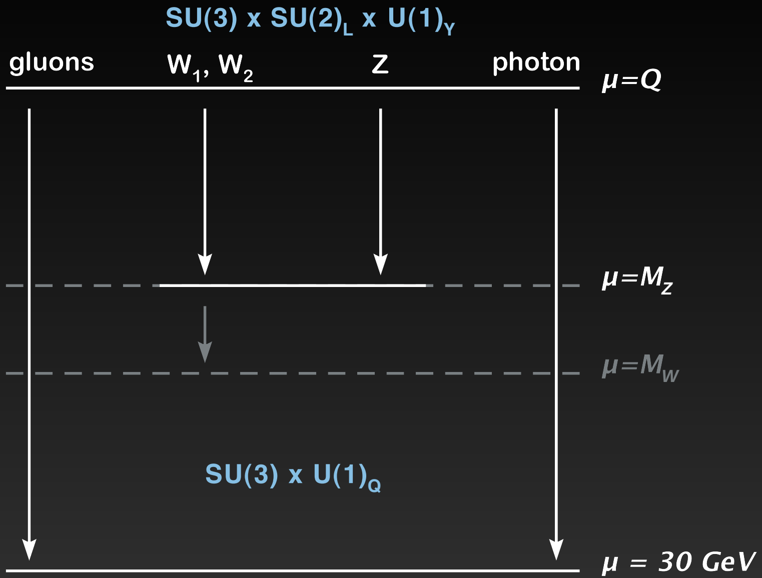
in SU(2) toy theory,
taking $m_s = 500 GeV$



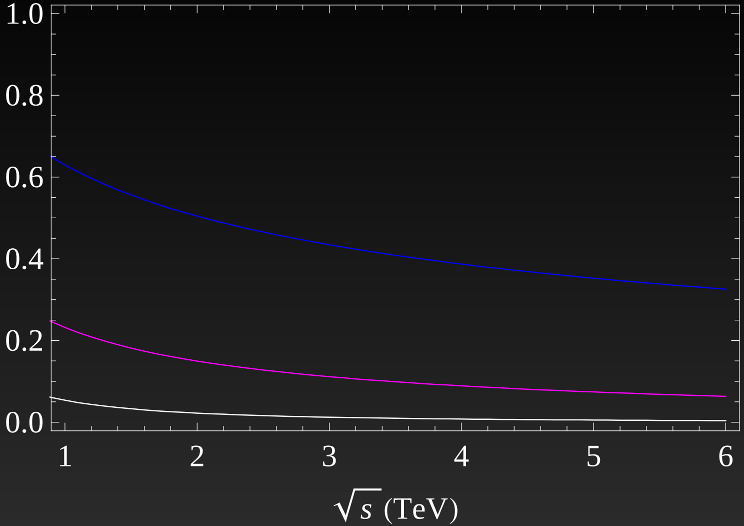


t=-0.5s t=-0.3s t=-0.1s

Standard Model (light quarks):



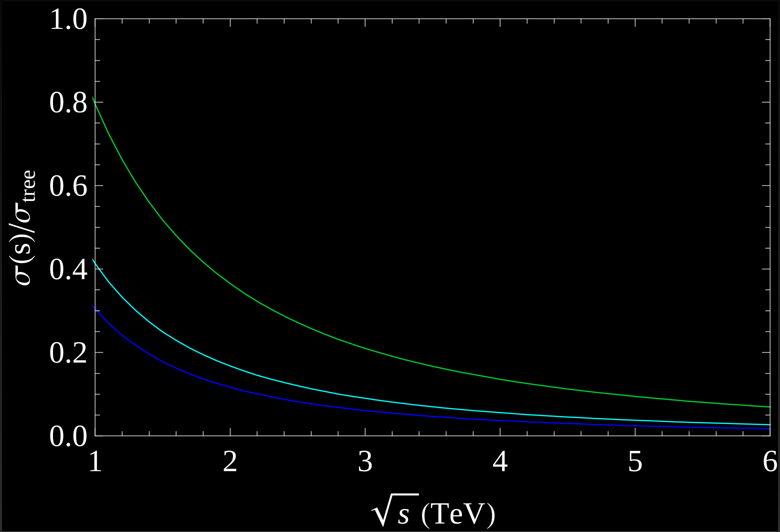
light quarks Sudakov vs. $u\bar{u}_L \to c\bar{c}_L$



F_E $|C_2|$ for t=-0.5s $|C_2|^2$

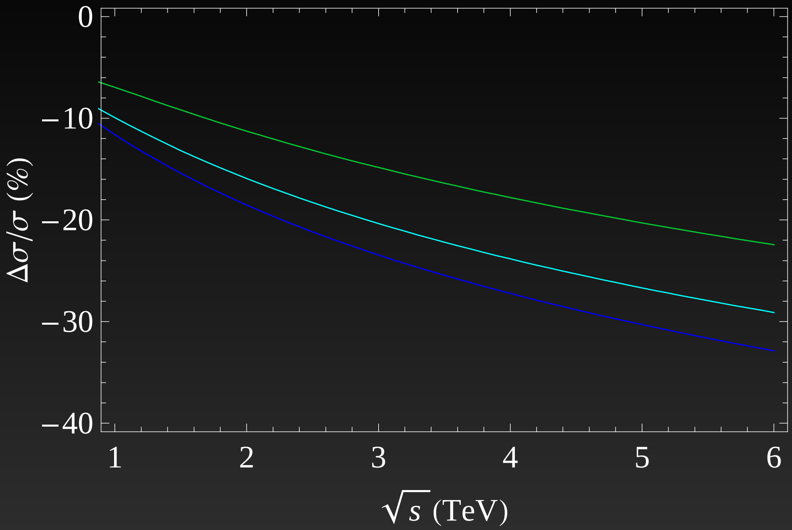
$u\bar{u}_L \longrightarrow c\bar{c}_L$

$$\sigma = \left(\frac{2}{9}|C_1|^2 + |C_2|^2\right) \times \sigma_0$$



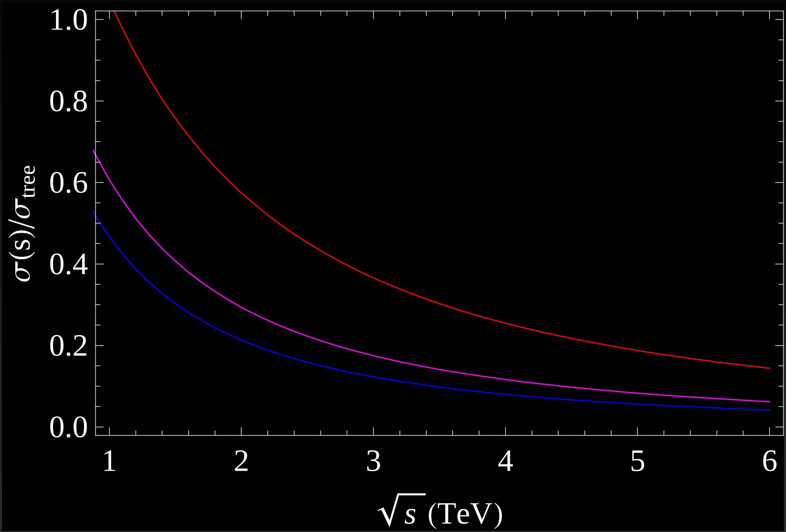
t=-0.5s t=-0.3s t=-0.1s

$u\bar{u}_L \longrightarrow c\bar{c}_L$, contribution from EW



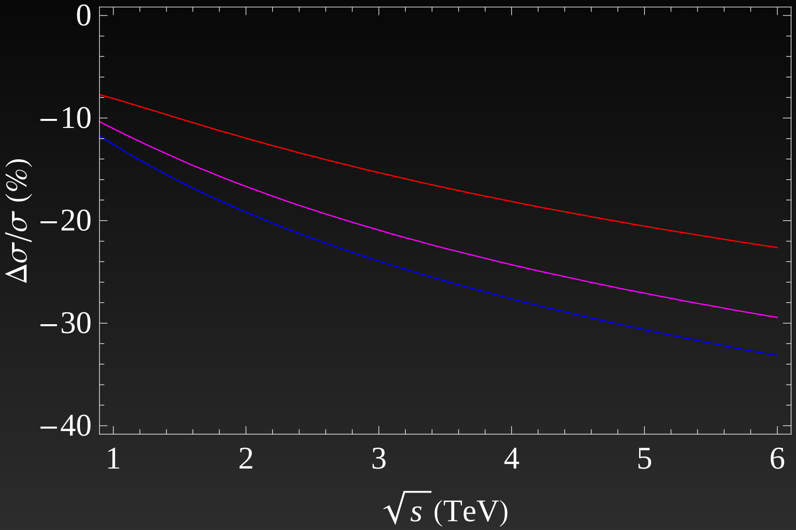
t=-0.5s t=-0.3s t=-0.1s

$u\bar{u}_L \longrightarrow t\bar{t}_L$



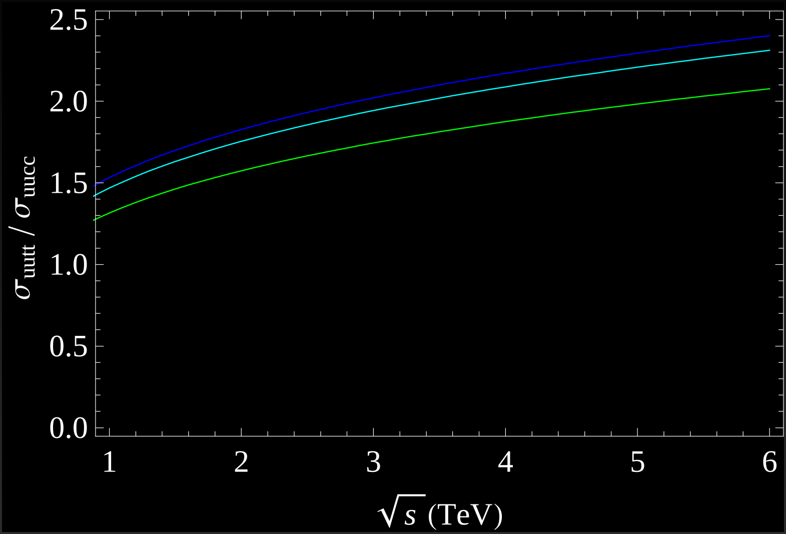
t=-0.5s t=-0.3s t=-0.1s

$u\bar{u} \rightarrow t\bar{t}_L$, contribution from EW



t=-0.5s   t=-0.3s   t=-0.1s

heavy quark mass effect



t=-0.5s t=-0.3s t=-0.1s

Conclusion

- Processes calculated in toy model:
  - $u\bar{u} \longrightarrow \tilde{q}\tilde{q}^*$
  - $u\bar{u} \longrightarrow c\bar{c}$
- Processes calculated in SM using SCET including
  - Sudakov form factor for electrons, u and t quarks
  - $u\bar{u} \longrightarrow c\bar{c}$
  - $u\bar{u} \longrightarrow t\bar{t}$
- Ability to include mass effects due to differences in M_Z, M_W, M_h and m_t.
- The results have been easily generalized to include:
  - production of SUSY particles
  - 4-particle processes in ILC ($e^+e^- \rightarrow u\bar{u}$, $t\bar{t}$, $\tilde{t}\tilde{t}^*$ etc)

Standard Model (heavy quarks):