Threshold resummation for Higgs production

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Overview

- Why threshold resummation?
- Higgs an Drell-Yan
- The effective Cross-section
- Resumming large-π terms
- Phenomenological Analysis
- Conclusion

Why threshold resummation?

- Cross section for prod. of heavy Particles receive large contributions from partonic threshold regions

$$\hat{s} = x_1 x_2 s \geq M^2 \qquad z = \frac{M^2}{\hat{s}} \to 1$$

- Large logarithms arise $\alpha_s^n \frac{\ln^{2n-1}(1-z)}{1-z}$ → threshold resummation

- Physical reason: strong fall-off of parton luminosities

- Examples:

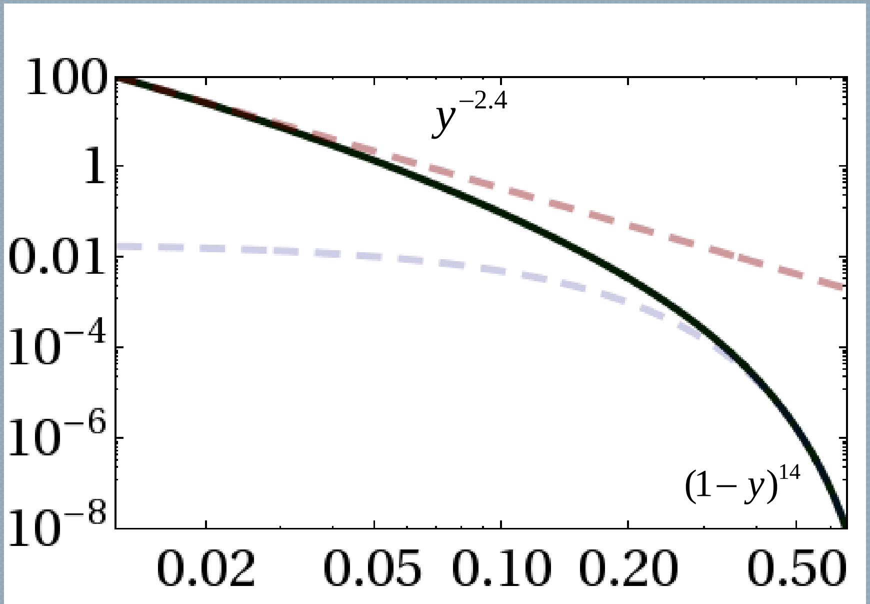
DY production $p + \bar{p} \to \mu^+ \mu^- + X$

Higgs production at Tevatron/LHC $p + p \to H + X$

Fall-off of the gluon-gluonluminosity funcion

$$gg(y,\mu_f) = \int_y^1 \frac{dx}{x}[\, f_{g/N_1}(x,\mu_f)\, f_{g/N_2}(y/x,\mu_f)]$$

$gg(y,\mu_f)$



y

Due to the steep fall-off of the luminosity function
Threshold resummation can be justified even if $\tau = \frac{M^2_H}{s^2}$ is not near 1.

Drell Yan- and Higgs-production

$$\frac{d\sigma^{thresh}}{dM^2} = \frac{4\pi\alpha^2}{3N_c M^2 s}\int_t^1 \frac{dz}{z}\, C_{DY}(z, M, \mu_f)\, ff(\tau / z, \mu_f)$$

$$\downarrow$$

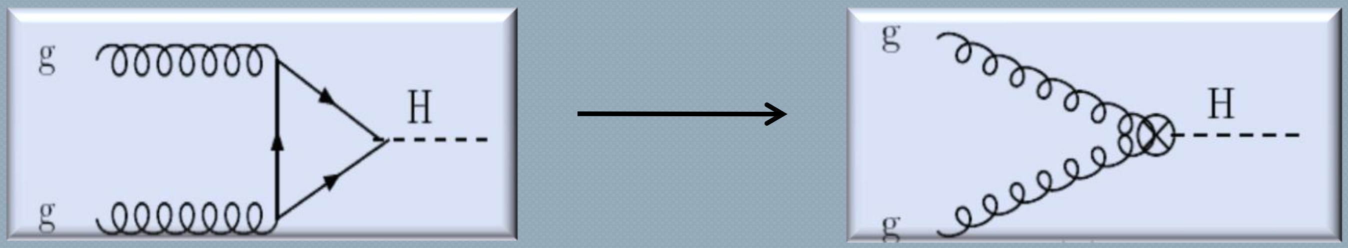
$$\sigma^{thresh} = \sigma_0 \tau \int_t^1 \frac{dz}{z}\, C(z, M_H, \mu_f)\, gg(\tau / z, \mu_f)$$

$$\tilde{C}(z, M, \mu_f) = \left|\tilde{C}_H(M, \mu_f)\right|^2 S(\sqrt{\hat{s}}(1-z), \mu_f)$$

$$\rightarrow \quad C(z, M_H, \mu_f) = \left|C_{EW}(m_t, \mu_f)\right|^2 \left|C_S(M_H, \mu_f)\right|^2 S_H\, \sqrt{\hat{s}}(1-z), \mu_f)$$

- Similar overall structure
- Equivalent factorization of the scattering kernel
- Enables us to use conclusions from Drell-Yan production, as the exact solutions for the evolution equations

Effective theory



First matching: $\mathcal{L}_{eff} = -\frac{C_{EW}}{4v} H\, G^{\mu\nu} G_{\mu\nu}$

$$C_{EW} = -\frac{\alpha_s}{3\pi}\left(1 + \frac{11}{4}\frac{\alpha_s}{4\pi} + \left(\frac{\alpha_s}{4\pi}\right)^2 \left[\frac{2777}{18} - 19\ln\left(\frac{m_t^2}{\mu^2}\right) + n_f\left(-\frac{67}{6} - \frac{16}{3}\ln\left(\frac{m_t^2}{\mu^2}\right)\right)\right]\right)$$

Chetyrkin, Kniehl and Steinhauser, `97

Evolution of C_{EW}

$$\frac{d}{d\ln\mu} C_{EW}(\mu) = \alpha_s \frac{d}{d\alpha_s}\left(\frac{\beta(\alpha_s)}{\alpha_s}\right) C_{EW}(\mu)$$

$$= -2\left[\beta_0 \frac{\alpha_s}{4\pi} + 2\beta_1\left(\frac{\alpha_s}{4\pi}\right)^2 + 3\beta_2\left(\frac{\alpha_s}{4\pi}\right)^3 + \ldots.\right] C_{EW}(\mu)$$

Or through integrating the above equation

$$C_{EW}(\mu) = C_{EW}(\mu_t)\frac{\beta(\alpha_s(\mu))/\alpha_s(\mu)}{\beta(\alpha_s(\mu_t))/\alpha_s(\mu_t)}, \qquad \mu_t = 2m_t$$

Inami, Kubota &Okada, `69

Hard and soft factorization

$$C(z, M_H, \mu_f) = \left|C_{EW}(m_t, \mu_f)\right|^2 \left|C_S(M_H, \mu_f)\right|^2 S_H(\sqrt{\hat{s}}(1-z), \mu_f)$$

Gluon-gluon form factor

Moch, Vermaseren and Vogt, `05

$$\left|1 + C_A \frac{\alpha_s}{4\pi}\left[-L^2 + \varsigma_2\right] + \ldots\right|^2, \qquad L = \ln\left(\frac{M^2{}_H}{\mu^2{}_f}\right) - i\pi$$

Soft part is given by matrix element of Two Wilson lines in gluon directions n_μ and $\bar{n}_\mu$.

Formula for the cross-section

$$\sigma^{thresh} = \sigma_0 \tau \int_t^1 \frac{dz}{z} C(z, M_H, \mu_f)\; gg\,(\tau / z, \mu_f)$$

Where: $$C(z, M_H, \mu_f) = \left|C_{EW}(m_t, \mu_h)\right|^2 \left|C_S(M_H, \mu_h)\right|^2 U(M_H, \mu_h, \mu_s, \mu_f)$$

$$\times \hat{s}_H \left(\ln\left(\frac{M_H^2}{\mu_s^2}\right) + \partial_\eta, \mu_s\right) \frac{z^{-\eta}}{(1-\eta)^{1-2\eta}} \frac{e^{-2\gamma_E \eta}}{\Gamma(2\eta)}$$

and $$U(M_H, \mu_h, \mu_s, \mu_f) = \left(\frac{M_H^2}{\mu_h^2}\right)^{-2a_\Gamma(\mu_H, \mu_s)} \exp[4S(\mu_h, \mu_s) - 2a_\gamma(\mu_h, \mu_s) + 4a_{\gamma^\phi}(\mu_s, \mu_f)]$$

Resummation of π-terms

Look at: $$C_S(-M_H^2,\mu^2)/C_s(M_H^2,\mu^2) = 1 + C_A \frac{\alpha_s(\mu^2)}{4\pi}\left[\pi^2 + 2i\pi \ln\frac{M_H^2}{\mu^2}\right] + \ldots$$

$$= \quad 1 + \frac{\alpha_s(\mu^2)}{4\pi}\left[\frac{\Gamma_0}{4}\pi^2 + \frac{\Gamma_0}{2} i\pi \ln\frac{M_H^2}{\mu^2}\right] + \ldots$$

Observation: 'large-π' terms are avoided if we evaluate the Wilson coefficients at time-like values of μ

Example $N_C = 3,\ n_f = 5,\ \mu = Q$

$$C_S(Q^2,Q^2) = 1 + 0.393\,\alpha_s(Q^2) - 0.152\ \alpha_s^2\ (Q^2)$$

$$C_S(-Q^2,Q^2) = 1 + 2.75\ \alpha_s(Q^2) + (4.8 + 2.1i)\ \alpha_s^2(Q^2)$$

Evolving from $-\mu^2$ to μ^2

$$C_S(-M_H^2,\mu^2) = \exp[2S(-\mu^2,\mu^2) - a_{\gamma^s}(-\mu^2,\mu^2)] \left(\frac{M_H^2}{\mu^2}\right)^{-a_\Gamma(-\mu^2,\mu^2)} C_S(-M_H^2,-\mu^2)$$

contains 'large-π'

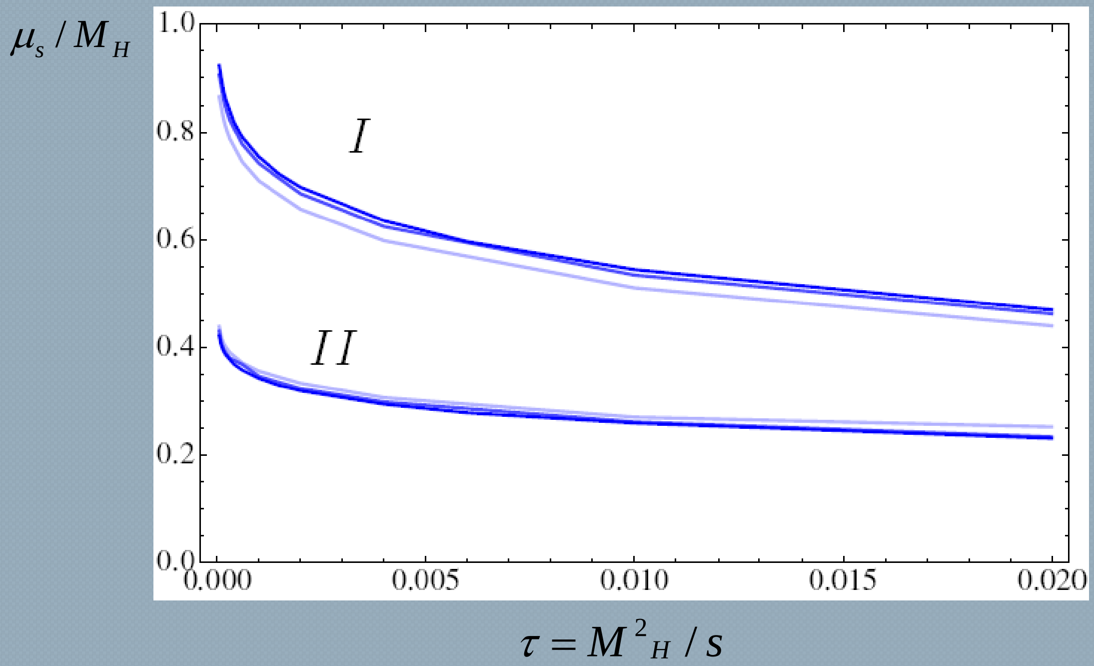
good convergence

In principle one should resum these π-terms in all proceces with time like momentum transfer.

Phenemenological Analysis

- Scale setting and variation
- Matching with fixed order results for the cross-section and factorisation scale
- Resluts for Tevatron and LHC

Choosing the soft scale



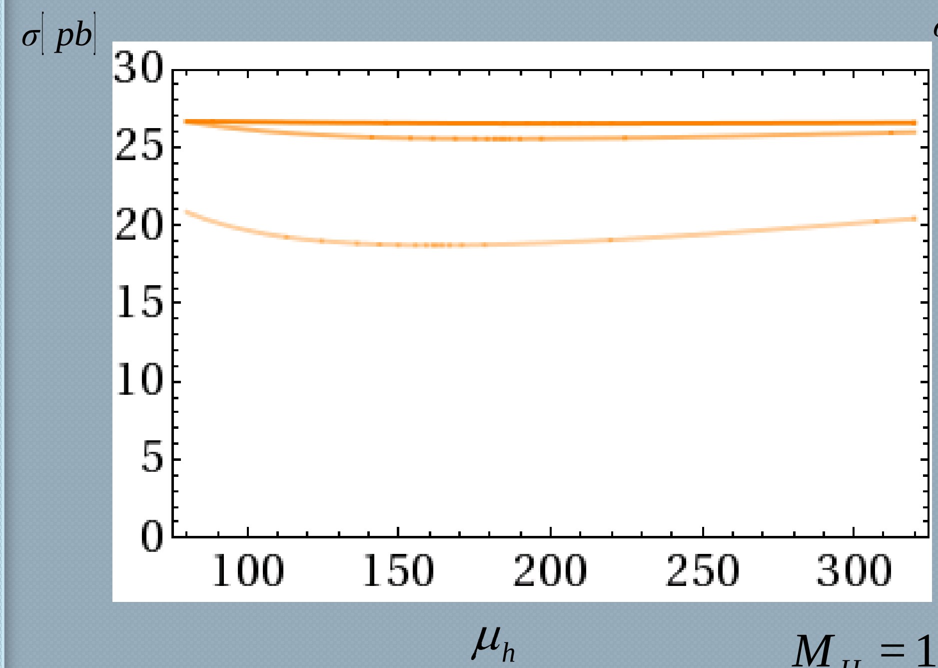
I . Starting from a high scale, we determine the value of μ_s at which the one-loop correction of the soft funcion drops below 15%.

II. We choose the value of μ_s for which the one-loop contribution of the soft function is minimal.

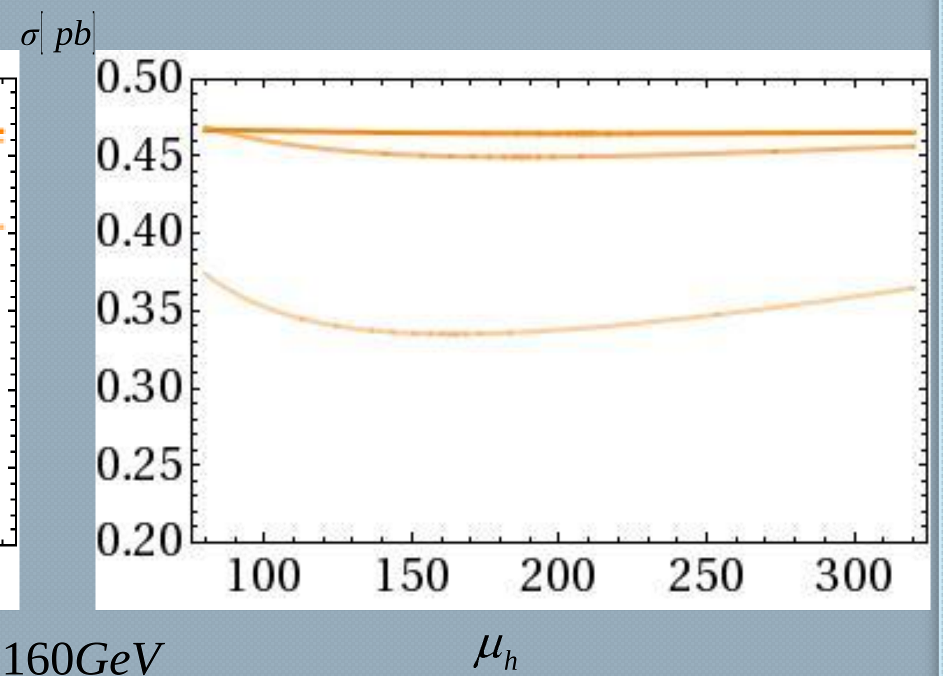
As default we use a superposition of both.

Hard scale dependence

LHC



Tevatron

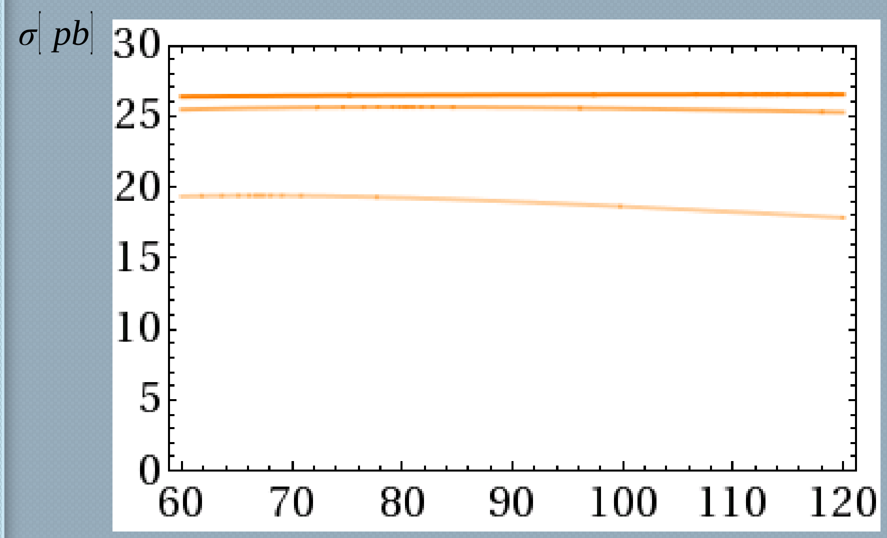


$M_H = 160 GeV$

- Nearly no dependence at NNLO!

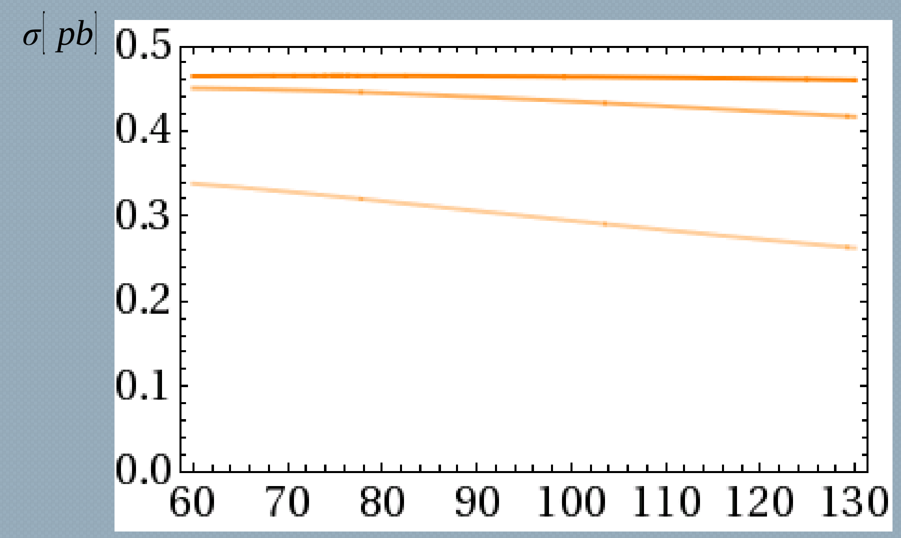
Soft scale dependence

LHC



μ_s

Tevatron



μ_s

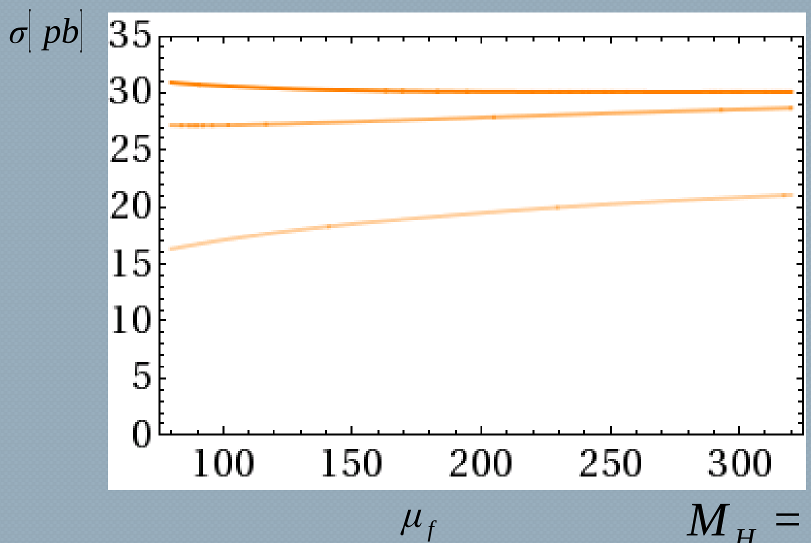
$M_H = 160 GeV$

- Nearly no dependence at NNLO!

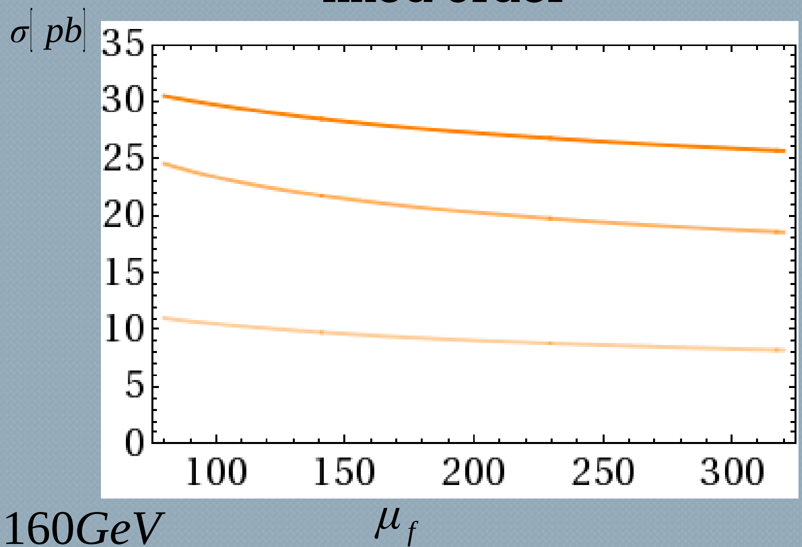
Dependenc on the factorization scale

LHC:

matched



fixed order



$M_H = 160 GeV$

- Scale dependence is reduced
- Convergence is improved

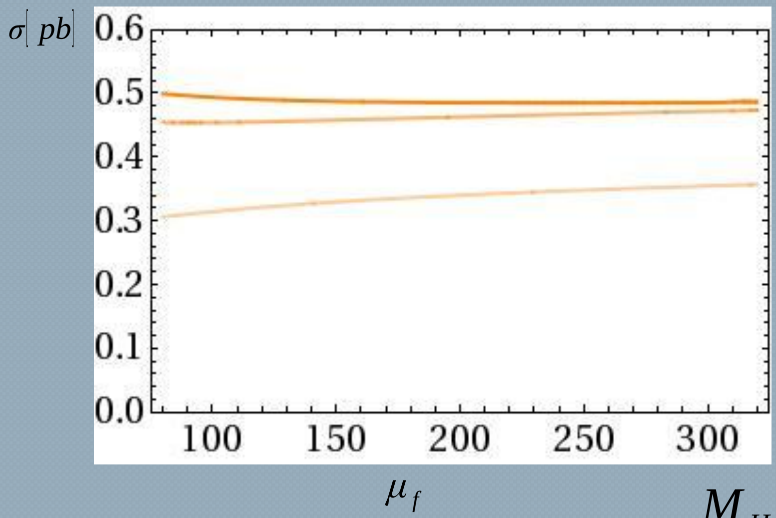
With:

$$\sigma^{matched} = \sigma^{thresh}_{\mu_h,\mu_s,\mu_f} + (\sigma^{fixed\ order}_{\mu_f} - \sigma^{thresh}_{\mu_h=\mu_s=\mu_f})$$

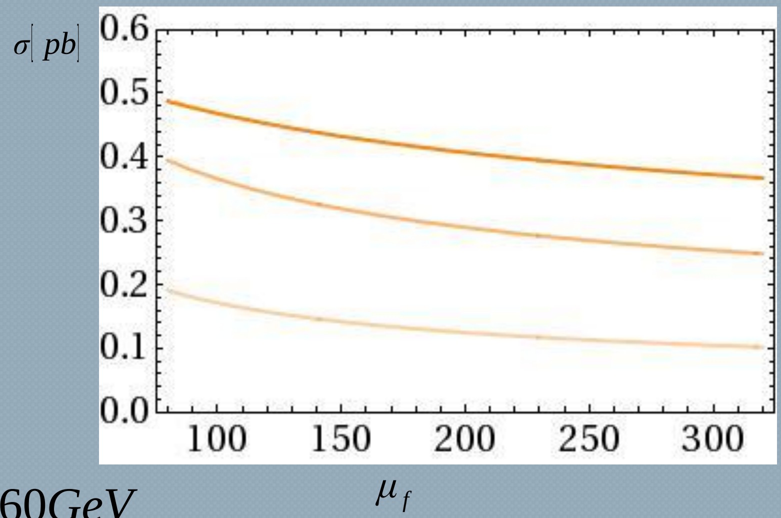
Dependenc on the factorization scale

Tevatron:

matched



fixed order



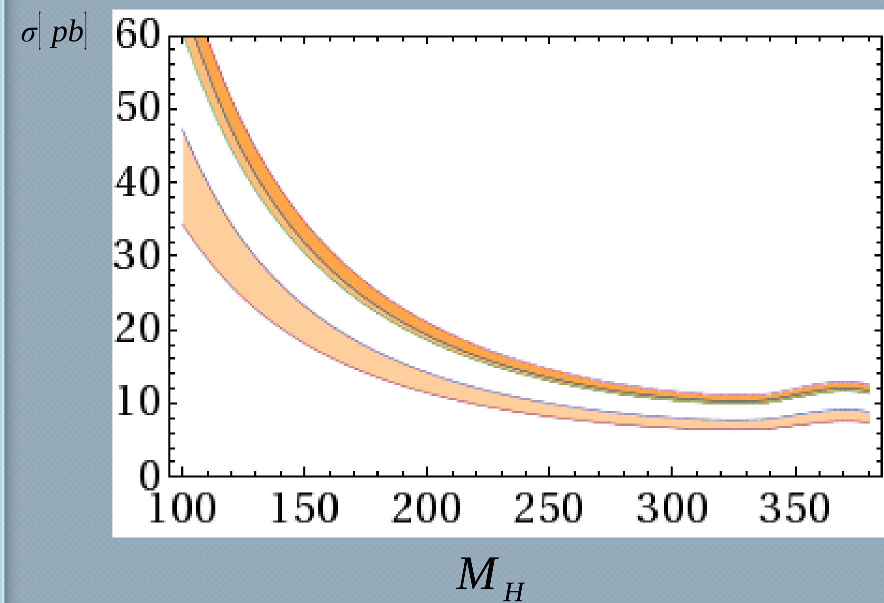
$M_H = 160 GeV$

$\sigma^{NLO} / \sigma^{LO}(\mu_f = M_H) \approx 1.3$

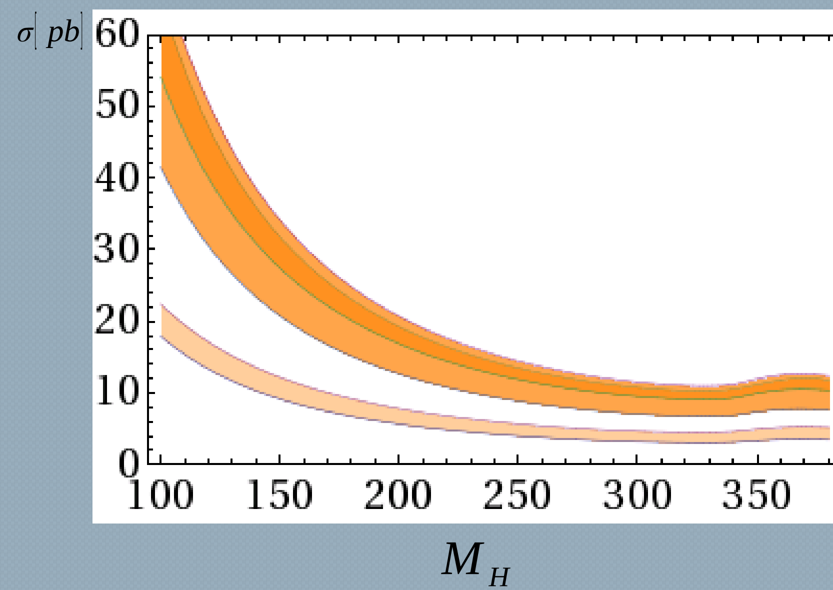
$\sigma^{NLO} / \sigma^{LO}(\mu_f = M_H) \approx 2.3$

Cross section in LHC

matched



fixed order



Cross section for LO, NLO and NNLO fixedvs. resummed.
(color gets darkeras order increases)

Conclusion…so far

- Resummation improves the convergence
- Resummation of π-terms lessens the jump in the cross-section between

  LO and NLO
- Scale dependence is improved

Work in progress