Theoretical Physics 3 — Quantum Mechanics
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Problem Sheet No. 7

Hand-out: 7.12.2016 Hand-in: 16.12.2016 Discussion: 19.-23.12.2016

Problem 12. Non-Hermitian Hamilton Operators (3 Points) We consider a non-relativistic quantum mechanical system with a non-hermitian Hamilton operator. All other postulates of quantum mechanics remain valid. The Hilbert space of the system is two-dimensional with a complete orthonormal basis $|\phi_1\rangle$, $|\phi_2\rangle$. The Hamilton operator of the system is given by $H = \epsilon_1|\phi_1\rangle\langle\phi_1| + (\epsilon_2 - i\epsilon_3)|\phi_2\rangle\langle\phi_2|$ with $\epsilon_{1,2,3} \in \mathbb{R}$ and $\epsilon_3 > 0$. At time $t = 0$ the system is described by the state vector $|\psi_0\rangle = \lambda_1|\phi_1\rangle + \lambda_2|\phi_2\rangle$ with $|\lambda_1|^2 + |\lambda_2|^2 = 1$.

(a) At time $t > 0$ the sytem is described by the state $|\psi(t)\rangle$. Write down this state as a linear combination of the basis states $|\phi_1\rangle$ and $|\phi_2\rangle$ and compute the probabilities to find the system in one of these states at time $t > 0$. In addition compute the (squared) norm $\langle\psi(t)|\psi(t)\rangle$. **2P**

(b) Find an interpretation for your results of part a); Which system could be described by this Hamilton operator? **1P**

13. Properties of the parity operator (8 points) Consider a particle with 1 spacial degree of freedom $x \in \mathbb{R}$. The parity transformation $\hat{P}$ is defined as the transformation of the coordinates $x \to x' = \hat{P}x := -x$. The corresponding parity operator Π in the space of wave functions $\mathcal{H}$ is defined by its action on a wave function $\Pi\Psi(x) = \Psi(-x)$. Show that

a) $\Pi^{-1} = \Pi^\dagger = \Pi$ holds; **2P**

b) Π can only have the eigenvalues $\pi = +1, -1$; **1P**

c) $(\Psi_1, \hat{T}\Psi_2) = 0$, if Ψ_1 and Ψ_2 are eigenvectors of Π of the same eigenvalue π and $\hat{T}$ is an odd operator defined in $\mathcal{H}$, i.e. an operator which fulfills $\Pi\hat{T}\Pi^\dagger = -\hat{T}$; **2P**

d) the position operator Q and the momentum operator P of the particle are odd operators; **1P**

e) Π and the Hamilton operator share common eigenvectors if the potential is symmetric, $V(x) = V(-x)$. **2P**

14. Harmonic oscillator in three dimensions (7 points) Consider a particle of mass m moving in the three dimensional oscillator potential $V(\vec{x}) = m\sum_{k=1}^{3}\omega_k^2 x_k^2/2$. Compute the eigenvalues of the Hamilton operator as explained in the lecture. In particular, answer the following questions:

a) How can the ladder operators be defined using the postion and momentum operators? **1P**

b) What are the commutator relations of the ladder operators? **1P**

c) Give an expression for the Hamilton operator in terms of the ladder operators. **1P**

d) Compute the energy eigenvalues and show how the ladder operators act on the energy eigenstates. **2P**

d) What is the degeneracy of the energy eigenstates in the case of $\omega_1 = \omega_2 \neq \omega_3$. **2P**

Please tell us how much time it took for you to solve the problems.