Contributions to $b \to s\gamma$ in the Randall-Sundrum Model

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Contents

Introduction

The Standard Model of Particle Physics is in complete agreement with the present experimental data. Nevertheless, it is believed to leave many questions unanswered, and this belief has resulted in numerous attempts to discover a more fundamental underlying theory.

Radiative B decays are a powerful tool for testing models of New Physics. In particular, the rare decay $b \to s\gamma$ has found an increasing attention in the recent years [1]. The decay, like other B meson decays, does not arise at tree-level in the Standard Model. The leading order processes are one-loop electroweak penguin diagrams in which the top quark is the dominant particle. In theories beyond the Standard Model, new virtual particles may appear in the penguin loop and lead to sizeable contributions to the branching ratio. The $b \to s\gamma$ decay thus provides a window to possible extensions of the Standard Model and has been investigated in Two-Higgs-Doublet models [2, 3] and supersymmetric theories [4, 5, 6].

In this thesis, we will examine the $b \to s\gamma$ decay in another well-known extension of the Standard Model, namely the Randall-Sundrum model [7]. It was first proposed by Lisa Randall and Raman Sundrum as an attempt to solve the Hierarchy Problem. In its original formulation, all Standard Model fields were constrained to reside on the low-energy boundary of the five dimensional space-time. Soon it was realized that gauge [8, 9, 10, 11] and matter fields [11, 12] can live in the extra dimension, which has the virtue of admitting a natural explanation of the flavor structure of the Standard Model.

This thesis will be organized as follows: In the first chapter I will briefly summarize some basic aspects of the Standard Model, focusing on the construction of the Standard Model Lagrangian out of basic principles. In the second chapter I will introduce the idea of effective field theories and outline the formal framework to describe weak hadron decays. In chapter 3 together with some basic properties of extra dimensions the original Randall-Sundrum idea will be introduced. This leads directly to chapter 4 where I will present the specific Randall-Sundrum model, which provides the basis for the calculations in this thesis. In chapter 5 I will come back to the Standard Model again. I will discuss the magnetic dipole operators, which govern the $b \to s\gamma$ and $b \to sg$ decays and outline the calculation of the corresponding Wilson coefficients. In chapter 6 I will examine which new contributions arise in the Randall-Sundrum framework and compute these analytically for single Kaluza-Klein excitations. Furthermore, I will study the numerical impact of the new contributions on the

Standard Model Wilson coefficients. In the last chapter I will address the question whether the infinite sum over the entire Kaluza-Klein tower leads to finite results on a simpler but similar example.

Chapter 1

The Standard Model of Particle Physics

The Standard Model of Particle Physics (SM), formulated in the 1960s and 1970s, is the currently accepted and experimentally well-tested theory, which describes all the particles and their fundamental interactions[1] in our universe. In this chapter I will give a short overview of some aspects of the SM, which will be relevant for this thesis.

1.1 Basic Principles

Let us start with the main principles which determine the structure of the Standard Model: the gauge principle and the requirement of renormalizability.

1.1.1 Gauge Principle

The Standard Model [13, 14, 15] is a non-abelian gauge theory. A gauge theory is a quantum field theory which is based on the principle of local gauge invariance, or Gauge Principle (GP).

The GP provides a method to transform a Lagrangian that is invariant with respect to a global symmetry of some non-abelian symmetry group $SU(N)$ into a Lagrangian that is invariant with respect to a local symmetry, or gauge invariant. Let $\mathcal{L}(\Psi(x), \partial_\mu \Psi(x))$ be a Lagrangian which is invariant under global $SU(N)$ transformations

$$\Psi(x) \to U\Psi(x) \qquad , \quad U^{-1} = U^\dagger. \tag{1.1}$$

Our aim is to construct a theory that is invariant under local $SU(N)$ transformations as well

$$\Psi(x) \to U(x)\Psi(x) \qquad , \quad U = e^{i\alpha^a(x)\tau^a}. \tag{1.2}$$

The problem is that the Lagrangian is no more locally invariant. In order to restore the invariance we replace the conventional derivatives ∂_μ by the so-called covariant derivatives

[1]Except those due to Gravity.

$D_\mu = \partial_\mu - igA^a_\mu \tau^a$, which transform like the field itself

$$D_\mu \Psi(x) \to (D_\mu \Psi(x))' = U(x)(D_\mu \Psi(x)). \tag{1.3}$$

The arbitrary constant g can be identified with the coupling constant, A^a_μ are a set of vector fields (gauge fields) and τ^a are the generators of the symmetry group, which obey the commutation rule

$$\left[\tau^a, \tau^b\right] = if^{abc}\tau^c. \tag{1.4}$$

The real numbers f^{abc} are the structure constants of the symmetry group. To preserve gauge invariance the gauge fields should transform as well

$$A^a_\mu \to A^{a\prime}_\mu = U(x)(A^a_\mu + \frac{i}{g}\partial_\mu)U^\dagger(x). \tag{1.5}$$

Finally, one has to add a kinetic term for the gauge field: a locally invariant term that depends on A_μ and its derivatives, but not on Ψ. Looking at the field strength tensor $F_{\mu\nu}$, defined as

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + gf^{abc}A^b_\mu A^c_\nu, \tag{1.6}$$

we find that the combination $F^a_{\mu\nu}F^{a,\mu\nu}$ satisfies these conditions, and thus may appear in the Lagrangian. The new locally invariant Lagrangian reads

$$\mathcal{L} = \mathcal{L}(\Psi(x), D_\mu\Psi(x)) - \frac{1}{4}F_{\mu\nu}F^{\mu\nu}. \tag{1.7}$$

It is important to note that the GP not only extends a global to a local symmetry, but determines uniquely the interaction of the gauge field with itself (through the kinetic term) and with the matter fields (through the covariant derivative). Thus it determines not only the symmetry, but also the dynamics.

A serious problem is however that mass terms for the gauge fields $\frac{1}{2}m^2A_\mu A^\mu$ break the gauge invariance, since these terms are not invariant under the transformations eq.(1.5). This is problematic because if non-abelian gauge theories are to be applied to the description of the physical interactions, the gauge fields must be identified with the observable vector fields. Since the weak gauge bosons are found to be massive, the theory needs massive bosons. Mass terms are introduced in the theory through a different mechanism, the spontaneous symmetry breaking or Higgs mechanism (see section 1.3).

1.1.2 Renormalizability

The Standard Model is also a renormalizable quantum field theory. Renormalizable quantum field theories are theories in which the infinities, which arise from loop calculations, can be eliminated by absorbing them into a redefinition, or a renormalization, of a finite number of physical parameters.

The demand of renormalizability is an enormous restriction to the Lagrangian of the theory. Roughly speaking, the renormalizability of the theory is determined by the mass dimension of the couplings in the Lagrangian. If the theory is to be renormalizable, the coupling constant c in the Lagrangian

$$\mathcal{L} \propto c\,\mathcal{O} \tag{1.8}$$

should have positive mass dimension, where $\mathcal{O}$ are operators built out of the fields which appear in the theory and their derivatives. Since the mass dimension of the Lagrangian itself is d, the dimension of space-time, which is four, only operator products with dimension $d \leq 4$ are allowed in the Lagrangian. Operators with higher mass dimension equivalently coupling constants with negative mass dimension lead to non-renormalizable interactions.

For a long time it was accepted that only the renormalizable theories can describe nature correctly, because of their great predictive power. A non-renormalizable theory would require an infinite number of experimental inputs. The concept of Effective Field Theory (EFT) however provides a new understanding of renormalization. In the framework of EFT non-renormalizable interactions are not forbidden. Operators with mass dimension greater than 4 always contribute. Their effects are simply numerically suppressed by powers of the fundamental scale of the theory, which can be much larger than the typical energies achievable experimentally. Non-renormalizable theories still retain predictive power. There is however a finite accuracy associated with these predictions. We can regard a renormalizable theory, such as the SM, as an EFT, where the non-renormalizable terms have been neglected.

1.2 The Structure of the Standard Model

The Standard Model combines the theory of strong interactions, Quantum Chromodynamics (QCD), based on the gauge group $SU(3)_C$, and the Glashow-Weinberg-Salam theory of electroweak interactions, based on the $SU(2)_L \times U(1)_Y$ gauge group. Thus the Standard Model has the product gauge group $SU(3)_C \times SU(2)_L \times U(1)_Y$. The most general SM Lagrangian, which obeys the principle of gauge symmetry and the requirement of renormalizability, is given by[2]

$$\mathcal{L}_{SM} = \mathcal{L}_{Gauge} + \mathcal{L}_{Fermion}. \tag{1.9}$$

[2] Before spontaneous symmetry breaking.

1.2.1 Gauge Sector

The gauge sector contains 12 gauge fields, according to the dimension of the SM gauge group, which mediate the interactions between the fermion fields. The Lagrangian reads

$$\mathcal{L}_{Gauge} = -\frac{1}{4} B_{\mu\nu} B^{\mu\nu} - \frac{1}{4} W^a_{\mu\nu} W^{a,\mu\nu} - \frac{1}{4} G^a_{\mu\nu} G^{a,\mu\nu}, \tag{1.10}$$

where the field strength tensors are given by

$$\begin{aligned} U(1)_Y : B_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu, \\ SU(2)_L : W^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g_2 \epsilon^{abc} W^b_\mu W^c_\nu, \\ SU(3)_C : G^a_{\mu\nu} &= \partial_\mu G^a_\nu - \partial_\nu G^a_\mu + g_3 f^{abc} G^b_\mu G^c_\nu. \end{aligned} \tag{1.11}$$

Here B_μ, W^a_μ $(a = 1,2,3)$, G^a_μ $(a = 1,\ldots,8)$ are the gauge fields belonging to the weak-hypercharge $U(1)_Y$, the weak-isospin $SU(2)_L$ and the color symmetry group $SU(3)_C$, respectively, g_1, g_2 and g_3 are the corresponding gauge couplings, and ϵ^{abc} and f^{abc} the structure constants. It is important to note that in contrast to the abelian field strength tensor $B_{\mu\nu}$, the non-abelian field strength tensors $W^a_{\mu\nu}$ and $G^a_{\mu\nu}$ contain quadratic terms in the gauge fields, which lead to gauge self-interactions. These interactions are a specific aspect of non-abelian gauge theories and are responsible for the asymptotic freedom in QCD.

1.2.2 Fermion Sector

The fermions appear in three generations. Each generation consists of a neutrino, a charged lepton, and an up- and a down-type quark. Furthermore, we group the fermions into left-handed fermions, which are doublets with respect to $SU(2)_L$, and right-handed fermions, which are singlets with respect to $SU(2)_L$. The doublets are given by

$$\begin{aligned} E^i_L &= \left(\begin{pmatrix} \nu_e \\ e \end{pmatrix}_L, \begin{pmatrix} \nu_\mu \\ \mu \end{pmatrix}_L, \begin{pmatrix} \nu_\tau \\ \tau \end{pmatrix}_L \right), \\ Q^i_L &= \left(\begin{pmatrix} u \\ d \end{pmatrix}_L, \begin{pmatrix} c \\ s \end{pmatrix}_L, \begin{pmatrix} t \\ b \end{pmatrix}_L \right), \end{aligned} \tag{1.12}$$

whereas the singlets are given by

$$\begin{aligned} e^i_R &= (e_R, \mu_R, \tau_R)\,, \\ u^i_R &= (u_R, c_R, t_R)\,, \\ d^i_R &= (d_R, s_R, b_R)\,. \end{aligned} \tag{1.13}$$

The right-handed neutrinos are not included, because they would be neutral under all three gauge groups. With this the Lagrangian of the fermion sector reads

$$\mathcal{L}_{Fermion} = \bar{E}^i_L i \not{D}_E E^i_L + \bar{Q}^i_L i \not{D}_Q Q^i_L + \bar{e}^i_R i \not{D}_e e^i_R + \bar{u}^i_R i \not{D}_u u^i_R + \bar{d}^i_R i \not{D}_d d^i_R, \tag{1.14}$$

where we sum over the three fermion generations and with the definition $\not{D} = D^\mu \gamma_\mu$. The covariant derivatives are given by

$$\begin{aligned} D^\mu_E &= \partial_\mu - i g_1 Y_L B^\mu - i g_2 \frac{\sigma^a}{2} W^{a,\mu}, \\ D^\mu_Q &= \partial_\mu - i g_1 Y_Q B^\mu - i g_2 \frac{\sigma^a}{2} W^{a,\mu} - i g_3 t^a G^{a,\mu}, \\ D^\mu_e &= \partial_\mu - i g_1 Y_e B^\mu, \\ D^\mu_q &= \partial_\mu - i g_1 Y_q B^\mu - i g_3 t^a G^{a,\mu}, \qquad q = u, d. \end{aligned} \tag{1.15}$$

Here σ^a are the Pauli matrices, Y is the hypercharge, and t^a are the generators of $SU(3)_C$, which are related to the Gell-Mann matrices according to $t^a = \frac{\lambda^a}{2}$.

1.3 Higgs Mechanism

Gauge invariance does not allow mass terms in the Lagrangian for the gauge bosons and the chiral fermions. This is however a clear contradiction with the experiments, which show that the weak gauge bosons and all fermions except the neutrinos are massive particles. As we do not want to abandon the concept of gauge theories, we face the problem of how to incorporate masses into gauge theories. The solution is provided by the concept of spontaneous symmetry breaking, also known as the Higgs mechanism. The idea is to add a new complex scalar field Φ to the theory, which is a doublet with respect to $SU(2)_L$ and has a hypercharge $Y_\Phi = \frac{1}{2}$

$$\Phi = \begin{pmatrix} \phi_1 + i\phi_2 \\ h + i\phi_0 \end{pmatrix}. \tag{1.16}$$

This is done by supplementing the SM Lagrangian with an extra term

$$\mathcal{L}_{Higgs} = \left(D_\mu \Phi\right)^\dagger \left(D^\mu \Phi\right) - V\left(\Phi\right), \tag{1.17}$$

where the covariant derivative D_μ is given by

$$D_\mu = \partial_\mu - i \frac{g_1}{2} B_\mu - i g_2 \frac{\sigma^a}{2} W^a_\mu \tag{1.18}$$

and

$$V\left(\Phi\right) = -\mu^2 \Phi^\dagger \Phi + \frac{\lambda}{2} \left(\Phi^\dagger \Phi\right)^2 \tag{1.19}$$

is the Higgs potential, which involves two new real parameters μ and λ. For $\mu^2 > 0$ and $\lambda > 0$ the potential develops a non-zero vacuum expectation value (VEV), or a degeneracy of minima satisfying the equation

$$\Phi^\dagger \Phi = \frac{\mu^2}{2\lambda} \equiv \frac{v^2}{2}. \tag{1.20}$$

All these minima are physically equivalent. However, each of them selects a direction in the representation space. Choosing one of them as the ground state of the theory spontaneously breaks the symmetry. This means that the Lagrangian is still invariant under the $SU(2)_L \times U(1)_Y$ symmetry, the ground state however not. Without loss of generality, we can make a $SU(2)$ rotation, so that it is the lower component of Φ which acquires a non-vanishing VEV

$$\langle \Phi \rangle = \frac{1}{\sqrt{2}} \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{1.21}$$

With this particular choice, the $SU(2)_L \times U(1)_Y$ is broken to the remaining $U(1)_{EM}$

$$SU(2)_L \times U(1)_Y \quad \longrightarrow \quad U(1)_{EM}. \tag{1.22}$$

The fields $\phi_\pm$ and ϕ_0, the so-called would-be Goldstone bosons, do not appear in the theory as independent physical particles. The easiest way to see this is to make a particular choice of gauge, called the unitary gauge. However, the would-be Goldstone bosons do not disappear from the theory completely, they essentially reappear in the gauge sector, providing the longitudinal modes for the W and Z bosons. One says that the gauge bosons acquire their mass by eating the would-be Goldstone bosons.

1.3.1 Gauge Boson Masses

The gauge boson mass terms come from the kinetic term of the Higgs Lagrangian, evaluated at the scalar field VEV. The relevant terms are

$$\mathcal{L}_{Higgs} \ni \frac{1}{2} \begin{pmatrix} 0 & v \end{pmatrix} \left(g_2 W_\mu^a \frac{\sigma^a}{2} + \frac{1}{2} g_1 B_\mu \right) \left(g_2 W_\mu^b \frac{\sigma^b}{2} + \frac{1}{2} g_1 B_\mu \right) \begin{pmatrix} 0 \\ v \end{pmatrix}. \tag{1.23}$$

Evaluating the matrix product explicitly leads to three massive gauge bosons

$$\begin{aligned} W_\mu^\pm &= \frac{1}{\sqrt{2}} \left(W_\mu^1 \mp i W_\mu^2 \right) \qquad \text{with} \qquad m_W^2 = \frac{1}{4} g_2^2 v^2, \\ Z_\mu^0 &= W_\mu^3 \cos\theta_W - B_\mu \sin\theta_W \qquad \text{with} \qquad m_Z^2 = \frac{1}{4} \left(g_1^2 + g_2^2 \right) v^2. \end{aligned} \tag{1.24}$$

The fourth vector field, orthogonal to Z^0_μ, remains massless

$$A_\mu = W^3_\mu \sin\theta_W + B_\mu \cos\theta_W \qquad \text{with} \qquad m_A^2 = 0, \tag{1.25}$$

where the Weinberg angle θ_W is given by

$$\cos\theta_W = \frac{g_2}{\sqrt{g_1^2+g_2^2}} \qquad\qquad \sin\theta_W = \frac{g_1}{\sqrt{g_1^2+g_2^2}}. \tag{1.26}$$

The remaining massless gauge field A_μ can be identified with the photon. It is a consequence of the fact that the $SU(2)_L \times U(1)_Y$ is broken to the $U(1)_{EM}$ symmetry. In the basis of the new fields the fermionic Lagrangian takes the form

$$\begin{aligned}\mathcal{L}_{Fermion} =& \bar{E}^i_L i\not{\partial}_E E^i_L + \bar{Q}^i_L i\not{\partial}_Q Q^i_L + \bar{e}^i_R i\not{\partial}_e e^i_R + \bar{u}^i_R i\not{\partial}_u u^i_R + \bar{d}^i_R i\not{\partial}_d d^i_R \\ &+ g_2\left(W^+_\mu J^{+\mu}_W + W^-_\mu J^{-\mu}_W + Z^0_\mu J^\mu_Z\right) + g_1 A_\mu J^\mu_{EM},\end{aligned} \tag{1.27}$$

where we define the following currents

$$\begin{aligned}J^\mu_{EM} &= (-1)\left(\bar{e}\gamma^\mu e\right) + \left(\frac{2}{3}\right)\left(\bar{u}\gamma^\mu u\right) + \left(-\frac{1}{3}\right)\left(\bar{d}\gamma^\mu d\right), \\ J^{+\mu}_W &= \frac{1}{\sqrt{2}}\left(\bar{\nu}_L\gamma^\mu e_L + \bar{u}_L\gamma^\mu d_L\right), \\ J^{-\mu}_W &= \frac{1}{\sqrt{2}}\left(\bar{e}_L\gamma^\mu \nu_L + \bar{d}_L\gamma^\mu u_L\right), \\ J^\mu_Z =& \frac{1}{\cos\theta_W}\left[\left(\frac{1}{2}\right)\left(\bar{\nu}_L\gamma^\mu\nu_L\right) + \left(-\frac{1}{2}+\sin^2\theta_W\right)\left(\bar{e}_L\gamma^\mu e_L\right) + \sin^2\theta_W\left(\bar{e}_R\gamma^\mu e_R\right)\right. \\ &+ \left(\frac{1}{2}-\frac{2}{3}\sin^2\theta_W\right)\left(\bar{u}_L\gamma^\mu u_L\right) + \left(-\frac{2}{3}\sin^2\theta_W\right)\left(\bar{u}_R\gamma^\mu u_R\right) \\ &\left.+ \left(-\frac{1}{2}+\frac{1}{3}\sin^2\theta_W\right)\left(\bar{d}_L\gamma^\mu d_L\right) + \left(\frac{1}{3}\sin^2\theta_W\right)\left(\bar{d}_R\gamma^\mu d_R\right)\right].\end{aligned} \tag{1.28}$$

1.3.2 Fermion Masses

In order to produce mass terms for the fermions we extend the Lagrangian with the couplings between the fermion doublets and the field Φ. These new terms, called Yukawa interactions, are allowed by the symmetries

$$\mathcal{L}_{Yukawa} = -\left(\bar{E}^i_L \cdot \Phi\right)\mathbf{Y}^{ij}_e e^j_R - \left(\bar{Q}^i_L \cdot \Phi\right)\mathbf{Y}^{ij}_d d^j_R - \left(\bar{Q}^i_L \cdot \tilde{\Phi}\right)\mathbf{Y}^{ij}_u u^j_R + \text{h.c.}, \tag{1.29}$$

where the $\mathbf{Y}$'s are 3×3 complex matrices, the so-called Yukawa matrices, and $\tilde{\Phi} = i\sigma_2\Phi$. After spontaneous symmetry breaking the Lagrangian reads

$$\mathcal{L}_{Yukawa} = -\frac{v}{\sqrt{2}}\bar{e}_L\mathbf{Y}_e e_R - \frac{v}{\sqrt{2}}\bar{d}_L\mathbf{Y}_d d_R - \frac{v}{\sqrt{2}}\bar{u}_L\mathbf{Y}_u u_R + \text{interactions} + \text{h.c.}\ . \tag{1.30}$$

We diagonalize the Yukawa matrices using biunitary transformations[3]

$$\mathbf{Y}_q = \mathbf{U}_q \mathbf{D}_q \mathbf{W}_q^\dagger \qquad , \quad q = u, d, \tag{1.31}$$

where $\mathbf{D}_q$ are diagonal matrices which contain the eigenvalues of the Yukawa matrices. Then we perform field redefinitions to eliminate the unitary matrices

$$\begin{aligned} (u_L)_i &\to (U_u)_{ij}(u_L)_j, \qquad & (u_R)_i &\to (W_u)_{ij}(u_R)_j, \\ (d_L)_i &\to (U_d)_{ij}(d_L)_j, \qquad & (d_R)_i &\to (W_d)_{ij}(d_R)_j. \end{aligned} \tag{1.32}$$

These transformations convert the quark fields to the basis of mass eigenstates. The masses of the quarks are related to the Higgs VEV and the diagonal elements of the Yukawa couplings by

$$m_u^i = \frac{v}{\sqrt{2}} D_u^{ii} \qquad , \qquad m_d = \frac{v}{\sqrt{2}} D_d^{ii}. \tag{1.33}$$

The change of variables doesn't affect the kinetic term of the fermionic Lagrangian, but leads to the appearance of the Cabibo-Kobayashi-Maskawa (CKM) mixing matrix, defined as

$$\mathbf{V}_{CKM} = \mathbf{U}_u^\dagger \mathbf{U}_d \tag{1.34}$$

in the interaction term (see section 2.1), which allows weak-interaction transitions between quark generations.

1.3.3 The Higgs Boson

The Higgs mechanism, which provides the masses for the weak gauge bosons and fermions, has as a consequence the prediction of a new particle: the Higgs Boson. This must be scalar and electrically neutral. The Lagrangian for the Higgs boson reads

$$\mathcal{L}_{Higgs} = \frac{1}{2}(\partial_\mu h)(\partial^\mu h) - \frac{1}{2} m_h^2 h^2 + \text{interactions}, \tag{1.35}$$

where the Higgs boson mass is given by $m_h = \sqrt{\frac{\lambda}{2}} v$. The interaction terms contain both self-interactions and interactions with fermions and gauge bosons. One of the most important points about the Higgs mechanism is that all of the couplings of the Higgs boson are completely determined in terms of coupling constants and masses.

The Higgs boson has not been observed yet, despite large efforts invested in accelerator experiments at CERN and Fermilab. The Higgs mass is a free parameter in the Standard Model, which can not be predicted. Theoretical bounds however imply that if the SM is to be perturbative up to the GUT scale $\sim 10^{16}$ GeV, the Higgs mass should be within about

[3] The discussion contains only the quark sector. The extension to the lepton sector is straightforward. Since right-handed neutrinos are omitted, there are no mass terms for the neutrinos in the SM.

130 and 180 GeV. The discovery of a Higgs boson with mass below 130 GeV would suggest the onset of new physics at a scale below Λ_{GUT} [16]. It is expected that the Large Hadron Collider (LHC) will be able to confirm or reject the existence of the Higgs boson.

1.4 Gauge Fixing and Ghosts

The quantization of the SM requires, as the quantization of all non-abelian gauge theories, the introduction of a gauge fixing term and of Faddeev-Popov fields. Looking at the functional integral

$$\mathcal{Z}[0] = \int \mathcal{D}A \, e^{i \int d^4x \mathcal{L}} \tag{1.36}$$

the Lagrangian is unchanged along an infinite number of directions in the space of field configurations. The integration over the gauge equivalent fields leads to a divergent integral. In order to remove the divergence, we must factor out these integrations, constraining the remaining integral to a much smaller space. Therefore we impose a gauge fixing condition, which adds a new contribution to the Lagrangian $\mathcal{L}_{GF}$. The procedure invokes also another additional term, which can be written as a Lagrangian for anti-commuting scalar fields

$$\mathcal{L}_{FP} = \bar{c}^a(-\partial^2\delta^{ac} - g\partial^\mu f^{abc} A^b_\mu)c^c. \tag{1.37}$$

The first term is a kinetic term, whereas the second one describes the interaction with the gauge fields. Note that in abelian gauge theories $f^{abc} = 0$ and the ghosts do not interact with the gauge fields. Since the ghost fields violate the spin-statistic relation, they do not correspond to any real particles or external states, but appear only as virtual particles in Feynman diagrams.

With this we obtain the complete Lagrangian of the SM

$$\mathcal{L}_{SM} = \mathcal{L}_{Gauge} + \mathcal{L}_{Fermion} + \mathcal{L}_{Yukuwa} + \mathcal{L}_{Higgs} + \mathcal{L}_{GF} + \mathcal{L}_{FP}. \tag{1.38}$$

1.5 Problems of the Standard Model

Despite numerous successes, there remain a number of important questions which the SM does not answer.

The Fermion Problem. All matter can be constructed out of the fermions of the first generation ν_e, e, u, d. Yet we know from experimental studies that there are two additional generations, which are heavier copies of the first generation with no obvious role in nature[4].

[4]The presence of at least three generations of quarks is however a necessary condition for the CP violation in the SM.

The Standard Model gives no explanation for the existence of these heavier families. Furthermore, there is no explanation or prediction of the fermion masses which are observed to occur in a hierarchical pattern, which varies over 5 orders of magnitude between the top quark and the electron. Even more mysterious are the neutrinos which are many orders of magnitude lighter. It is not even certain whether the neutrinos are Majorana or Dirac particles. A related difficulty is that while the CP violation observed in the laboratory is well accounted for by the phase in the CKM matrix, there is no SM source of CP breaking adequate to explain the baryon asymmetry of the universe.

The Hierarchy Problem. Another problem, which inspires most extensions of the SM, is the Hierarchy Problem. It concerns the two fundamental scales in nature: the electroweak scale $m_{EW} \sim 1\,TeV$, and the Planck scale $M_{Pl} \sim 10^{19}\,GeV$, where quantum gravitational effects can no longer be neglected, if not earlier. There is a gap of 16 orders of magnitude between the two scales. The Hierarchy Problem is the question why there is such a large gap between them, or why the Planck scale is so much larger than the electroweak scale.

More technically, the question is why the Higgs boson is so much lighter than the Planck mass. Going beyond tree level, the bare Higgs boson mass receives quadratically-divergent corrections

$$m_h^2 = (m_h^2)_0 + \mathcal{O}(\Lambda^2), \tag{1.39}$$

where Λ is the cutoff scale of the theory, i.e., the scale up to which the theory is valid. If it is the Planck scale, then the Higgs boson mass should be of the order of the Planck mass. However we expect the Higgs mass to be of the order of the electroweak scale. Extreme fine-tuning must occur to explain this enormous difference.

These and other open questions provide a good justification to develop theories, which expand and complete the Standard Model. These new theories should contain the SM as a limiting case and provide solutions to at least some of its unsolved problems.

Chapter 2

Flavor Physics

In this chapter I will come back to the fermion sector of the Standard Model and in particular the quark sector, and discuss some of its important features. I will introduce the concept of effective field theory and discuss the basic formalism to describe weak decays.

2.1 The CKM matrix

As discussed in the previous chapter, the field redefinitions eq.(1.32) lead to the appearance of the CKM matrix eq.(1.34) in the current that couples to the W boson field

$$J_W^{+\mu} = \frac{1}{\sqrt{2}} \bar{u}_L \gamma^\mu d_L \longrightarrow \frac{1}{\sqrt{2}} \bar{u}_L \mathbf{V}_{CKM} \gamma^\mu d_L. \tag{2.1}$$

The CKM matrix is a 3×3 unitary matrix. It can be parametrized by three angels and one phase. This phase leads to an imaginary part in the CKM matrix, which is the source of all CP-violating phenomena in the Standard Model.

2.1.1 Standard Parametrization

The CKM matrix is often given in the standard parametrization [16]

$$\begin{aligned} \mathbf{V}_{CKM} &= \begin{pmatrix} V_{ud} & V_{us} & V_{ub} \\ V_{cd} & V_{cs} & V_{cb} \\ V_{td} & V_{ts} & V_{tb} \end{pmatrix} \\ &= \begin{pmatrix} c_{12}c_{13} & s_{12}s_{13} & s_{13}e^{-i\delta} \\ -s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & c_{12}s_{23} - s_{12}s_{23}s_{23}e^{i\delta} & s_{23}c_{13} \\ s_{12}c_{23} - c_{12}s_{23}s_{13}e^{i\delta} & -s_{23}c_{12} - s_{12}c_{23}s_{13}e^{i\delta} & c_{23}c_{13} \end{pmatrix}, \end{aligned} \tag{2.2}$$

where $c_{ij} = \cos\theta_{ij}$ and $s_{ij} = \sin\theta_{ij}$, $(i, j = 1, 2, 3)$, and δ is the phase, which varies in the range $0 \leq \delta \leq 2\pi$. The four independent parameters can be taken as

$$s_{12} = |V_{us}| \qquad s_{13} = |V_{ub}| \qquad s_{23} = |V_{cb}| \qquad \text{and} \quad \delta. \tag{2.3}$$

The first three can be extracted from tree level decays mediated by the transitions $s \to u$, $b \to u$ and $b \to c$, respectively. The phase can be extracted from CP-violating transitions or loop processes sensitive to $|V_{td}|$. The standard parametrization is perfectly suited for numerical calculations.

2.1.2 Wolfenstein Parametrization

However if we want to see the structure of the result more transparently, the Wolfenstein parametrization is the right one, where each element is expanded as a power series in the small parameter $\lambda = |V_{us}| = 0.22$ [16]

$$\mathbf{V}_{CKM} = \begin{pmatrix} 1-\frac{\lambda^2}{2} & \lambda & A\lambda^3\left(\rho - i\eta\right) \\ -\lambda & 1-\frac{\lambda^2}{2} & A\lambda^2 \\ A\lambda^3\left(1-\rho-i\eta\right) & -A\lambda^2 & 1 \end{pmatrix} + \mathcal{O}\left(\lambda^4\right). \tag{2.4}$$

The new independent parameters are

$$\lambda \qquad A \qquad \rho \qquad \text{and} \quad \eta \tag{2.5}$$

and the connection to the parameters of the standard parametrization is established by

$$s_{12} \simeq \lambda \qquad s_{23} \simeq A\lambda^2 \qquad s_{13}e^{i\delta} \simeq A\lambda^3\left(\rho - i\eta\right). \tag{2.6}$$

The next order terms can be found in the literature [17]. The Wolfenstein parametrization is often used to exhibit the hierarchical structure of the CKM matrix

$$\mathbf{V}_{CKM} \sim \begin{pmatrix} 1 & \lambda & \lambda^3 \\ \lambda & 1 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} + \mathcal{O}(\lambda^4). \tag{2.7}$$

We observe that the off-diagonal elements, corresponding to transitions between the generations, are small, whereas the diagonal elements, corresponding to quark transitions within a generation, are close to one.

2.2 Weak Hadron Decays

The weak decays of hadrons are mediated through weak interactions between quarks, which take place at energies much lower than the scale of weak interactions $\mathcal{O}(M_{W,Z})$. Thus they can be described by an effective low energy theory.

2.2.1 Effective Field Theory

Effective Field Theory (EFT) [18, 19, 20] is a tool in quantum field theory, which provides a method to deal with multi-scale problems. Consider a quantum field theory with a characteristic energy scale M, and suppose we are interested in the physics at some much lower scale $E \ll M$. To construct an EFT we choose a cutoff Λ slightly below M and integrate out the heavy degrees of freedom from the theory, i.e., remove the particles which are heavier with respect to the cutoff scale. The EFT contains only the relevant light degrees of freedom and thus can be regarded as a low energy limit of the full theory. The effective Lagrangian takes the form

$$\mathcal{L}_{\text{eff}} = \sum_{n \geqslant 0} C_n \mathcal{O}_n. \tag{2.8}$$

It is an infinite sum over all local operators $\mathcal{O}_n$ which are allowed by the symmetries of the theory, multiplied by coupling constants C_n, the so-called Wilson coefficients.

One may wonder how such a theory can be predictive. To answer this question we replace the coupling constants with dimensionless constants c_{i_n}. With this we can rewrite the Lagrangian

$$\mathcal{L}_{\text{eff}} = \mathcal{L}^{(0)} + \sum_{n>0} \sum_{i_n} \frac{c_{i_n}}{M^n} \mathcal{O}_{i_n}. \tag{2.9}$$

The advantage of the new formula is easily seen. The higher the dimension of the operator, the more powers of M it is suppressed by. In other words, the lowest dimensional operators will be the most important ones. Depending on the precision goal one can truncate the series and thus only a finite number of operators and couplings need to be retained.

2.2.2 Operator Product Expansion

The formal framework to describe the weak interactions of quarks is the Operator Product Expansion (OPE). We can study this technique on the simple example of the weak decay $c \to s u \bar{d}$, see figure 2.1.

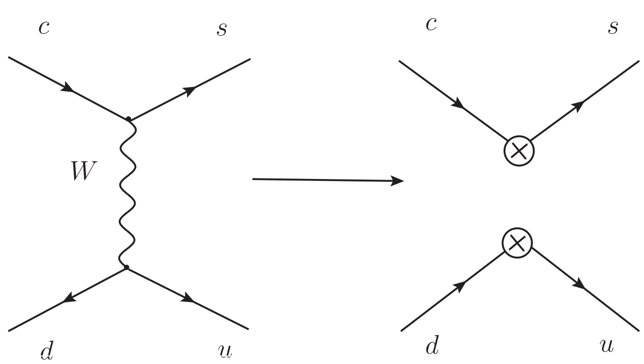


Figure 2.1: $c \to s u \bar{d}$ in the full (left) and the effective (right) theory

The amplitude for the decay is given by

$$\begin{aligned} A_{\text{full}} &= \frac{g_2^2}{8} V_{cs}^* V_{ud} \left[\bar{u}_s(p_s)\gamma^\mu(1-\gamma^5)u_c(p_c)\right] \frac{g_{\mu\nu}}{k^2 - M_W^2} \left[\bar{u}_u(p_u)\gamma^\nu(1-\gamma^5)u_d(p_d)\right] \\ &= \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[\bar{u}_s(p_s)\gamma^\mu(1-\gamma^5)u_c(p_c)\right] \frac{M_W^2}{k^2 - M_W^2} \left[\bar{u}_u(p_u)\gamma_\mu(1-\gamma^5)u_d(p_d)\right] \end{aligned} \tag{2.10}$$

with the Fermi constant G_F

$$\frac{G_F}{\sqrt{2}} = \frac{g_2^2}{8M_W^2}. \tag{2.11}$$

We can expand the amplitude to $\mathcal{O}\left(k^2/M_W^2\right)$ as

$$A_{\text{full}} = -\frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[\bar{u}_s(p_s)\gamma^\mu(1-\gamma^5)u_c(p_c)\right] \left[\bar{u}_u(p_u)\gamma_\mu(1-\gamma^5)u_d(p_d)\right] + \mathcal{O}\left(\frac{k^2}{M_W^2}\right) \tag{2.12}$$

Since k, the momentum transfer through the W propagator, is small compared to M_W, terms of the order $\mathcal{O}\left(k^2/M_W^2\right)$ can be safely neglected and the full amplitude can be approximated by the first term of the r.h.s of eq.(2.12). The same result can be obtained from the effective Hamiltonian

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{cs}^* V_{ud} \left[\bar{s}\gamma^\mu(1-\gamma^5)c\right] \left[\bar{u}\gamma_\mu(1-\gamma^5)d\right] + \text{higher dimensional operators}, \tag{2.13}$$

which corresponds to a low energy theory, in which the heavy particles have been integrated out. The higher dimensional operators correspond to the terms of order $\mathcal{O}\left(k^2/M_W^2\right)$.

This example shows the idea of the OPE: the non-local product of two charged current operators can be expanded into a series of local operators, whose contributions are weighted by effective coupling constants, the Wilson coefficients.

2.2.3 Effective Hamiltonian

The basic starting point for discussing weak decays of hadrons is the effective Hamiltonian, which has the following generic form

$$\mathcal{H}_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i V_{CKM}^i C_i(\mu) \mathcal{O}_i(\mu), \tag{2.14}$$

where $\{\mathcal{O}_i\}$ is a complete set of dimension six local operators relevant for the process. The CKM matrix elements and the Wilson coefficients describe the strength with which the operators enter the Hamiltonian. We observe that both the Wilson coefficients and the local operators depend on the cutoff scale μ[1]. All physics above this scale (high-energy effects) is absorbed into the Wilson coefficients, whereas low-energy effects are contained in the local

[1] From now on μ takes the role of the cutoff scale Λ.

operators. In other words, the problem is separated in high- and low-energy regimes. This is the most important property of the OPE.

2.2.4 Wilson Coefficients

In order to compute the Wilson coefficients we have to choose an operator basis, i.e., a set of operators, so that the effective Hamiltonian of each process can be expressed as a linear combination of these operators. Then the coefficients can be obtained by the requirement that the amplitude A_{full} of the full theory is equal to the amplitude of the effective theory

$$A_{\text{full}} \stackrel{!}{=} A_{\text{eff}} = \frac{G_F}{\sqrt{2}} \sum_i V^i_{CKM} C_i(\mu) \langle \mathcal{O}_i(\mu) \rangle \,, \tag{2.15}$$

where the brackets denote the matrix elements of the corresponding operators $\mathcal{O}_i(\mu)$. This procedure is called matching of the full theory to the effective theory. The full theory is the theory in which all particles appear as dynamical degrees of freedom, whereas the effective theory is constructed by integrating out the heavy (with respect to the cutoff scale) degrees of freedom. If the scale μ is large enough, the matching can be done in perturbation theory. The Wilson coefficients will, in general, depend on the masses of the particles, which were integrated out.

Back to our example. Matching the full to the effective theory

$$A_{\text{full}} \stackrel{!}{=} A_{\text{eff}} = \frac{G_F}{\sqrt{2}} V^*_{cs} V_{ud} C \langle \mathcal{O} \rangle \qquad \text{with} \qquad O = \left[\bar{s}\gamma^\mu(1-\gamma^5)c\right] \left[\bar{u}\gamma_\mu(1-\gamma^5)d\right] \tag{2.16}$$

we see that the Wilson coefficient is equal to 1. In the example we discarded QCD effects. For a complete analysis they must be also included. In our special case QCD effects lead to a second operator, which differs in its colour structure. I will not discuss these complications any further, since they won't be considered in this thesis.

2.3 Flavor Changing Neutral Currents

In the Standard Model, flavor changing neutral currents (FCNCs) at tree level are forbidden. There is for example no direct coupling between the b quark and the s or d quarks. To understand why let us look at the Z boson current in the fermionic Lagrangian eq.(1.27)

$$J^\mu_Z \ni \bar{d}_L \gamma^\mu d_L \tag{2.17}$$

When we apply the field redefinitions, the unitary matrices just cancel out and thus there are no transitions between quarks of different generations. The same is true for the electromagnetic current. As we have already seen in eq.(2.1), there are flavor changing charged currents in the Standard Model. At loop level FCNCs can be induced through a W boson exchange. There are two types of diagrams, which allow flavor changing neutral transitions: penguin and box diagrams.

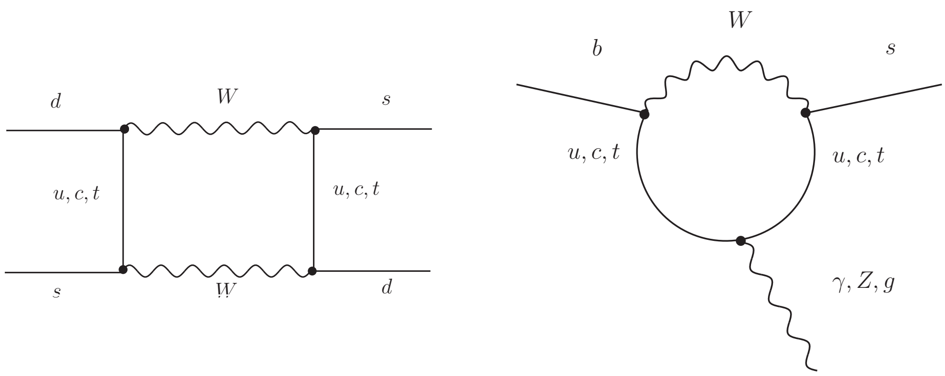


Figure 2.2: Box (left) and penguin (right) diagrams

In the following discussion I will confine myself to the penguin diagrams, since they will be the topic of this thesis.

The FCNCs are important features of flavor physics. They not only allow measurements of the CKM matrix elements, but are also highly sensitive to New Physics. FCNCs are however strongly suppressed in the SM. This is ensured through the GIM mechanism.

2.4 GIM Mechanism

The GIM mechanism [21] was discovered 1970 by S. L. Glashow, J. Iliopolus and L. Maiani. Its discovery involved the introduction of a fourth quark, the charm quark, which was still unknown at that time.

We can study the GIM mechanism on the example $b \to s\gamma$, see figure 2.2. The overall amplitude is the sum of the diagrams with u, c and t in the loop.

$$A = A(m_u^2)V_{ub}V_{us}^* + A(m_c^2)V_{cb}V_{cs}^* + A(m_t^2)V_{tb}V_{ts}^*. \tag{2.18}$$

From the unitarity of the CKM matrix we obtain

$$V_{ub}V_{us}^* + V_{cb}V_{cs}^* + V_{tb}V_{ts}^* = 0. \tag{2.19}$$

Thus if the quarks would have equal masses $m_u = m_c = m_t$, the amplitude will be zero and the FCNCs will be forbidden even at loop level. However, we know that the quarks differ in their masses and especially $m_t \gg m_u, m_c$. So the cancellation is broken by the quark mass difference and the amplitude is proportional to $\ln\left(m_t^2/m_W^2\right)$. Since the top quark is so massive, the loop diagrams are not so strongly suppressed and are expected to have substantial rates, which can be used in testing the Standard Model, or in searching for New Physics.

Chapter 3

Extra Dimensions

3.1 Introduction

The idea of extra (spatial) dimensions (EDs) was first introduced 1914 by G. Nordström, as an attempt to unify the electromagnetism and a scalar version of gravity [22, 23]. After the invention of the General Relativity, T. Kaluza found in 1919 that a five-dimensional (5D) generalization of the Einstein theory can simultaneously describe gravitational and electromagnetic interactions [24]. 1926 O. Klein proposed that the ED is compactified on a circle with a small radius in order to explain its absence in the macroscopic world [25]. With the discovery of the other two fundamental forces and the development of the quantum field theory, the Kaluza-Klein idea was pushed into the background.

In the recent decades however the interest in extra dimensions has increased enormously. EDs are fundamental ingredient of String Theory, since all versions of the theory are consistently formulated only in a space-time of more than four dimensions (actually 10). More recently it was realized that EDs can also be used to address the Hierarchy Problem. Some of the most important scenarios in this regard are the ADD Model [26] by Arkani-Hamed, Dimopoulus and Dvali, and the Randall-Sundrum Model [7], which differ in the number and the geometry of the additional dimensions.

3.2 Kaluza-Klein Decomposition

Mathematically, EDs are described by compact manifolds, since they must have a finite size. Now we want to explore the consequences of the compactification for the fields which propagate through the EDs.

Consider a scalar field on an ED, which is compactified on an interval $[0, R]$. The action reads

$$S = \frac{1}{2}\int d^4x \int_0^R dy\ \partial_M\Phi(x,y)\partial^M\Phi(x,y), \tag{3.1}$$

where $\partial_M\Phi\partial^M\Phi = \partial_\mu\Phi\partial^\mu\Phi - \partial_y\Phi\partial_y\Phi$. In this case it is convenient to decompose the field in a sum of fields which depend on the 4D coordinates, multiplied by functions which depend only on the extra dimension coordinate

$$\Phi(x_\mu, y) = \sum_{n=0}^{\infty} \phi_n(x_\mu) f_n(y). \tag{3.2}$$

The decomposition is the so-called Kaluza-Klein (KK) decomposition. The set of functions $\{f_n\}$ forms a complete set of functions on the interval $[0, R]$

$$\int_0^R dy\ f_m(y) f_n(y) = \delta_{mn}. \tag{3.3}$$

Inserting the KK decomposition into the action and using the orthonormality relation, leads to an effective 4D action

$$\begin{aligned} S = &\frac{1}{2}\int d^4x\ \sum_n \partial_\mu \phi_n(x) \partial^\mu \phi_n(x) \\ &- \frac{1}{2}\int d^4x\ \sum_{n,m} \phi_n(x)\phi_m(x)(f_m(R) f_n'(R) - f_m(0) f_n'(0)) \\ &+ \frac{1}{2}\int d^4x\ \int_0^R dy\ \sum_{n,m} \phi_n(x)\phi_m(x) f_m(y) \partial_y^2 f_n(y) \\ \\ &= \frac{1}{2}\int d^4x\ \sum_n \partial_\mu \phi_n(x) \partial^\mu \phi_n(x) - m_n^2 \phi_n^2(x), \qquad m_n^2 = a_n^2 \end{aligned} \tag{3.4}$$

if

$$\begin{aligned} \partial_y f_n(y) &= -a_n^2 f_n(y), \\ f_n(y) = 0 \quad \text{and} \quad f'(y) &= 0, \qquad y = 0, R. \end{aligned} \tag{3.5}$$

In this way the 5D scalar field decomposes into a tower of 4D scalar fields with different masses, the so-called KK tower. The differential equation eq.(3.5) describes the behaviour of the fields along the fifth dimension. The KK decomposition is an important tool to match a 5D theory to a 4D effective theory. It will be used later in this thesis to derive the field profiles in the Randall-Sundrum model.

3.3 The Randall-Sundrum Scenario

As already mentioned, the Randall-Sundrum (RS) model is a higher-dimensional scenario, whose original purpose was the solution of the Hierarchy Problem. It is based on the five dimensional space-time $R^4 \times M$ with the metric

$$G_{MN} = \begin{pmatrix} e^{-2\sigma(\phi)}\eta_{\mu\nu} & 0 \\ 0 & -r^2 \end{pmatrix} \qquad M, N = 1, 2, 3, 4, 5 \tag{3.6}$$

with

$$\sigma(\phi) = kr|\phi|, \qquad \eta_{\mu\nu} = \text{diag}(1, -1, -1, -1). \tag{3.7}$$

There r is the compactification radius, k a constant of the order of the Planck mass, connected to the curvature of the space, and $\phi \in [0, \pi]$ the coordinate of the extra dimension. The exponential factor in the metric is called warp factor and as we will see later, it is the source of the hierarchy between the Planck and the electroweak scale.

The ED is compactified on an orbifold $M = S^1/Z_2$, which corresponds to a circle where the opposite points are identified, $\phi \to -\phi$. In this way, we obtain an interval $[0, \pi]$.

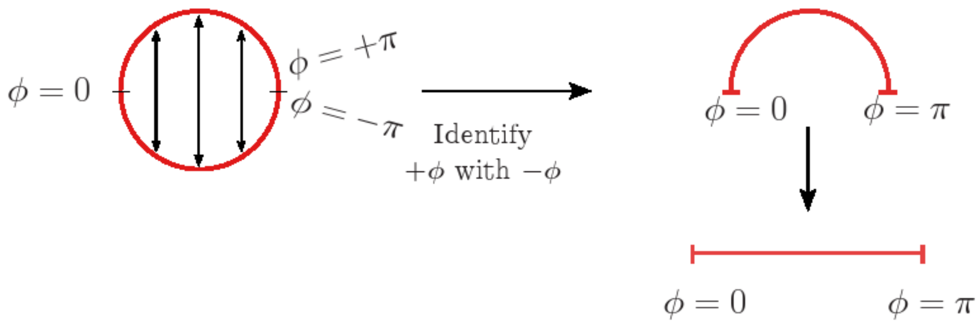


Figure 3.1: Orbifold. Picture taken from [27].

The orbifold fixed points at $\phi = 0, \pi$ are taken as the locations of two 3-branes, extending in the x_μ-directions, so that they are the boundaries of the five dimensional space-time. The 3-branes can support (3+1)-dimensional field theories. The space-time between the two branes is called bulk. The brane at $\phi = 0$ is the so-called visible or infrared (IR) brane, whereas the brane at $\phi = \pi$ is referred to as the hidden or ultraviolet (UV) brane. The 4D metric on the branes is given by

$$g^{vis}_{\mu\nu}(x^\mu) = G_{\mu\nu}(x^\mu, \phi = \pi), \qquad g^{hid}_{\mu\nu}(x^\mu) = G_{\mu\nu}(x^\mu, \phi = 0). \tag{3.8}$$

The SM fields are confined on the visible brane, whereas gravity is allowed to propagate into the fifth dimension.

Now let us see how this scenario solves the Hierarchy Problem. Consider the action of the Higgs field on the visible brane

$$S_{vis} \supset \int d^4x \sqrt{-g_{vis}} \left(g^{\mu\nu}_{vis} \partial_\mu H^\dagger \partial_\nu H - \lambda(|H|^2 - v_0^2)^2 \right), \tag{3.9}$$

where v_0 is the fundamental Higgs VEV. With $g^{\mu\nu}_{vis} = e^{2kr\pi}\eta^{\mu\nu}$ and $\sqrt{-g_{vis}} = \det(-g^{\mu\nu}_{vis}) = e^{-4kr\pi}$ the action can be written as

$$S_{vis} \supset \int d^4x \; e^{-4kr\pi} \left(e^{2kr\pi} \eta^{\mu\nu} \partial_\mu H^\dagger \partial_\nu H - \lambda(|H|^2 - v_0^2)^2 \right). \tag{3.10}$$

After rescaling the Higgs field, $H \to e^{kr\pi}H$, we obtain

$$S_{vis} \supset \int d^4x \; \left(\eta^{\mu\nu} \partial_\mu H^\dagger \partial_\nu H - \lambda(|H|^2 - e^{-2kr\pi} v_0^2)^2 \right). \tag{3.11}$$

We observe that the Higgs VEV in the effective theory is exponentially suppressed compared to the VEV in the fundamental theory

$$v \equiv v_0 e^{-kr\pi}. \tag{3.12}$$

Thus the Higgs gets a TeV scale VEV even though the parameter v_0 in the 5D action is of order M_{Pl}. This is a general result in the RS model. Mass parameters m_0 in the action correspond to physical masses

$$m \equiv m_0 e^{-kr\pi}. \tag{3.13}$$

If we choose $kr \approx 12$, the exponential suppression reduces a mass of order M_{Pl} to only 1 TeV. Thus the ratio of the weak scale to the Planck scale is explained through the exponential factor in the 5D metric

$$\frac{M_{EW}}{M_{Pl}} = e^{-kr\pi} \equiv \epsilon \tag{3.14}$$

and no large ratios appear anywhere else in the model. It has been shown by Goldberger and Wise [28] that values of $kr \approx 12$ can be generated dynamically with reasonable tuning using the Goldberger-Wise stabilisation mechanism, and we will assume that such mechanism is at work.

Chapter 4

The Specific Randall-Sundrum Model

In this chapter I will present a model which is based on the RS idea. Whereas in the original RS scenario only gravity is allowed in the fifth dimension, in this specific RS model all fermions and gauge bosons are promoted to be 5D field. The only field constrained to reside on the visible brane is the Higgs field. Localizing the Higgs field on or near the IR brane is indeed necessary in order to solve the Hierarchy Problem. Furthermore, localizing fermions in the bulk provides a natural explanation of the flavor structure of the SM.

4.1 The Boson Sector

I will begin with an analysis of the gauge sector, considering the simplest case for which the gauge group is the same as in the SM

$$G = SU(3)_C \times SU(2)_L \times U(1)_Y . \tag{4.1}$$

We will start by writing down the 5D action for the boson sector and using the already discussed KK decomposition of the gauge fields, we are going to match the 5D action to the 4D effective action. This will provide certain matching conditions, from which the boson profiles can be derived. The discussion in this chapter will follow [29].

4.1.1 The Boson Action

The boson action can be written as

$$S_{gauge} = \int d^4x\, r \int_{\pi}^{-\pi} d\phi \;\left(\mathcal{L}_{Gauge} + \mathcal{L}_{Higgs} + \mathcal{L}_{GF} + \mathcal{L}_{FP}\right) \tag{4.2}$$

where

$$\begin{aligned} \mathcal{L}_{Gauge} &= \mathcal{L}_C + \mathcal{L}_{W,B} \\ &= \frac{\sqrt{g}}{r} g^{KM} g^{LN} \left(-\frac{1}{4} G^a_{KL} G^a_{MN}\right) + \frac{\sqrt{g}}{r} g^{KM} g^{LN} \left(-\frac{1}{4} W^a_{KL} W^a_{MN} - \frac{1}{4} B_{KL} B_{MN}\right) \end{aligned} \tag{4.3}$$

is the 5D generalization of the SM gauge Lagrangian eq.(1.10) with the field strength tensors G^a_{KM}, W^a_{KM} and B_{KM} defined as in eq.(1.11). Since $SU(3)_C$ remains unbroken in the 5D theory, I will confine myself to the electroweak Lagrangian $\mathcal{L}_{W,B}$. The Higgs Lagrangian is given by

$$\mathcal{L}_{Higgs} = \frac{\delta(|\phi| - \pi)}{r}\left[(D_\mu\Phi)^\dagger(D^\mu\Phi) - V(\Phi)\right], \qquad V(\Phi) = -\mu^2\Phi^\dagger\Phi + \lambda(\Phi^\dagger\Phi)^2. \tag{4.4}$$

It is localized on the IR brane ($\phi = \pi$) by the delta function, defined through the limiting procedure

$$\delta(|\phi| - \pi) \equiv \lim_{\theta\to 0^+} \frac{1}{2}\left[\delta(\phi - \pi + \theta) + \delta(\phi + \pi - \theta)\right]. \tag{4.5}$$

This definition allows to integrate by parts in the action without encountering boundary terms. This is important because otherwise the Lagrangian is not hermitian.
The $SU(2)_L \times U(1)_Y$ gauge group is spontaneously broken by the Higgs VEV $v \approx$ 264 GeV. After electroweak symmetry breaking (EWSB), we decompose the Higgs doublet as

$$\Phi(x) = \frac{1}{\sqrt{2}}\begin{pmatrix} -i\sqrt{2}\varphi^+(x) \\ v + h(x) + i\varphi^3(x)\end{pmatrix}, \qquad \varphi^\pm = \frac{1}{\sqrt{2}}(\varphi^1 \mp i\varphi^2). \tag{4.6}$$

With the usual field redefinitions

$$\begin{aligned} W^\pm_M &= \frac{1}{\sqrt{2}}\left(W^1_M \mp iW^2_M\right), \\ Z_M &= \frac{1}{\sqrt{g_5^2 + g_5'^2}}\left(g_5 W^3_M - g_5' B_M\right), \\ A_M &= \frac{1}{\sqrt{g_5^2 + g_5'^2}}\left(g_5' W^3_M + g_5 B_M\right), \end{aligned} \tag{4.7}$$

the $W^\pm$ and Z^0 bosons acquire masses

$$M_W = \frac{g_5 v}{2}, \qquad M_Z = \frac{\sqrt{g_5^2 + g_5'^2}\,v}{2}, \tag{4.8}$$

whereas the photon remains massless $M_A = 0$. The couplings g_5 and g_5' are the 5D gauge couplings of $SU(2)_L$ and $U(1)_Y$, respectively, which are related to the 4D couplings through the relations

$$g_2 = \frac{g_5}{\sqrt{2\pi r}}, \qquad g_1 = \frac{g_5'}{\sqrt{2\pi r}}. \tag{4.9}$$

The kinetic terms of the gauge and the Higgs Lagrangian contain mixed terms between the gauge fields and the scalar components $W^\pm_\phi$, Z_ϕ and A_ϕ, and between the gauge and the scalar fields $\varphi^\pm$ and φ^3. These terms can be removed by a proper choice of the gauge-fixing

Lagrangian. The gauge-fixing terms are chosen to be

$$
\begin{aligned}
\mathcal{L}_{GF} = &-\frac{1}{2\xi}\left(\partial^\mu A_\mu - \xi\left[\frac{\partial_\phi e^{-2\sigma(\phi)}A_\phi}{r^2}\right]\right)^2 \\
&-\frac{1}{2\xi}\left(\partial^\mu Z_\mu - \xi\left[\frac{\delta(|\phi|-\pi)}{r}M_Z\varphi^3 + \frac{\partial_\phi e^{-2\sigma(\phi)}Z_\phi}{r^2}\right]\right)^2 \\
&-\frac{1}{\xi}\left(\partial^\mu W^+_\mu - \xi\left[\frac{\delta(|\phi|-\pi)}{r}M_W\varphi^+ + \frac{\partial_\phi e^{-2\sigma(\phi)}W^+_\phi}{r^2}\right]\right) \\
&\times\left(\partial^\mu W^-_\mu - \xi\left[\frac{\delta(|\phi|-\pi)}{r}M_W\varphi^- + \frac{\partial_\phi e^{-2\sigma(\phi)}W^-_\phi}{r^2}\right]\right).
\end{aligned}
\tag{4.10}
$$

Note that the expressions contain squares of delta function. These terms are however not problematic, since they will be precisely cancelled by other terms which arise later. The form of the Faddeev-Popov Lagrangian $\mathcal{L}_{FP}$ will not be discussed here; it can be found in [30].

4.1.2 Kaluza-Klein Decomposition

In the next step we write the KK decompositions of the gauge fields in the form

$$
\begin{aligned}
A_\mu(x,\phi) &= \frac{1}{\sqrt{r}}\sum_n A^{(n)}_\mu(x)\chi^A_n(\phi), & A_\phi(x,\phi) &= \frac{1}{\sqrt{r}}\sum_n a^A_n\varphi^{(n)}_A(x)\partial_\phi\chi^A_n(\phi), \\
Z_\mu(x,\phi) &= \frac{1}{\sqrt{r}}\sum_n Z^{(n)}_\mu(x)\chi^Z_n(\phi), & Z_\phi(x,\phi) &= \frac{1}{\sqrt{r}}\sum_n a^Z_n\varphi^{(n)}_Z(x)\partial_\phi\chi^Z_n(\phi), \\
W^\pm_\mu(x,\phi) &= \frac{1}{\sqrt{r}}\sum_n W^{\pm(n)}_\mu(x)\chi^W_n(\phi), & W^\pm_\phi(x,\phi) &= \frac{1}{\sqrt{r}}\sum_n a^W_n\varphi^{\pm(n)}_W(x)\partial_\phi\chi^W_n(\phi).
\end{aligned}
\tag{4.11}
$$

We choose the vector components W^a_μ and B_μ to be even under the Z_2 orbifold symmetry and the scalar components W^a_ϕ and B_ϕ to be odd. This ensures that the lightest mass eigenstates, the zero modes, correspond to the SM gauge bosons. The profiles χ^a_n form complete sets of functions on the orbifold, which obey the orthonormality relation

$$
\int_{-\pi}^{\pi} d\phi\;\chi^a_m(\phi)\chi^a_n(\phi) = \delta_{mn}. \tag{4.12}
$$

The 4D scalar fields can expanded as

$$
\varphi^\pm(x) = \sum_n b^W_n\varphi^{\pm(n)}_W(x), \qquad \varphi^3(x) = \sum_n b^Z_n\varphi^{(n)}_Z(x). \tag{4.13}
$$

Inserting these decompositions into the action and matching the 5D theory to the 4D theory, leads to the equation of motion (EOM)

$$-\frac{1}{r^2}\partial_\phi e^{-2\sigma(\phi)}\partial_\phi \chi_n^a(\phi) = (m_n^a)^2\chi_n^a(\phi) - \frac{\delta(|\phi|-\pi)}{r}M_a^2\chi_n^a(\phi), \tag{4.14}$$

where m_n^a denotes the masses of the 4D vector fields. From this we obtain boundary conditions by integrating over an infinitesimal interval

$$\begin{aligned} \partial_\phi\chi_n^a(0) &= 0 \qquad \text{UV brane,} \\ \partial_\phi\chi_n^a(\pi^-) &= -\frac{rM_a^2}{2\epsilon^2}\chi_n^a(\pi) \qquad \text{IR brane.} \end{aligned} \tag{4.15}$$

The matching procedure fixes the constants as well

$$a_n^a = -\frac{1}{m_n^a}, \qquad b_n^a = \frac{M_a}{\sqrt{r}}\frac{\chi_n^a(\pi^-)}{m_n^a}. \tag{4.16}$$

At the end of the day we obtain a 4D theory, which contains a tower of massive gauge bosons, accompanied by a tower of massive scalars, as well as the Higgs field.

4.1.3 Some Definitions

It is often convenient to introduce a new coordinate t [12]

$$t = \epsilon e^{\sigma(\phi)}, \qquad t|_{\phi=0} = \epsilon\ , \qquad t|_{\phi=\pi} = 1. \tag{4.17}$$

Integrals over the orbifold are obtained using

$$\int_{-\pi}^{\pi} d\phi \to \frac{2\pi}{L}\int_\epsilon^1 \frac{dt}{t}, \qquad \int_{-\pi}^{\pi} d\phi\ e^{\sigma(\phi)} \to \frac{2\pi}{L\epsilon}\int_\epsilon^1 dt. \tag{4.18}$$

We use the notation

$$f(\pi^-) \equiv \lim_{\theta\to 0^+} f(\pi-\theta) \tag{4.19}$$

to indicate the value of a function f that is discontinuous at $|\phi| = \pi$.

4.1.4 The Boson Profiles

The boson profiles are the solutions of the EOM eq.(4.14). In the bulk $\phi \neq \pi$, the EOM is the same as in the case of the unbroken gauge symmetry [8]. In t notation we obtain a Bessel differential equation of first order

$$\left(t^2\frac{d^2}{dt^2} + t\frac{d}{dt} + \left((x_n t)^2 - 1\right)\right)\chi^{(n)} = 0, \tag{4.20}$$

where $x_n \equiv \frac{m_n}{M_{KK}}$ are dimensionless parameters related to the masses of the gauge bosons and their KK excitations. The general solution is a linear combination of Bessel functions in the form

$$\chi_n = \frac{1}{N_n}\left[a^{(n)}J_1(x_n t) + b^{(n)}Y_1(x_n t)\right], \tag{4.21}$$

where the coefficients $a^{(n)}$ and $b^{(n)}$ are determined by the boundary conditions. The same solution remains valid in the case of spontaneously broken symmetry. We will adopt the more convenient shorthand notation, which already fulfills the boundary condition on the UV brane $c_n^-(\epsilon) = 0$

$$\chi_n(\phi) = N_n\sqrt{\frac{L}{\pi}}tc_n^+(t), \tag{4.22}$$

$$\begin{aligned} c_n^+(t) &= Y_0(x_n\epsilon)J_1(x_n t) - J_0(x_n\epsilon)Y_1(x_n t), \\ c_n^-(t) &= \frac{1}{x_n t}\frac{d}{dt}\left[tc_n^+(t)\right] = Y_0(x_n\epsilon)J_0(x_n t) - J_0(x_n\epsilon)Y_0(x_n t). \end{aligned} \tag{4.23}$$

Using the orthonormality relations, we obtain the normalization constant

$$N_n^{-2} = [c_n^+(1)]^2 + [c_n^-(1)]^2 - \frac{2}{x_n}c_n^+(1)c_n^-(1) - \epsilon^2[c_n^+(\epsilon)]^2. \tag{4.24}$$

In t notation the boundary condition on the IR brane eq.(4.15) can be written as

$$x_n c_n^-(1) = -\frac{g^2v^2}{4M_{KK}^2}Lc_n^+(1) \tag{4.25}$$

where g is the 4D coupling constant. With this one can derive the eigenvalues x_n and the masses of the gauge bosons. Without EWSB ($v = 0$), there is a massless zero mode with flat profile

$$\chi_{\gamma,g}(\phi) = \frac{1}{\sqrt{2\pi}}. \tag{4.26}$$

In the case $m_0 \neq 0$, the profiles for the W and Z boson are given in the approximation $x_0 \ll 1$ by

$$\chi_{W,Z}(\phi) = \frac{1}{\sqrt{2\pi}}\left[1 + \frac{m_{W,Z}^2}{4M_{KK}^2}\left(1 - \frac{1-\epsilon^2}{L} + t^2(1 - 2L - 2\,\mathrm{ln}t)\right) + \mathcal{O}\left(\frac{m_{W,Z}^4}{M_{KK}^4}\right)\right]. \tag{4.27}$$

4.2 The Fermion Sector

In this section I will proceed with a discussion of the fermion sector, concentrating on the quark sector. The discussion will follow [29] again.

To describe 5D fermions we need a representation of the 5D Clifford algebra

$$\{\Gamma_M, \Gamma_N\} = 2\eta_{MN}. \tag{4.28}$$

We know that the matrix γ^5 anti-commutes with the other Dirac matrices and choose

$$\Gamma_M = (\gamma_\mu, i\gamma_5). \tag{4.29}$$

Since γ^5 is now part of the Clifford algebra, it can not be used to construct projection operators $P_{L,R} = \frac{1\mp\gamma^5}{2}$. There is only one irreducible representation of the Lorenz group in 5 dimensions. Thus there are only Dirac spinors and it is not possible to generate chiral fermions[1]. A solution to this problem is the compactification on the orbifold S^1/Z_2. The fields must respect the Z_2 symmetry, so that they are either Z_2-even ($\sim$ cos $(n\phi)$) or Z_2-odd ($\sim$ sin $(n\phi)$). In this way we can distinguish two types of fermions corresponding to the two chiralities.

Each 5D Dirac spinor is now decomposed into two components. The left-handed (right-handed) components of the $SU(2)_L$ doublet Q are chosen to be even (odd) under the Z_2 orbifold symmetry, whereas for the components of the singlet q^c the reversed transformation behaviour is required

$$\begin{aligned} Q &= Q_L + Q_R \qquad \text{where} \quad Z_2\, Q_L = Q_L \quad Z_2\, Q_R = -Q_R \\ q^c &= q^c_L + q^c_R \qquad \text{where} \quad Z_2\, q^c_L = -q^c_L \quad Z_2\, q^c_R = q^c_R. \end{aligned} \tag{4.30}$$

The assignment of the Z_2 parity is such that the zero modes of the even fields correspond to the SM particles. The zero modes of the odd profiles vanish, so that each SM fermion is accompanied by two towers of heavy KK states.

[1] This is in general true for theories with odd dimensions.

4.2.1 The Fermion Action

The quadratic terms in the action can be written in the form

$$
\begin{aligned}
S_{ferm,2} = \int d^4x\, r \int_{-\pi}^{\pi} d\phi \Bigg\{ & e^{-3\sigma(\phi)} \left(\bar{Q} i\partial\!\!\!/ Q + \sum_{q=u,d} \bar{q}^c i\partial\!\!\!/ q^c \right) \\
& - e^{-4\sigma(\phi)} \mathrm{sgn}(\phi) \left(\bar{Q}\mathbf{M}_Q Q + \sum_{q=u,d} \bar{q}^c \mathbf{M}_q q^c \right) \\
& - \frac{e^{-2\sigma(\phi)}}{r} \Big[\bar{Q}_L \partial_\phi e^{-2\sigma(\phi)} Q_R - \bar{Q}_R \partial_\phi e^{-2\sigma(\phi)} Q_L \\
& \quad + \sum_{q=u,d} \left(\bar{q}^c_L \partial_\phi e^{-2\sigma(\phi)} q^c_R - \bar{q}^c_R \partial_\phi e^{-2\sigma(\phi)} q^c_L \right) \Big] \\
& - \delta(|\phi| - \pi) e^{-3\sigma(\phi)} \frac{v}{\sqrt{2}r} \left[\bar{u}_L \mathbf{Y}_u^{(5D)} u^c_R + \bar{d}_L \mathbf{Y}_d^{(5D)} d^c_R + \text{h.c.} \right] \Bigg\}
\end{aligned}
\tag{4.31}
$$

where $\mathbf{M}_{Q,q}$ are diagonal matrices containing the bulk masses, and $\mathbf{Y}_q^{(5D)}$ are the 5D Yukawa matrices, defined by

$$
\mathbf{Y}_q^{(5D)} \equiv \frac{2\mathbf{Y}_q}{k}, \qquad q = u, d. \tag{4.32}
$$

4.2.2 Kaluza-Klein Decomposition

The next step is to perform KK decompositions for the spinor fields

$$
\begin{aligned}
Q_L(x,\phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \mathbf{C}_n^{(Q)}(\phi) a_n^{(U,D)} q_L^{(n)}(x), \qquad & Q_R(x,\phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \mathbf{S}_n^{(Q)}(\phi) b_n^{(U,D)} q_R^{(n)}(x), \\
q^c_L(x,\phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \mathbf{S}_n^{(q)}(\phi) b_n^{(q)} q_L^{(n)}(x), \qquad & q^c_R(x,\phi) &= \frac{e^{2\sigma(\phi)}}{\sqrt{r}} \sum_n \mathbf{C}_n^{(q)}(\phi) a_n^{(q)} q_R^{(n)}(x),
\end{aligned}
\tag{4.33}
$$

where Q and q^c are the 5D Dirac spinors, whereas q^n are the 4D spinor fields. The index n labels not only KK excitations but also fermion generations. The profiles $\mathbf{C}_n^{(Q,q)}$ and $\mathbf{S}_n^{(Q,q)}$ are diagonal $N \times N$ matrices, where each entry refers to different bulk mass parameter. The quantities $a_n^{(U,D,q)}$ and $b_n^{(U,D,q)}$ are N-component vectors in flavor space. It is important to note the following subtlety. While the profiles $\mathbf{C}_n^{(Q)}$ and $\mathbf{S}_n^{(Q)}$ are the same for the up- and down-type quarks belonging to the same $SU(2)_L$ doublet, the vectors $a_n^{(U)}$ and $a_n^{(D)}$ associated with these profiles in the KK decomposition of the up- and down-type 5D quark fields will be different objects.

From now on only the up sector will be discussed, in order to keep the notation as simple as possible. The discussion for the down sector is completely analogous.

Inserting the KK decompositions into the action and requiring that the resulting 4D action reduces to the desired SM form

$$S_{ferm,2} = \sum_n \int d^4x \left[\bar{u}^{(n)}(x) i\partial\!\!\!/ u^{(n)}(x) - m_n \bar{u}^{(n)}(x) u^{(n)}(x)\right], \tag{4.34}$$

we derive the equations of motion for the fermion profiles

$$\begin{aligned}\left(\frac{1}{r}\partial_\phi - \mathbf{M}_Q \mathrm{sgn}(\phi)\right)\mathbf{C}_n^{(Q)}(\phi) a_n^{(U)} &= -m_n e^{\sigma(\phi)} \mathbf{S}_n^{(Q)}(\phi) a_n^{(U)},\\ \left(-\frac{1}{r}\partial_\phi - \mathbf{M}_Q \mathrm{sgn}(\phi)\right)\mathbf{S}_n^{(Q)}(\phi) b_n^{(U)} &= -m_n e^{\sigma(\phi)} \mathbf{C}_n^{(Q)}(\phi) b_n^{(U)}\\ &\quad + \delta(|\phi| - \pi) e^{\sigma(\phi)} \frac{\sqrt{2}v}{kr} \mathbf{Y}_u \mathbf{C}_n^{(u)}(\phi) a_n^{(u)},\end{aligned} \tag{4.35}$$

and

$$\begin{aligned}\left(-\frac{1}{r}\partial_\phi - \mathbf{M}_u \mathrm{sgn}(\phi)\right)\mathbf{C}_n^{(u)}(\phi) a_n^{(u)} &= -m_n e^{\sigma(\phi)} \mathbf{S}_n^{(u)}(\phi) a_n^{(u)},\\ \left(\frac{1}{r}\partial_\phi - \mathbf{M}_u \mathrm{sgn}(\phi)\right)\mathbf{S}_n^{(u)}(\phi) b_n^{(u)} &= -m_n e^{\sigma(\phi)} \mathbf{C}_n^{(u)}(\phi) b_n^{(u)}\\ &\quad + \delta(|\phi| - \pi) e^{\sigma(\phi)} \frac{\sqrt{2}v}{kr} \mathbf{Y}_u^\dagger \mathbf{C}_n^{(Q)}(\phi) a_n^{(U)}.\end{aligned} \tag{4.36}$$

The boundary conditions read

$$\mathbf{S}_n^{(Q,u)}(0) = 0 \tag{4.37}$$

on the UV brane and

$$\begin{aligned}\mathbf{S}_n^{(Q)}(\pi^-) b_n^{(U)} &= \frac{v}{\sqrt{2}M_{KK}} \mathbf{Y}_u C_n^{(u)}(\pi) a_n^{(u)},\\ -\mathbf{S}_n^{(u)}(\pi^-) b_n^{(u)} &= \frac{v}{\sqrt{2}M_{KK}} \mathbf{Y}_u^\dagger C_n^{(Q)}(\pi) a_n^{(U)}\end{aligned} \tag{4.38}$$

on the IR brane. Without the brane-localized Yukawa terms, the profiles $\mathbf{C}_n^{(Q,u)}$ and $\mathbf{S}_n^{(Q,u)}$ form a complete sets of even and odd functions on the orbifold. The delta functions in the EOMs however lead to inconsistencies with the orthonormality relations. Thus we impose the generalized orthonormality relations

$$\begin{aligned}\int_{-\pi}^{\pi} d\phi\; e^{\sigma(\phi)} \mathbf{C}_m^{(Q,u)}(\phi) \mathbf{C}_n^{(Q,u)}(\phi) &= \delta_{mn}\mathbf{1} + \Delta\mathbf{C}_{mn}^{(Q,u)},\\ \int_{-\pi}^{\pi} d\phi\; e^{\sigma(\phi)} \mathbf{S}_m^{(Q,u)}(\phi) \mathbf{S}_n^{(Q,u)}(\phi) &= \delta_{mn}\mathbf{1} + \Delta\mathbf{S}_{mn}^{(Q,u)}.\end{aligned} \tag{4.39}$$

The action reduces to eq.(4.34) if and only if the relations

$$a_n^{(U,u)} = b_n^{(U,u)}, \qquad a_n^{(U)\dagger} a_n^{(U)} + a_n^{(u)\dagger} a_n^{(u)} = 1, \tag{4.40}$$

$$a_m^{(U,u)\dagger} \Delta\mathbf{C}_{mn}^{(Q,u)} a_n^{(U,u)} + a_m^{(u,U)\dagger} \Delta\mathbf{S}_{mn}^{(u,Q)} a_n^{(u,U)} = 0 \tag{4.41}$$

hold. With these one can now compute the $\Delta\mathbf{C}_{mn}^{(Q,u)}$ and $\Delta\mathbf{S}_{mn}^{(Q,u)}$ terms and simplify the boundary conditions on the IR brane

$$\begin{aligned} \mathbf{S}_n^{(Q)}(\pi^-) a_n^{(U)} &= \frac{v}{\sqrt{2} M_{KK}} \mathbf{Y}_u C_n^{(u)}(\pi) a_n^{(u)}, \\ -\mathbf{S}_n^{(u)}(\pi^-) a_n^{(u)} &= \frac{v}{\sqrt{2} M_{KK}} \mathbf{Y}_u^\dagger C_n^{(Q)}(\pi) a_n^{(U)}. \end{aligned} \tag{4.42}$$

Observing that the matrices $\mathbf{C}_n^{(Q,u)}$ and $\mathbf{S}_n^{(Q,u)}$ are non-singular, we can replace the boundary conditions by the decoupled equations

$$\begin{aligned} \mathbf{S}_n^{(Q)}(\pi^-) a_n^{(U)} &= -\frac{v^2}{2M_{KK}^2} \mathbf{Y}_u \mathbf{C}_n^{(u)}(\pi) \left[\mathbf{S}_n^{(u)}(\pi^-)\right]^{-1} \mathbf{Y}_u^\dagger \mathbf{C}_n^{(Q)}(\pi) a_n^{(U)}, \\ \mathbf{S}_n^{(u)}(\pi^-) a_n^{(u)} &= -\frac{v^2}{2M_{KK}^2} \mathbf{Y}_u^\dagger \mathbf{C}_n^{(Q)}(\pi) \left[\mathbf{S}_n^{(Q)}(\pi^-)\right]^{-1} \mathbf{Y}_u \mathbf{C}_n^{(u)}(\pi) a_n^{(u)}. \end{aligned} \tag{4.43}$$

Now we can compute the mass eigenvalues m_n by solving

$$\det\left(\mathbf{1} - \frac{v^2}{2M_{KK}^2} \left[\mathbf{S}_n^{(Q)}(\pi^-)\right]^{-1} \mathbf{Y}_u \mathbf{C}_n^{(u)}(\pi) \left[-\mathbf{S}_n^{(u)}(\pi^-)\right]^{-1} \mathbf{Y}_u^\dagger \mathbf{C}_n^{(Q)}(\pi)\right) = 0, \tag{4.44}$$

and the eigenvectors $a^{(U,u)}$ by solving eq.(4.43). Note that while it is always possible to work with real profiles $\mathbf{C}_n^{(Q,u)}$ and $\mathbf{S}_n^{(Q,u)}$, the eigenvectors are in general complex-valued objects.

4.2.3 The Fermion Profiles

As in the case of the gauge bosons, the fermion profiles can be derived from the equations of motion eq.(4.35) and eq.(4.36). In the bulk they reduce to the equations first obtained in [12]

$$\begin{aligned} (t\partial_t - \nu) f_n^L &= -x_n t f_n^R, \\ (-t\partial_t - \nu) f_n^R &= -x_n t f_n^L, \end{aligned} \tag{4.45}$$

where $\nu = M_{Q,q}/k$ is a dimensionless parameter derived from the bulk mass terms. The system of two coupled first order differential equations implies the second order equation

$$\left[t^2 \partial_t^2 + x_n^2 t^2 - \nu(\nu \mp 1)\right] f_n^{L,R}(t) = 0, \tag{4.46}$$

which is a Bessel differential equation with the general solution

$$f_n^{L,R}(t) = \sqrt{x_n t}\left(a_{L,R}^{(n)} J_{\frac{1}{2}\pm\nu}(x_n t) + b_{L,R}^{(n)} J_{-\frac{1}{2}\mp\nu}(x_n t)\right). \tag{4.47}$$

With the replacement $c_{Q,q} = \pm\nu$ the solution can be written as

$$\begin{aligned} C_n^{(Q,q)}(\phi) &= \mathcal{N}_n(c_{Q,q})\sqrt{\frac{L\epsilon t}{\pi}} f_n^+(t, c_{Q,q}), \\ S_n^{(Q,q)}(\phi) &= \pm\mathcal{N}_n(c_{Q,q})\,\mathrm{sgn}(\phi)\sqrt{\frac{L\epsilon t}{\pi}} f^-(t, c_{Q,q}), \end{aligned} \tag{4.48}$$

where

$$f_n^{\pm}(t,c) = J_{-\frac{1}{2}-c}(x_n\epsilon) J_{\mp\frac{1}{2}+c}(x_n t) \pm J_{\frac{1}{2}+c}(x_n\epsilon) J_{\pm\frac{1}{2}-c}(x_n t). \tag{4.49}$$

The orthonormality relations eq.(4.39) fix the normalization constant

$$\mathcal{N}_n^{-2}(c) = \left[f_n^+(1,c)\right]^2 + \left[f_n^-(1,c)\right]^2 - \frac{2c}{x_n} f_n^+(1,c) f_n^-(1,c) - \epsilon^2\left[f_n^+(\epsilon,c)\right]^2. \tag{4.50}$$

Let us look at the zero-mode $x_n = 0$ explicitly. The solution reads

$$C_0^{(Q,q)}(\phi) = F(c_{Q,q}) t^{c_{Q,q}} \tag{4.51}$$

where

$$F(c) \equiv \mathrm{sgn}[\cos(\pi c)]\sqrt{\frac{1+2c}{1-\epsilon^{1+2c}}} \tag{4.52}$$

denotes the zero-mode profile [11, 12]. The important feature of the zero mode is that the overlap with the IR brane is strongly dependent on the bulk mass parameter c

$$C_0(\pi) = F(c) \approx \sqrt{|1+2c|}\begin{cases} 1, & c > -\frac{1}{2} \\ \epsilon^{-(c+\frac{1}{2})}, & c < -\frac{1}{2} \end{cases}. \tag{4.53}$$

As will see in the next section, the zero-mode profile plays a crucial role by establishing the fermion mass hierarchies.

4.3 Fermion Mass Hierarchy

As already discussed in the first chapter, the large mass hierarchies in the fermion sector can not be explained within the SM. The framework of the RS model, however, provides a natural explanation to this problem. To understand this, we work in the "zero-mode approximation" (ZMA), in which one first solves the equations of motion for the fermion profiles without the Yukawa terms and than treats these as a perturbation [12, 11, 31]. In the ZMA the fermion profiles on the IR brane read

$$C_n^{(Q,q)}(\pi) \to \sqrt{\frac{L\epsilon}{\pi}} F(c_{Q,q}), \qquad S_n^{(Q,q)}(\pi^-) \to \sqrt{\frac{L\epsilon}{\pi}} \frac{x_n}{F(c_{Q,q})}. \tag{4.54}$$

With these the boundary conditions eq.(4.42) can be written in the form

$$\frac{\sqrt{2}m_n}{v}\hat{a}_n^{(U)} = \mathbf{Y}_u^{\text{eff}}\hat{a}_n^{(u)} = 0, \qquad \frac{\sqrt{2}m_n}{v}\hat{a}_n^{(u)} = (\mathbf{Y}_u^{\text{eff}})^\dagger\hat{a}_n^{(U)} = 0 \tag{4.55}$$

where we introduced effective Yukawa matrices and rescaled the a vectors

$$\left(Y_u^{\text{eff}}\right)_{ij} \equiv F(c_{Q_i})(Y_u)_{ij}F(c_{u_j}), \qquad \hat{a}_n^{(A)} \equiv \sqrt{2}a_n^{(A)}. \tag{4.56}$$

This equation shows how the fermion mass hierarchies are generated in the RS model: Starting with completely anarchic 5D Yukawa couplings, i.e., complex-valued matrices $\mathbf{Y}_q$ with random elements, the mass hierarchies can be reproduced by assuming a hierarchical structure of the zero-mode profiles $F(c)$

$$|F(c_{A_1})| < |F(c_{A_2})| < |F(c_{A_3})|. \tag{4.57}$$

Such a hierarchy is completely natural and can be generated by small $\mathcal{O}(1)$ differences in the bulk mass parameters c_{A_i}.

The mass eigenvalues m_n can be derived from the equation

$$\det\left(m_n^2\mathbf{1} - \frac{v^2}{2}\mathbf{Y}_u^{\text{eff}}(\mathbf{Y}_u^{\text{eff}})^\dagger\right) = 0. \tag{4.58}$$

The eigenvectors $\hat{a}_n^{(Q)}$ and $\hat{a}_n^{(q)}$ of the matrices $\mathbf{Y}_q^{\text{eff}}(\mathbf{Y}_q^{\text{eff}})^\dagger$ and $(\mathbf{Y}_q^{\text{eff}})^\dagger\mathbf{Y}_q^{\text{eff}}$ form the columns of the unitary matrices $\mathbf{U}_q$ and $\mathbf{W}_q$, which appear in the singular-value decomposition

$$\mathbf{Y}_q^{\text{eff}} = \mathbf{U}_q\boldsymbol{\lambda}_q\mathbf{W}_q^\dagger, \tag{4.59}$$

where

$$\boldsymbol{\lambda}_u = \frac{\sqrt{2}}{v}\text{diag}(m_u, m_c, m_t), \qquad \boldsymbol{\lambda}_d = \frac{\sqrt{2}}{v}\text{diag}(m_d, m_s, m_b) \tag{4.60}$$

contain the mass eigenvalues. With the effective Yukawa matrices eq.(4.56), the hierarchies of the fermion masses and the mixings follow from the Froggatt-Nielsen mechanism, a proposal to explain the disparate fermion masses in the SM by introducing a spontaneously broken family (horizontal) symmetry. Although it doesn't contain extra dimensions, the structure of this scenario turns out to be similar to the one in our case [32]. Taking the determinant of eq.(4.60), we obtain

$$
\begin{aligned}
m_u m_c m_t &= \frac{v^3}{2\sqrt{2}}\,|\det(\mathbf{Y}_u)| \prod_{i=1,2,3} |F(c_{Q_i})F(c_{u_i})|, \\
m_d m_s m_b &= \frac{v^3}{2\sqrt{2}}\,|\det(\mathbf{Y}_d)| \prod_{i=1,2,3} |F(c_{Q_i})F(c_{d_i})|.
\end{aligned}
\tag{4.61}
$$

One can now evaluate all mass eigenvalues

$$
\begin{aligned}
m_u &= \frac{v}{\sqrt{2}}\frac{|\det(\mathbf{Y}_u)|}{|(M_u)_{11}|}\,|F(c_{Q_1})F(c_{u_1})|, & m_d &= \frac{v}{\sqrt{2}}\frac{|\det(\mathbf{Y}_d)|}{|(M_d)_{11}|}\,|F(c_{Q_1})F(c_{d_1})|, \\
m_c &= \frac{v}{\sqrt{2}}\frac{|(M_u)_{11}|}{|(Y_u)_{33}|}\,|F(c_{Q_2})F(c_{u_2})|, & m_s &= \frac{v}{\sqrt{2}}\frac{|(M_d)_{11}|}{|(Y_d)_{33}|}\,|F(c_{Q_2})F(c_{d_2})|, \\
m_t &= \frac{v}{\sqrt{2}}\,|(Y_u)_{33}|\,|F(c_{Q_3})F(c_{u_3})|, & m_b &= \frac{v}{\sqrt{2}}\,|(Y_d)_{33}|\,|F(c_{Q_3})F(c_{d_3})|,
\end{aligned}
\tag{4.62}
$$

where $(M_q)_{ij}$ is the minor of $\mathbf{Y}_q$. We observe that the mass eigenvalues are proportional to combinations of the zero-mode profiles of the left- and right-handed fermions. The elements of the unitary matrices $\mathbf{U}_q$ and $\mathbf{W}_q$ are given to leading order in hierarchies by

$$
(U_q)_{ij} = (u_q)_{ij} \begin{cases} \frac{F(c_{Q_i})}{F(c_{Q_j})}, & i \le j, \\ \\ \frac{F(c_{Q_j})}{F(c_{Q_i})}, & i > j, \end{cases} \qquad (W_q)_{ij} = (w_q)_{ij} e^{i\phi_j} \begin{cases} \frac{F(c_{q_i})}{F(c_{q_j})}, & i \le j, \\ \\ \frac{F(c_{q_j})}{F(c_{q_i})}, & i > j. \end{cases}
\tag{4.63}
$$

The phase factor appearing in $\mathbf{W}_q$ allows to make the diagonal elements $(U_q)_{ii}$ real. The coefficient matrices $\mathbf{u}_q$ and $\mathbf{w}_q$ are expressed through the elements $(Y_q)_{ij}$ and their minors. Explicitly, they read

$$
\mathbf{u}_q = \begin{pmatrix} 1 & \frac{(M_q)_{21}}{(M_q)_{11}} & \frac{(Y_q)_{13}}{(Y_q)_{33}} \\ -\frac{(M_q)^*_{21}}{(M_q)^*_{11}} & 1 & \frac{(Y_q)_{23}}{(Y_q)_{33}} \\ -\frac{(M_q)^*_{31}}{(M_q)^*_{11}} & \frac{(Y_q)^*_{23}}{(Y_q)^*_{33}} & 1 \end{pmatrix}, \qquad \mathbf{w}_q = \begin{pmatrix} 1 & \frac{(M_q)^*_{21}}{(M_q)^*_{11}} & \frac{(Y_q)^*_{31}}{(Y_q)^*_{33}} \\ -\frac{(M_q)_{21}}{(M_q)_{11}} & 1 & \frac{(Y_q)^*_{32}}{(Y_q)^*_{33}} \\ \frac{(M_q)_{13}}{(M_q)_{11}} & -\frac{(Y_q)_{32}}{(Y_q)_{33}} & 1 \end{pmatrix}.
\tag{4.64}
$$

We can now calculate the CKM matrix

$$
\mathbf{V}_{CKM} = \mathbf{U}_u^\dagger \mathbf{U}_d
\tag{4.65}
$$

to leading order in hierarchies. Note that the leading order $\mathbf{U}_q$ and thus the CKM matrix do not depend on the right-handed profiles. For the Wolfenstein parameters we obtain the useful expressions

$$\lambda = \frac{|F(c_{Q_1})|}{|F(c_{Q_2})|}\left|\frac{(M_d)_{21}}{(M_d)_{11}} - \frac{(M_u)_{21}}{(M_u)_{11}}\right|, \quad A = \frac{|F(c_{Q_2})|^3}{|F(c_{Q_1})|^2|F(c_{Q_3})|}\left|\frac{\frac{(Y_d)_{23}}{(Y_d)_{33}} - \frac{(Y_u)_{23}}{(Y_u)_{33}}}{\left[\frac{(M_d)_{21}}{(M_d)_{11}} - \frac{(M_u)_{21}}{(M_u)_{11}}\right]^2}\right|, \tag{4.66}$$

$$\bar{\rho} - i\bar{\eta} = \frac{(Y_d)_{33}(M_u)_{31} - (Y_d)_{23}(M_u)_{21} + (Y_d)_{13}(M_u)_{11}}{(Y_d)_{33}(M_u)_{11}\left[\frac{(Y_d)_{23}}{(Y_d)_{33}} - \frac{(Y_u)_{23}}{(Y_u)_{33}}\right]\left[\frac{(M_d)_{21}}{(M_d)_{11}} - \frac{(M_u)_{21}}{(M_u)_{11}}\right]}.$$

Note that the size of the CP violation describing quantities $\bar{\rho}$ and $\bar{\eta}$ are to first order independent of the zero mode profiles $F(c_{A_i})$, so that their exact size remains to be explained by some more fundamental theory.

We saw that within the RS model the mass hierarchies of the fermions can be explained through small differences in the bulk mass parameters c. Interpreting the masses as overlaps of the zero-mode profiles with the IR brane, where the Higgs reside, we can generate different masses through different localizations in the bulk. Fermions with large masses are localized near the IR brane, whereas fermions with small masses are localized near the Planck brane.

However, the precise value of the c parameter remains unexplained. Thus although the RS model gives a natural explanation to the mass hierarchies in the fermion sector, it can not predict the precise values of the fermion masses.

Chapter 5

The Decay $b \to s\gamma$ in the Standard Model

Before exploring the $b \to s\gamma$ decay in the RS model, in this chapter I will briefly come back to the SM once again and recapitulate the basic formalism to describe rare B decays and in particular the $b \to s\gamma$ decay. I will discuss the SM contributions to the Wilson coefficients of the electromagnetic and chromomagnetic dipole operators, which play the crucial role in this decay, and outline some basic steps of their calculation.

5.1 Basic Formalism of B decays

Rare B decays are usually described within the framework of an effective low-energy theory obtained by integrating out the heavy degrees of freedom, which in this case are the top quark and the $W^\pm$ bosons. We only take into account operators up to dimension six and neglect the s quark mass. This is justified due to the strong suppression of higher dimension operators in the operator product expansion.

The effective Hamiltonian relevant for the $b \to s\gamma$ transition at scales $\mu_b = \mathcal{O}(m_b)$ can be written as [17]

$$\mathcal{H}_{\text{eff}} = -\frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} \left[\sum_{i=1}^{6} C_i(\mu_b) \mathcal{O}_i + C_{7\gamma}(\mu_b) \mathcal{O}_{7\gamma} + C_{8g}(\mu_b) \mathcal{O}_{8g} \right] \tag{5.1}$$

where $\mathcal{O}_i$ are the relevant local operators and $C_i(\mu_b)$ are the corresponding Wilson coefficients, which contain the complete top- and W-mass dependence.

Note the common factor $V_{ts}^* V_{tb}$. Originally, we have terms with $V_{cs}^* V_{cb}$ and $V_{us}^* V_{ub}$ too, eq.(2.14), since u and c quarks may also appear in the loop. However, $V_{us}^* V_{ub}$ is suppressed by λ^2 relative to $V_{ts}^* V_{tb}$ and can be neglected. Furthermore, we can use the unitarity of the CKM matrix eq.(2.19) to eliminate $V_{cs}^* V_{cb}$:

$$V_{cs}^* V_{cb} = -V_{ts}^* V_{tb} \left(1 + \frac{V_{us}^* V_{ub}}{V_{ts}^* V_{tb}} \right) = -V_{ts}^* V_{tb} + \mathcal{O}(\lambda^2). \tag{5.2}$$

We use the following basis of local operators:

Current-Current Operators

$$\begin{aligned} \mathcal{O}_1 &= (\bar{s}_\alpha c_\alpha)_{V-A}(\bar{c}_\beta b_\beta)_{V-A} \\ \mathcal{O}_2 &= (\bar{s}_\alpha c_\beta)_{V-A}(\bar{c}_\beta b_\alpha)_{V-A} \end{aligned} \tag{5.3}$$

QCD Penguin Operators

$$\begin{aligned} \mathcal{O}_3 &= (\bar{s}_\alpha b_\alpha)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\beta)_{V-A} \\ \mathcal{O}_4 &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V-A} \\ \mathcal{O}_5 &= (\bar{s}_\alpha b_\alpha)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\beta)_{V+A} \\ \mathcal{O}_6 &= (\bar{s}_\alpha b_\beta)_{V-A} \sum_{q=u,d,s,c,b} (\bar{q}_\beta q_\alpha)_{V+A} \end{aligned} \tag{5.4}$$

Magnetic Dipole Operators

$$\begin{aligned} \mathcal{O}_{7\gamma} &= -\frac{e m_b}{8\pi^2} \bar{s}_\alpha \sigma^{\mu\nu}(1+\gamma^5) b_\alpha F_{\mu\nu} \\ \mathcal{O}_{8g} &= -\frac{g_s m_b}{8\pi^2} \bar{s}_\alpha \sigma^{\mu\nu}(1+\gamma^5) T^a_{\alpha\beta} b_\beta G^a_{\mu\nu} \end{aligned} \tag{5.5}$$

where α and β are the color indices. e and $F_{\mu\nu}$ (g_s and $G^a_{\mu\nu}$) denote the electromagnetic (strong) coupling constant and field strength tensor, respectively, and T^a are the color generators. Furthermore, we used the shorthand notation $(\bar{q}_\alpha q_\beta)_{V\pm A} \equiv \bar{q}_\alpha \gamma^\mu (1+\gamma^5) q_\beta$.

The magnetic dipole operators originate from the mass insertion on the external b quark line in the QED and QCD penguin diagrams.

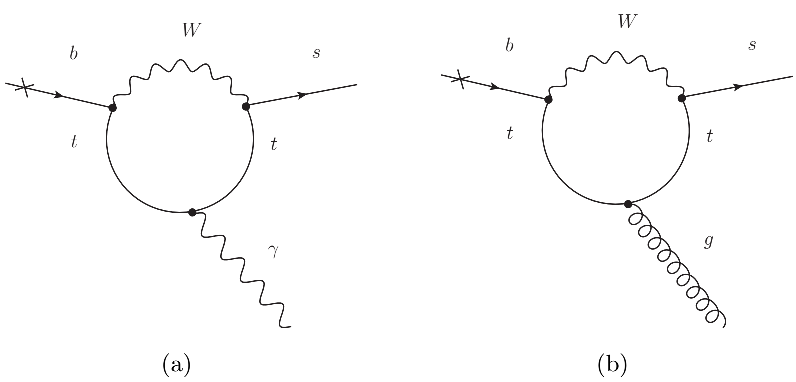


Figure 5.1: Magnetic Photon (a) and Gluon (b) Penguins

Since $m_s \ll m_b$, we do not include the corresponding contributions from the mass insertion on the external s quark line.

5.2 The Electromagnetic Dipole Operator

In the absence of QCD corrections the $b \to s\gamma$ decay is governed by the electromagnetic dipole operator $\mathcal{O}_{7\gamma}$. In order to calculate the relevant Wilson coefficient $C_{7\gamma}$, one has to match the amplitudes computed both in the full and the effective theory at the matching scale $\mu_W = \mathcal{O}(M_W)$. At one loop order the diagrams contributing to the decay in the full theory are shown in figure 5.2. The photon can be emitted from any of the lines. However, only the diagrams where the photon is emitted from the particles in the loop contribute to $C_{7\gamma}$.

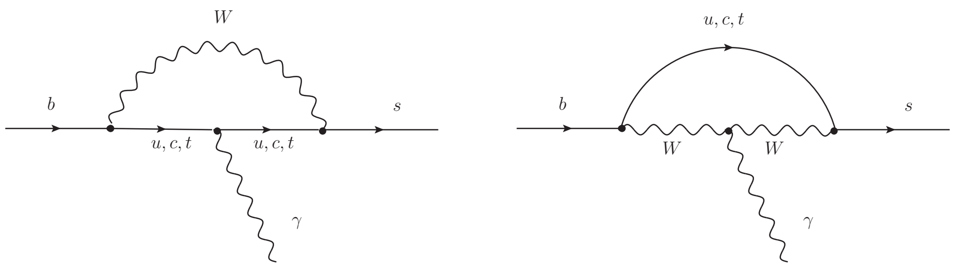


Figure 5.2: SM one loop contributions to $b \to s\gamma$.

We perform the calculation on-shell in Feynman gauge. This requires to consider diagrams in which Goldstone bosons are exchanged.

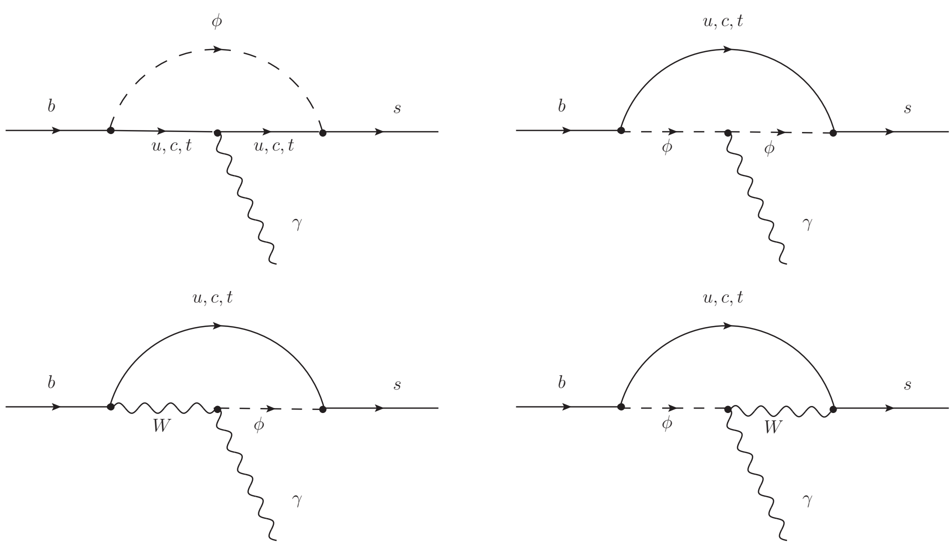


Figure 5.3: Goldstone boson contributions to $b \to s\gamma$ in the SM.

We set all the masses of the light particles to zero in the whole calculation. An exception is the b quark mass, which is being included up to linear order. The terms of order m_b^2 are neglected. The basic steps of the calculation of the amplitudes in the full theory can be found in Appendix A.

After computing the amplitude in the full theory, the Wilson coefficient $C_{7\gamma}$ can be obtained by the requirement

$$A_{\text{full}} \stackrel{!}{=} A_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} C_{7\gamma} \langle \mathcal{O}_{7\gamma} \rangle . \tag{5.6}$$

At the end of the day we have

$$C_{7\gamma}(\mu_W) = \frac{7x - 5x^2 - 8x^3}{24(x-1)^3} + \frac{3x^3 - 2x^2}{4(x-1)^4} \ln x \tag{5.7}$$

where $x = m_t^2/m_W^2$. The result agrees with [17].

Note that we neglected QCD effects. However, they play an important role in this decay too. They are known [33, 34, 35, 36] to enhance $C_{7\gamma}(\mu_b)$ substantially, so that the resulting branching ratio $\mathcal{B}(b \to s\gamma)$ turns out to be by a factor 3 higher than it would be without QCD effects. The leading order QCD correction with $m_t = 170$ GeV and $\mu_b = 5$ GeV gives us [17]

$$\begin{aligned} C_{7\gamma}(\mu_b) &= 0.695\, C_{7\gamma}(\mu_W) + 0.085\, C_{8g}(\mu_W) - 0.158\, C_2(\mu_W) \\ &= 0.695(-0.193) + 0.085(-0.096) - 0.158 = -0.300. \end{aligned} \tag{5.8}$$

We observe that the additive QCD correction, arising from $C_2(\mu_W)$, causes the enormous enhancement of the branching ratio. The multiplicative QCD correction (the factor 0.695) tends to suppress the rate, but fails in competition with the additive contributions.

5.3 The Chromomagnetic Dipole Operator

The QCD corrections require the Wilson coefficient of the chromomagnetic dipole operator $\mathcal{O}_{8g}$, which governs the $b \to sg$ decay. The Wilson coefficient C_{8g} can be calculated analogously to $C_{7\gamma}$. At one loop order the diagrams contributing to the decay $b \to sg$ in the full theory are shown in figure 5.4.
Matching the full to the effective theory at scales $\mu_W = \mathcal{O}(M_W)$, we arrive at

$$C_{8g}(\mu_W) = \frac{2x + 5x^2 - x^3}{8(x-1)^3} + \frac{-3x^2}{4(x-1)^4} \ln x, \tag{5.9}$$

where $x = m_t^2/m_W^2$. The result agrees with [17].

QCD effects lead to a similar enhancement as in the case of $C_{7\gamma}$. The leading order QCD

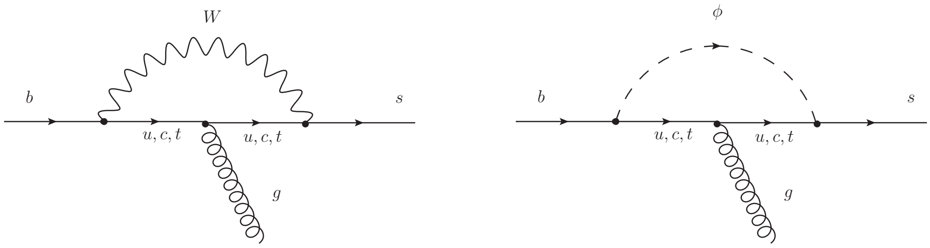


Figure 5.4: SM one loop contributions to $b \to sg$.

corrections gives us [17]

$$\begin{aligned} C_{8g}(\mu_b) &= 0.727\, C_{8g}(\mu_W) - 0.074\, C_2(\mu_W) \\ &= 0.695(-0.096) - 0.074 = -0.144. \end{aligned} \tag{5.10}$$

Chapter 6

The Decay $b \to s\gamma$ in the Randall-Sundrum Model

In this chapter we will explore the $b \to s\gamma$ decay in the RS model. We will discuss the new contributions to the Wilson coefficients $C_{7\gamma}$ and C_{8g}, which arise through the exchange of KK excitations. We will calculate these contributions analytically for the exchange of a single KK excitation and study their impact on the SM Wilson coefficients numerically. Finally, we will compare our results to previous calculations to the $b \to s\gamma$ decay in the RS model [37, 38, 39, 40].

6.1 Contributions to $C_{7\gamma}$ in the RS model

In the RS model the Wilson coefficient of the electromagnetic dipole operator receives new contributions from the KK excitations of the gauge bosons and the up- and the down-type quarks. The relevant diagrams are shown in figure 6.2. In order to determine these contributions we will proceed like in the SM case. We compute the amplitudes in the full and the effective theory and perform a matching at scales $\mu = \mathcal{O}(M_{KK})$ to extract the contributions to the Wilson coefficient $C_{7\gamma}$. The calculation is performed in each case for the exchange of a single KK mode.

6.1.1 Gluon Contributions

Let us start with the contributions from the exchange of a KK gluon but SM fermions in the loop. We follow the basic steps from the SM calculation, given in Appendix A. Applying the Feynman rules in the RS model, which can be found in [41], we obtain the following amplitude in the full theory

$$iA = g_s^2 C_F Q e I_{n2i}^{L,R} I_{ni3}^{L,R} \int d\tilde{k} \, \frac{\bar{u}_s(p_s)\gamma^\mu \left(\not{k} - \not{q} + m_i\right) \gamma^\alpha \left(\not{k} + m_i\right) \gamma_\mu u_b(p_b)}{[(k-q)^2 - m_i^2][k^2 - m_i^2][(k-p_b)^2 - M_n^2]}, \quad i = 1,2,3 \quad (6.1)$$

where m_i denotes the mass of a zero mode (SM) down-type quark and M_n the mass of a KK gluon. Furthermore, $I_{n2i}^{L,R}$ and $I_{ni3}^{L,R}$ denote the relevant elements of the overlap integral

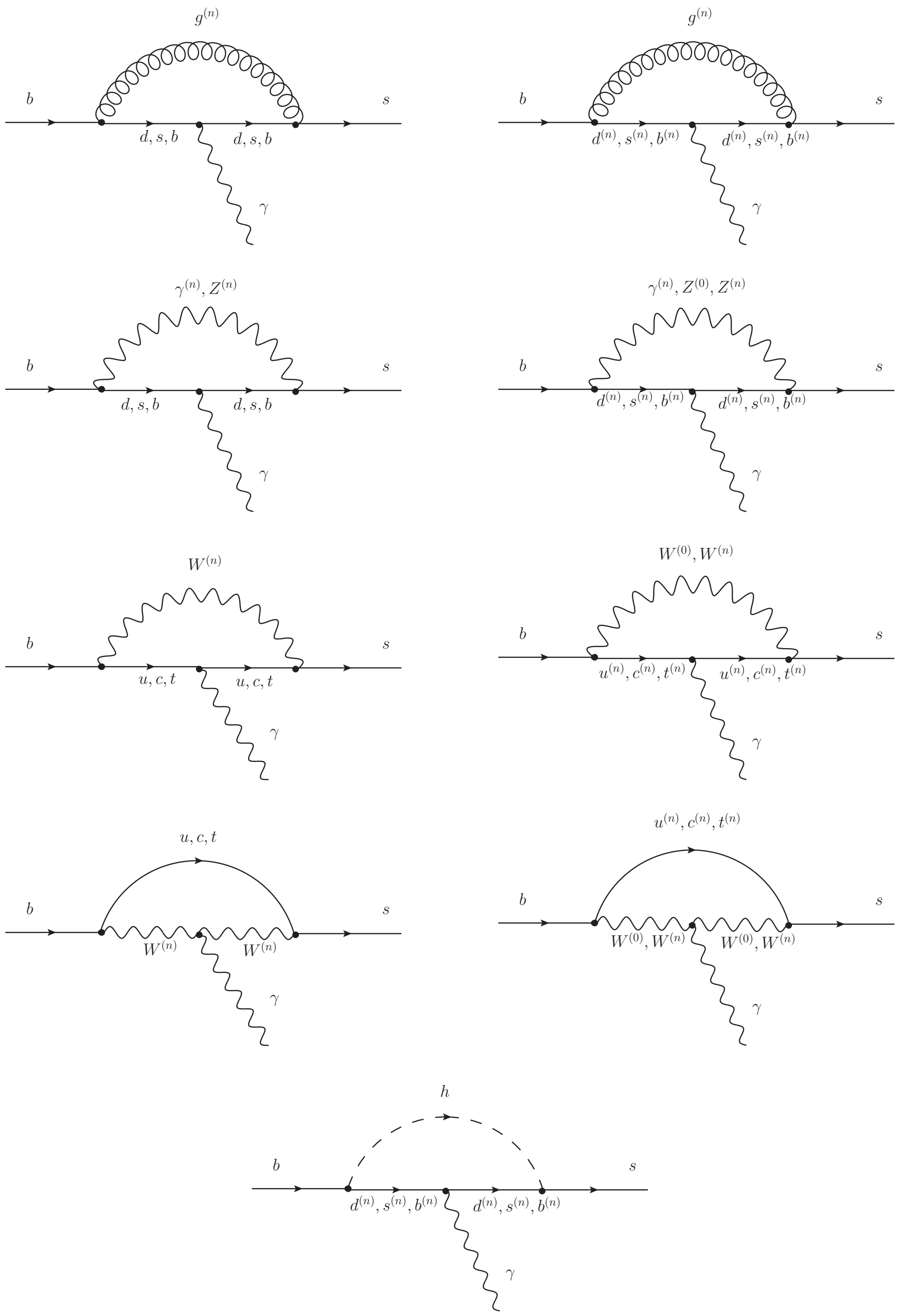


Figure 6.1: One loop diagrams for $b \to s\gamma$ in the RS model

$\mathbf{I}^{L,R}_{nml}$, which represent the fermion coupling (both left- and right-handed) to the n-th gluon KK mode. It is defined by

$$\mathbf{I}^{L,R}_{nml} = \begin{cases} \sqrt{2\pi}\int_{-\pi}^{\pi} d\phi\ \chi_n(\phi)\ e^{\sigma(\phi)} \left(a_m^{(D)\dagger}\mathbf{C}_m^{(Q)}(\phi)\mathbf{C}_l^{(Q)}(\phi)a_l^{(D)} + a_m^{(d)\dagger}\mathbf{S}_m^{(d)}(\phi)\mathbf{S}_l^{(d)}(\phi)a_l^{(d)}\right), \text{ L} \\ \sqrt{2\pi}\int_{-\pi}^{\pi} d\phi\ \chi_n(\phi)\ e^{\sigma(\phi)} \left(a_m^{(d)\dagger}\mathbf{C}_m^{(d)}(\phi)\mathbf{C}_l^{(d)}(\phi)a_l^{(d)} + a_m^{(D)\dagger}\mathbf{S}_m^{(Q)}(\phi)\mathbf{S}_l^{(Q)}(\phi)a_l^{(D)}\right), \text{ R} \end{cases} . \tag{6.2}$$

After performing the Feynman parametrization, the amplitude can be written as

$$iA = g_s^2 C_F Q e I^{L,R}_{n2i} I^{L,R}_{ni3}\, 2\int_0^1 dy \int_0^1 x\, dx \int d\tilde{l}\, \frac{\text{Num.}}{[l^2-\Delta]^3}, \tag{6.3}$$

where

$$\begin{aligned} l^\mu &= k^\mu - x\left[q^\mu y + (1-y)\, p_b^\mu\right], \\ \Delta &= (1-x+xy)\, m_i^2 + x\,(1-y)\, M_n^2. \end{aligned} \tag{6.4}$$

The numerator reads

$$\begin{aligned} &\gamma^\mu\left(\not{k} - \not{q} + m_i\right)\gamma^\alpha\left(\not{k} + m_i\right)\gamma_\mu \\ = &\gamma^\mu\left(\frac{1-\gamma^5}{2} + \frac{1+\gamma^5}{2}\right)\left(\not{k} - \not{q} + m_i\right)\gamma^\alpha\left(\not{k} + m_i\right)\gamma_\mu\left(\frac{1-\gamma^5}{2} + \frac{1+\gamma^5}{2}\right). \end{aligned} \tag{6.5}$$

It contains four terms

$$\begin{aligned} \mathbf{N1} &= I^L_{n2i} I^L_{ni3}\left[\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)\left(\not{k} - \not{q} + m_i\right)\gamma^\alpha\left(\not{k} + m_i\right)\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)\right] \\ &\ni \frac{1}{4} I^L_{n2i} I^L_{ni3}\left[2x^2 y\,(1-y) + 2x^2\,(1-y)^2\right] m_b\left[\gamma^\alpha, \not{q}\right]\left(1+\gamma^5\right), \\ \\ \mathbf{N2} &= I^R_{n2i} I^R_{ni3}\left[\gamma^\mu\left(\frac{1+\gamma^5}{2}\right)\left(\not{k} - \not{q} + m_i\right)\gamma^\alpha\left(\not{k} + m_i\right)\gamma_\mu\left(\frac{1+\gamma^5}{2}\right)\right] \\ &\ni \frac{1}{4} I^R_{n2i} I^R_{ni3}\left[2x^2 y\,(1-y) + 2x^2\,(1-y)^2\right] m_b\left[\gamma^\alpha, \not{q}\right]\left(1-\gamma^5\right), \\ \\ \mathbf{N3} &= I^L_{n2i} I^R_{ni3}\left[\gamma^\mu\left(\frac{1-\gamma^5}{2}\right)\left(\not{k} - \not{q} + m_i\right)\gamma^\alpha\left(\not{k} + m_i\right)\gamma_\mu\left(\frac{1+\gamma^5}{2}\right)\right] \\ &\ni -I^L_{n2i} I^R_{ni3}\, m_i x\,(1-y)\left[\gamma^\alpha, \not{q}\right]\left(1+\gamma^5\right), \\ \\ \mathbf{N4} &= I^R_{n2i} I^L_{ni3}\left[\gamma^\mu\left(\frac{1+\gamma^5}{2}\right)\left(\not{k} - \not{q} + m_i\right)\gamma^\alpha\left(\not{k} + m_i\right)\gamma_\mu\left(\frac{1-\gamma^5}{2}\right)\right] \\ &\ni -I^R_{n2i} I^L_{ni3}\, m_i x\,(1-y)\left[\gamma^\alpha, \not{q}\right]\left(1-\gamma^5\right). \end{aligned} \tag{6.6}$$

Consider the case with a b quark in the loop ($i = 3$). Only **N1** and **N3** contribute to $C_{7\gamma}$. After performing the momentum and Feynman integrations, we obtain the amplitude

$$A = -\frac{g_s^2}{(4\pi)^2} C_F Q e \left[I^L_{n23} I^L_{n33} \left(\frac{1}{6M_n^2} \right) + I^L_{n23} I^R_{n33} \left(-\frac{1}{2M_n^2} \right) \right] m_b \left[\gamma^\alpha, \not{q} \right] \left(1 + \gamma^5 \right). \tag{6.7}$$

Matching the amplitude to the one in the effective theory, gives the contribution to the Wilson coefficient

$$C^b_{7\gamma} = -\frac{1}{\kappa} \frac{g_s^2}{12M_n^2} C_F Q I^L_{n23} I^L_{n33} + \frac{1}{\kappa} \frac{g_s^2}{4M_n^2} C_F Q I^L_{n23} I^R_{n33}, \qquad \kappa = -\frac{G_F}{\sqrt{2}} V^*_{ts} V_{tb}. \tag{6.8}$$

In the case with a d or s quark in the loop only **N1** contributes, which leads to

$$C^{d,s}_{7\gamma} = -\frac{1}{\kappa} \frac{g_s^2}{12M_n^2} C_F Q I^L_{n2i} I^L_{ni3}, \qquad i = 1, 2. \tag{6.9}$$

Finally, we obtain the contribution from the diagram with a KK gluon and SM fermions in the loop

$$C_{7\gamma} = C^d_{7\gamma} + C^s_{7\gamma} + C^b_{7\gamma}. \tag{6.10}$$

We observe that these contributions go like $1/M^2_{KK}$. It turns out numerically that such contributions are strongly suppressed and lead to very small corrections to the SM value.

Now let us examine the case with down-type KK quarks in the loop. The amplitude reads

$$iA = g_s^2 C_F Q e I^{L,R}_{n2m} G^{L,R}_{0ml} I^{L,R}_{nl3} \int d\tilde{k} \, \frac{\bar{u}_s(p_s) \gamma^\mu \left(\not{k} - \not{q} + m_m \right) \gamma^\alpha \left(\not{k} + m_l \right) \gamma_\mu u_b(p_b)}{[(k-q)^2 - m_m^2][k^2 - m_l^2][(k-p_b)^2 - M_n^2]}, \tag{6.11}$$

where $\mathbf{G}^{L,R}_{0ml}$ is the overlap integral, which couples the m-th and the l-th KK down-type quark modes to the photon zero mode (SM photon), defined by

$$\mathbf{G}^{L,R}_{0ml} = \begin{cases} \sqrt{2\pi} \int_{-\pi}^{\pi} d\phi \, \chi_0(\phi) \, e^{\sigma(\phi)} \left(a_m^{(D)\dagger} \mathbf{C}^{(Q)}_m(\phi) \mathbf{C}^{(Q)}_l(\phi) a_l^{(D)} + a_m^{(d)\dagger} \mathbf{S}^{(d)}_m(\phi) \mathbf{S}^{(d)}_l(\phi) a_l^{(d)} \right), \text{L} \\ \sqrt{2\pi} \int_{-\pi}^{\pi} d\phi \, \chi_0(\phi) \, e^{\sigma(\phi)} \left(a_m^{(d)\dagger} \mathbf{C}^{(d)}_m(\phi) \mathbf{C}^{(d)}_l(\phi) a_l^{(d)} + a_m^{(D)\dagger} \mathbf{S}^{(Q)}_m(\phi) \mathbf{S}^{(Q)}_l(\phi) a_l^{(D)} \right), \text{R} \end{cases}. \tag{6.12}$$

Since the photon zero mode has a constant profile eq.(4.26) and the quark profiles obey the orthonormality condition, the overlap integral gives

$$\mathbf{G}^{L,R}_{0ml} = \delta_{ml} \mathbf{1}. \tag{6.13}$$

The numerator can be computed as before. Again only **N1** and **N3** ($i = m$) contribute to $C_{7\gamma}$. Whereas **N1** leads as in the previous case to contributions which are suppressed by M^2_{KK}, the contributions arising from **N3** go like $1/M_{KK}$ and thus give the leading contributions to

$C_{7\gamma}$. Restricting ourselves to them, we obtain the amplitude

$$A = -\frac{g_s^2}{(4\pi)^2} C_F Q e I^L_{n2m} I^R_{nm3} \left[\frac{1-x^2+2x\ln(x)}{2M_n^2(x-1)^3}\right] m_m \left[\gamma^\alpha, \not{q}\right] \left(1+\gamma^5\right), \qquad x = \frac{m_m^2}{M_n^2}. \quad (6.14)$$

Preforming the matching of the full to the effective amplitude, leads to

$$C_{7\gamma} = -\frac{1}{\kappa}\frac{g_s^2}{2M_n^2} C_F Q I^L_{n2m} I^R_{nm3} \left[\frac{1-x^2+2x\ln(x)}{2(x-1)^3}\right] \frac{m_m}{m_b} \quad (6.15)$$

where m_m denotes the mass of a KK fermion and M_n of a KK gluon.

6.1.2 Higgs Boson Contributions

Contributions suppressed only by M_{KK} can also be found in other diagrams, see appendix B. As an another example we will consider the diagram with a Higgs boson and KK fermions in the loop. The amplitude in the full theory can be written as

$$iA = -QeG^{L,R}_{2m} G^{L,R}_{m3} \int \frac{\bar{s}(\not{k} - \not{q} + m_m)\gamma^\alpha(\not{k}+m_m)b}{\left[(k-q)^2 - m_m^2\right]\left[k^2 - m_m^2\right]\left[(k-p)^2 - m_h^2\right]} d\tilde{k} \quad (6.16)$$

where $G^{L,R}_{2m}$ and $G^{L,R}_{m3}$ are the elements of the Higgs couplings $G^{L,R}_{ml}$ defined as

$$G^{L,R}_{nm} = \begin{cases} \frac{\sqrt{2}\pi}{\epsilon L}\left[a_n^{(D)\dagger}\mathbf{C}_n^{(Q)}(\pi)\mathbf{Y}_d\mathbf{C}_m^{(d)}(\pi)a_m^{(d)} + a_n^{(d)\dagger}\mathbf{S}_n^{(d)}(\pi^-)\mathbf{Y}_d^\dagger\mathbf{S}_m^{(Q)}(\pi^-)a_m^{(D)}\right], \text{ L} \\ \frac{\sqrt{2}\pi}{\epsilon L}\left[a_n^{(d)\dagger}\mathbf{C}_n^{(d)}(\pi)\mathbf{Y}_d\mathbf{C}_m^{(Q)}(\pi)a_m^{(D)} + a_n^{(D)\dagger}\mathbf{S}_n^{(Q)}(\pi^-)\mathbf{Y}_d^\dagger\mathbf{S}_m^{(d)}(\pi^-)a_m^{(d)}\right], \text{ R} \end{cases}. \quad (6.17)$$

After performing the Feynman parametrization, we obtain

$$iA = -QeG^{L,R}_{2m} G^{L,R}_{m3}\, 2\int_0^1 dy \int_0^1 x\, dx \int d\tilde{l}\, \frac{\text{Num.}}{\left[l^2 - \Delta\right]^3}, \quad (6.18)$$

where

$$\begin{aligned} l^\mu &= k^\mu - x\left[q^\mu y + (1-y)\, p^\mu\right] \\ \Delta &= (1 - x + xy)\, m_m^2 + x\,(1-y)\, m_h^2 \end{aligned} \quad (6.19)$$

As in the gluon case we can rewrite the numerator as

$$\begin{aligned} &(\not{k} - \not{q} + m_m)\,\gamma^\alpha\,(\not{k} + m_m) \\ &= \left(\frac{1-\gamma^5}{2} + \frac{1+\gamma^5}{2}\right)(\not{k} - \not{q} + m_m)\,\gamma^\alpha\,(\not{k} + m_m)\left(\frac{1-\gamma^5}{2} + \frac{1+\gamma^5}{2}\right). \end{aligned} \quad (6.20)$$

It contains four terms

$$\begin{aligned}
\mathbf{N1} &= \frac{1}{4}\left[(1-\gamma^5)(\not{k}-\not{q}+m_m)\gamma^\alpha(\not{k}+m_m)(1-\gamma^5)\right] \\
&\ni -\frac{1}{4}G^R_{2m}G^L_{m3}m_m x(1-y)\left[\gamma^\alpha,\not{q}\right]\left(1-\gamma^5\right)+\frac{1}{4}m_m\left[\gamma^\alpha,\not{q}\right]\left(1-\gamma^5\right), \\
\mathbf{N2} &= \frac{1}{4}\left[(1+\gamma^5)(\not{k}-\not{q}+m_m)\gamma^\alpha(\not{k}+m_m)(1+\gamma^5)\right] \\
&\ni -\frac{1}{4}G^L_{2m}G^R_{m3}m_m x(1-y)\left[\gamma^\alpha,\not{q}\right]\left(1+\gamma^5\right)+\frac{1}{4}m_m\left[\gamma^\alpha,\not{q}\right]\left(1+\gamma^5\right), \\
\mathbf{N3} &= \frac{1}{4}\left[(1+\gamma^5)(\not{k}-\not{q}+m_m)\gamma^\alpha(\not{k}+m_m)(1-\gamma^5)\right] \\
&\ni \frac{1}{4}G^L_{2m}G^R_{m3}\left[-(xy-1)x(1-y)-x^2(1-y)^2\right]m_b\left[\gamma^\alpha,\not{q}\right](1+\gamma^5), \\
\mathbf{N4} &= \frac{1}{4}\left[(1-\gamma^5)(\not{k}-\not{q}+m_m)\gamma^\alpha(\not{k}+m_m)(1+\gamma^5)\right] \\
&\ni \frac{1}{4}G^R_{2m}G^L_{m3}\left[-(xy-1)x(1-y)-x^2(1-y)^2\right]m_b\left[\gamma^\alpha,\not{q}\right](1-\gamma^5).
\end{aligned} \tag{6.21}$$

The leading contributions arise from **N2**. Thus we obtain

$$C_{7\gamma} = \frac{1}{\kappa}\frac{1}{2m_h^2}QG^L_{2m}G^R_{m3}\left[\frac{3-4x+x^2+2\ln(x)}{8(x-1)^3}\right]\frac{m_m}{m_b}, \qquad x=\frac{m_m^2}{m_h^2}. \tag{6.22}$$

Since the Higgs boson is much more lighter than the KK excitations, we can approximate the last formula by

$$C_{7\gamma} = \frac{1}{\kappa}QG^L_{2m}G^R_{m3}\frac{1}{8m_b}\frac{1}{m_m}. \tag{6.23}$$

6.2 Contributions to C_{8g} in the RS Model

The RS contributions to C_{8g} can be determined in the same way[1]. The one loop diagrams which contribute to the $b \to sg$ decay in the RS model are similar to the diagrams for the $b \to s\gamma$ decay, figure 6.1, where the SM photon is replaced by a SM gluon. Since the electroweak bosons do not couple to gluons, there are no diagrams where the SM gluon is emitted from the W boson. There is however a new diagram where the gluon is emitted from the KK gluon in the loop, see figure 6.2. Again the leading contributions are those suppressed by M_{KK}. Performing the usual calculation leads to

$$C_{8g} = \frac{1}{\kappa}\frac{g_s^2}{M_n^2}\frac{C_A}{4}I^L_{n2m}I^R_{nm3}\left[\frac{3+4x-7x^2+2x(x+4)\ln(x)}{8(x-1)^3}\right]\frac{m_m}{m_b}, \qquad x=\frac{m_m^2}{M_n^2}. \tag{6.24}$$

[1] For the most diagrams the RS contributions to C_{8g} can actually be obtained much more easily, namely from the contributions to $C_{7\gamma}$ using the replacement $Qe \to g_s t^a$.

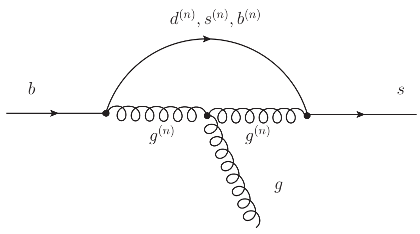


Figure 6.2: Additional KK gluon contribution to $b \to sg$ in the RS model.

6.3 Evolution of the Wilson Coefficients

We have calculated the contributions to the Wilson coefficients $C_{7\gamma}$ and C_{8g} in the RS model at scales $\mu = \mathcal{O}(M_{KK})$. On the other hand the SM contributions have been calculated at the scale $\mu_W = \mathcal{O}(M_W)$. In order to study the impact of the RS corrections on the SM values we must include renormalization group (RG) running into the numeric. The evolution of the Wilson coefficients from the scale M_{KK} to μ_W can be performed using the formulas [17]

$$\begin{aligned} C_{7\gamma}^{RS}(\mu_W) &= \eta^{\frac{16}{21}} C_{7\gamma}(M_{KK}) + \frac{8}{3}\left(\eta^{\frac{14}{21}} - \eta^{\frac{16}{21}}\right) C_{8g}(M_{KK}) \\ C_{8g}^{RS}(\mu_W) &= \eta^{\frac{14}{21}} C_{8g}(M_{KK}) \end{aligned} \tag{6.25}$$

with

$$\eta = \frac{\alpha_s(M_{KK})}{\alpha_s(\mu_W)}. \tag{6.26}$$

This renormalized contributions can now be added to the SM values

$$C_i^{tot}(\mu_W) = C_i^{SM}(\mu_W) + C_i^{RS}(\mu_W). \tag{6.27}$$

which inserted into eq.(5.8) and eq.(5.10) can be used to obtain $C_{7\gamma}(\mu_b)$ and $C_{8g}(\mu_b)$.

6.4 Numerical Analysis

In this section we are going to study the numerical impact of the RS contributions on the Wilson coefficients. Again we will confine ourselves to the contributions suppressed by M_{KK} arising from the Higgs boson and the KK gluon exchange, discussed in the previous sections.

Figure 6.3 shows the predictions for $R_8 = C_{8g}^{tot}/C_{8g}^{SM}$ vs. $R_7 = C_{7\gamma}^{tot}/C_{7\gamma}^{SM}$ in the RS model. The numerical analysis is performed for 1000 randomly generated parameter points, which reproduce the correct quark masses and mixings and furthermore, agree with the ϵ_k- and $Zb\bar{b}-$ constraints. The RS contributions lead to few percent corrections to the SM Wilson coefficents. Furthermore, we observe that C_{8g} receives higher contributions, which can be

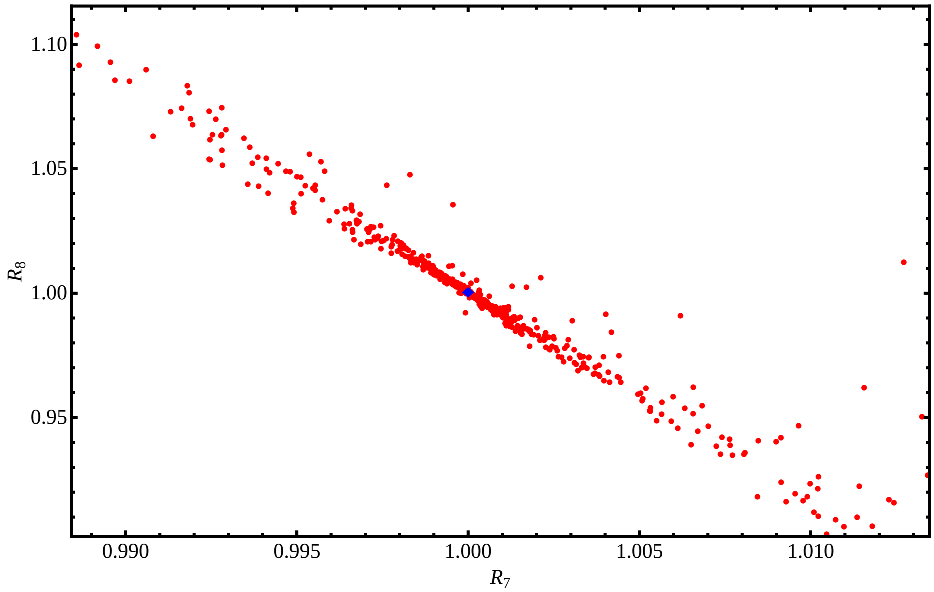


Figure 6.3: Predictions for R_8 vs. R_7. The diamond at (1.0,1.0) represents the SM value.

understood in the following way. Since $C_{8g}^{SM} \approx \frac{1}{2} C_{7\gamma}^{SM}$ and $\delta C_{8g}^{RS} \approx -3\, \delta C_{7\gamma}^{RS}$, we obtain

$$R_{8g} = 1 + \frac{\delta C_{8g}^{RS}}{C_{8g}^{SM}} \approx 1 - 6 \frac{\delta C_{7\gamma}^{RS}}{C_{7\gamma}^{SM}}. \tag{6.28}$$

That leads to the following relation between the both ratios

$$R_{8g} = 7 - 6\, R_{7\gamma} \tag{6.29}$$

which agrees with our observation. The small deviations from the expected behaviour are due to the additional contributions arising from the diagram in figure 6.2.

We can also analyse the contributions to the Wilson coefficients of the opposite chirality operators[2], defined as

$$\begin{aligned} \mathcal{O}'_{7\gamma} &= -\frac{e m_b}{8\pi^2} \bar{s}_\alpha \sigma^{\mu\nu}(1-\gamma^5) b_\alpha F_{\mu\nu}, \\ \mathcal{O}'_{8g} &= -\frac{g_s m_b}{8\pi^2} \bar{s}_\alpha \sigma^{\mu\nu}(1-\gamma^5) T^a_{\alpha\beta} b_\beta G^a_{\mu\nu}. \end{aligned} \tag{6.30}$$

In the SM the corresponding Wilson coefficients are given by

$$C'^{SM}_{7\gamma}(\mu_W) = \frac{m_s}{m_b} C^{SM}_{7\gamma}(\mu_W) \qquad \text{and} \qquad C'^{SM}_{8g}(\mu_W) = \frac{m_s}{m_b} C^{SM}_{8g}(\mu_W). \tag{6.31}$$

Due to the suppression by m_s/m_b, the Wilson coefficents of the opposite chirality operators

[2]In our computation these contributions follow from the terms proportional to $(1-\gamma^5)$.

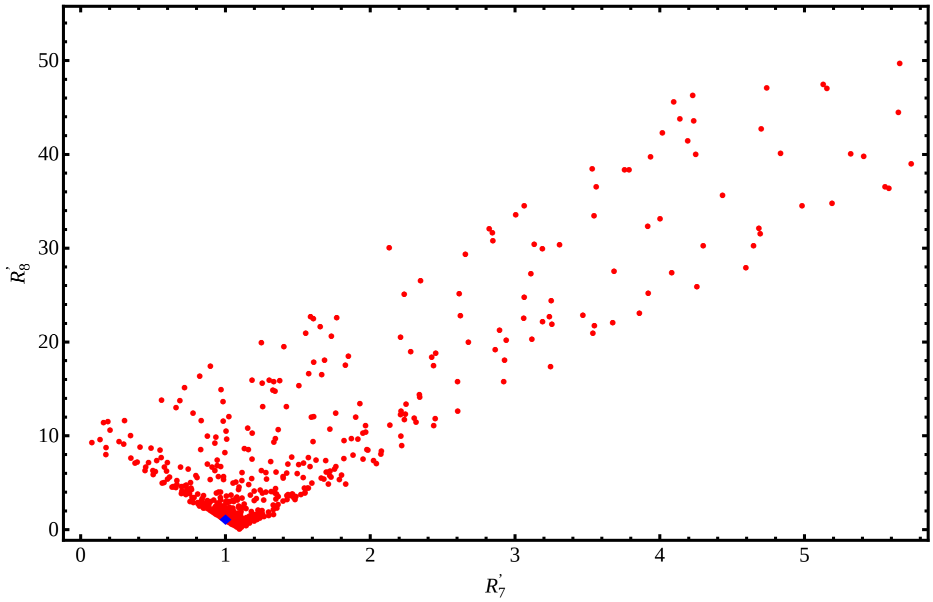


Figure 6.4: Predictions for R_8' vs. R_7'. The diamond at (1.0,1.0) represents the SM value.

are negligible in the SM. This is no longer the case in the RS model. Figure 6.4 shows the predictions for R_8' vs. R_7'. The Wilson coefficients $C_{7\gamma}'$ and C_{8g}' receive sizeable contributions in the RS model in agreement with the predictions made in [40].

These predictions however take into account only the contributions from the exchange of the first KK mode. In order to obtain the exact contributions we should sum over the entire KK tower. There arises the question if this sum converges, which will be discussed further in the next chapter.

6.5 Conclusion

In this chapter we examined the RS contributions to the Wilson coefficients $C_{7\gamma}$ and C_{8g} which arise through the exchange of KK excitations. We restricted ourselves however to the gluon and the Higgs contributions. These RS contributions have been already discussed in [37, 39, 40]. Considering the flavour structure of the contributing diagrams it was shown that the leading contributions arise from the diagrams containing a Higgs boson and KK fermions in the loop, whereas the diagrams with a KK gluon and KK fermions were shown to have small contributions. This can be understood in the following way. Using mass insertion approximation it can be shown that

$$C_{7,ij}^{G} \propto F(c_{Q_i}) Y_{ij}^{d} F(c_{d_j}). \tag{6.32}$$

Thus $C^G_{7,ij}$ has the same flavor structure as the quark mass matrix $m_{d,ij} \propto Y^d_{ij} F(c_{Q_i}) F(c_{d_j})$. Therefore, after unitary rotation into the mass eigenstates after EWSB, $C^G_{7,ij}$ will be approximately diagonal in flavour space and the contribution from KK gluon and KK fermion exchange to $b \to s\gamma$ will be suppressed. On the other hand the contributions from Higgs boson and KK fermion exchange are given by

$$C^H_{7,ij} \propto F(c_{Q_i}) Y^d_{ik} Y^d_{kl} Y^d_{lj} F(c_{d_j}). \tag{6.33}$$

It is obvious that $C^H_{7,ij}$ is not aligned with $m_{d,ij}$ and thus these diagrams should give the leading contributions.

We found however through exact calculation substantial contributions from both the gluon and the Higgs exchange.

Chapter 7

The Anomalous Magnetic Moment of the Electron

In the previous chapter we derived analytical expressions for the contributions to $C_{7\gamma}$ and C_{8g}, which arise from the exchange of a single KK mode. In order to obtain the exact contributions to the SM Wilson coefficients we have to sum over the entire KK tower. There arises the question if this infinite sum converges. In this section we will address this question on a simpler but similar example, namely the anomalous magnetic moment of the electron.

The anomalous magnetic moment is defined by

$$F_2(0) = \frac{g - 2F_1(0)}{2}, \tag{7.1}$$

where the form factors $F_1(q^2)$ and $F_2(q^2)$ are the coefficients of the possible Lorentz structures in the vertex function

$$\Gamma^\mu(p', p) = F_1(q^2)\gamma^\mu + F_2(q^2)\frac{i\sigma^{\mu\nu}q_\nu}{2m}. \tag{7.2}$$

At leading order $F_2(0) = 0$, whereas $F_1(0) = 1$ to all orders in perturbation theory. At one loop order the anomalous magnetic moment of the electron is given by

$$a_e \equiv \frac{g-2}{2} = \frac{\alpha}{2\pi}. \tag{7.3}$$

In the RS model additional contributions arise from the exchange of a tower of heavy KK excitations. We will compute these corrections using two setups in which the leptons are assumed to be localized on the IR brane, corresponding to 4D particles, or to live close to the UV brane, corresponding to light 5D particles.

Consider the electron vertex diagram, see figure 7.1, where heavy KK excitation of photon can propagate in the loop. The amplitude is given by

$$iA = -\sum_{n=0}^{\infty} ie_n^2 \int d\tilde{k}\, \frac{u(p')\gamma^\alpha(\not{p}' + \not{k} + m_e)\gamma^\mu(\not{p} + \not{k} + m_e)\gamma_\alpha u(p)}{[(p'+k)^2 - m_e^2][(p+k)^2 - m_e^2][k^2 - m_n^2]}, \tag{7.4}$$

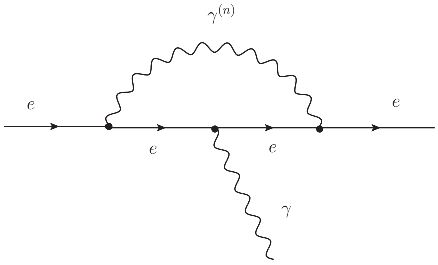


Figure 7.1: Contributions to the magnetic moment of the electron in the RS model.

where m_n ($n \geqslant 0$) denotes the mass of the n^{th} excitation of the photon and e_n contains the interaction of the electron with the photon KK modes. Calculating the amplitude in the usual way, we arrive at

$$F_2(0) = \sum_{n=0}^{\infty} \frac{e_n^2}{4\pi^2} \int_0^1 dy \, \frac{m_e^2 y^2 (1-y)}{(1-y) m_n^2 + y^2 m_e^2}. \tag{7.5}$$

Now we can extract the zero mode, which represents the SM case, and since the mass of the electron is much smaller than the mass of a KK photon, we expand in the small ratio $\tau = m_e^2/m_n^2$. After performing the integration over the Feynman parameters, we obtain

$$F_2(0) = \frac{\alpha}{2\pi} + \sum_{n=1}^{\infty} \frac{e_n^2}{8\pi^2} \left(\frac{2}{3}\tau + \mathcal{O}(\tau^2) \right). \tag{7.6}$$

The coupling is given by

$$e_n = e\sqrt{2\pi} \int_{-\pi}^{\pi} d\phi \, \chi_n(\phi) \, e^{\sigma(\phi)} a^\dagger C(\phi) C(\phi) a. \tag{7.7}$$

In the case we localize the electron on the IR brane, the profiles can be approximated by

$$C(\phi) \approx \sqrt{\frac{L\epsilon}{2\pi}} \, \delta^{1/2}(t-1), \tag{7.8}$$

where the factor $\sqrt{1/2}$ arises from the normalization of the fermion profiles in the way that $e_0 = e$. Thus we have

$$e_n = e\sqrt{2\pi} \int_{\epsilon}^{1} dt \, \chi_n(t) \, \delta(t-1) = \pm e\sqrt{2L}, \tag{7.9}$$

where the sign takes into account the different overlap of the even and the odd photon KK modes with the IR brane. With this we arrive at

$$F_2(0) = \frac{\alpha}{2\pi}\left(1 + \frac{4m_e^2}{3} L \sum_{n=1}^{\infty} \frac{1}{m_n^2}\right). \tag{7.10}$$

The infinite sum can be performed exactly [12] and yields

$$\sum_{n=1}^{\infty} \frac{1}{m_n^2} = \frac{1}{M_{KK}^2} \sum_{n=1}^{\infty} \frac{1}{x_n^2} = \frac{1}{M_{KK}^2} \frac{1}{2c+1}. \tag{7.11}$$

With $c = 1/2$ for the photon, we obtain the finite result

$$F_2(0) = \frac{\alpha}{2\pi}\left(1 + \frac{2}{3} L \frac{m_e^2}{M_{KK}^2}\right). \tag{7.12}$$

The localization of the electron on the IR brane simplifies the calculation enormously, but it is not a correct assumption, because it leads to dangerously large contributions from the leptonic diagrams. More realistically is to place the electron near the UV brane. In this case the profiles are given by

$$C(\phi) \approx \sqrt{\frac{L\epsilon}{2\pi}} F(c)\, t^c. \tag{7.13}$$

The coupling reads

$$e_n = e\sqrt{2\pi} F^2(c) \int_{\epsilon}^{1} dt\, \chi_n(t)\, t^{2c}. \tag{7.14}$$

Inserting the gauge boson profiles eq. (4.22), we can perform the integration and expand in ϵ. Since the electron is localized near the UV brane we can set $c = -1$ which gives $F^2(c) \approx \epsilon$. Thus only the terms proportional to $1/\epsilon$ will contribute. With this we obtain

$$e_n = e\sqrt{2L} N_n \frac{2}{\pi x_n} + \mathcal{O}(\epsilon), \tag{7.15}$$

where N_n is normalization constant of the gauge boson profiles which can be approximated by $N_n \approx [c_n^+(1)]^{-1}$. Thus we arrive at

$$F_2(0) = \frac{\alpha}{2\pi}\left(1 + \frac{16}{3} \frac{L}{\pi^2} \frac{m_e^2}{M_{KK}^2} \sum_{n=1}^{\infty} \frac{N_n^2}{x_n^4}\right). \tag{7.16}$$

The sum converges and can be approximated by

$$\sum_{n=1}^{\infty} \frac{N_n^2}{x_n^4} \approx \sum_{n=1}^{\infty} \frac{N_n^2}{(x_1 + n\pi)^4}. \tag{7.17}$$

Both scenarios lead to finite results which do not depend on unknown UV physics. Thus we can sum over the entire KK tower at least in this simple case. The computation in the

$b \to s\gamma(g)$ case should proceed in the same way and lead to finite results as well. It is however much more complicated due to the mixing between the quark generations.

For complete analysis we have to consider the case of heavy KK electrons in the loop. This case is much more complicated since we encounter double sums over the KK photon and electron excitations. The convergence behaviour of these infinite double sums however remains at present an open question.

Chapter 8

Conclusion and Outlook

In this thesis we calculated the new contributions to the Wilson coefficients of the magnetic dipole operators in the Randall-Sundrum model, which arise through the exchange of heavy KK gauge boson and fermion excitations.

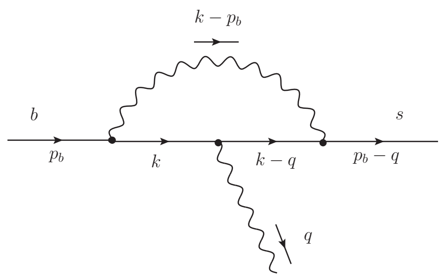
In the first chapter we discussed the construction of the SM Lagrangian using the two main principles, the gauge invariance and the requirement of renormalizability. We briefly reviewed the structure of the Standard Model and introduced the Higgs mechanism in order to incorporate masses into the gauge theory. In chapter 2 we introduced the concept of effective field theory, which is the formal framework to describe weak decays. We showed that the effective Hamiltonian for each weak decay can be written as a sum over some set of local operators, weighted with the Wilson coefficients, which can be calculated by matching the full to the effective theory. Although the Standard Model is in complete agreement with the experimental data, there are still open questions which give rise to numerous extensions. A popular extension of the Standard Model involves the implementation of extra dimensions. Chapter 3 gave a short introduction to extra dimensions and introduced the original Randall-Sundrum idea. In chapter 4 we discussed the actual Randall-Sundrum model, on which our calculations were based. We constructed the 5D Lagrangian and derived the profiles of the 5D gauge bosons and fermions. Furthermore, it was shown that in this particular framework the mass hierarchy of the quarks can be explained in terms of their localization in the bulk. In chapter 5 we discussed the magnetic dipole operators, which govern the $b \to s\gamma$ and $b \to sg$ decays and calculated the corresponding Wilson coefficients. In chapter 6 we discussed which new contributions to the SM Wilson coefficients arise in the framework of the Randall-Sundrum model. We calculated the contributions from the exchange of a KK gluon and a Higgs boson analytically and studied their impact on the SM Wilson coefficients numerically. We found only a few percent corrections to the SM values of $C_{7\gamma}$ and C_{8g}. On the other hand the Wilson coefficients of the opposite chirality operators $C'_{7\gamma}$ and C'_{8g} receive sizeable contributions in the RS model. We performed our computation however for the exchange of a single KK excitation. In order to obtain the exact contributions to the Wilson coefficients we have to sum over the entire KK tower. There arises the question if this sum converges. In chapter 8 we examined this question on a similar example, namely the anomalous magnetic moment of the electron. We calculated the RS contributions and showed that taking into account the entire photon KK tower at least for the case of SM electrons in the loop the infinite sum converges. Much more interesting is the case containing KK electrons in the loop. We encounter double sums whose convergence behaviour still has to be examined. It has to be shown whether these double sums are finite and if not whether there is a special mechanism at work which could lead to mutual cancellation of the heavy KK modes and thus could guarantee the finiteness of the RS contributions. Furthermore, the contributions arising from the exchange of the longitudinal modes of the Z and the W bosons still need to be calculated. They are expected to give sizeable contributions comparable to those from the Higgs boson exchange.

Appendix A

Collection of Calculations

Calculation of $C_{7\gamma}$ in the SM

Here I will give some basic steps in the calculation of the Wilson coefficient $C_{7\gamma}$ of the electromagnetic dipole operator in the SM. Consider the diagram with a top quark in the loop:



We perform the calculation on-shell, i.e., $q^2 = 0$, $p_s^2 = m_s^2$ and $p_b^2 = m_b^2$. Since $m_s \ll m_b$ we set the s quark mass to zero and keep the b quark mass only to linear order, i.e., $m_b^2 = 0$. For simplicity, we will use the following shorthand notation for the spinors $u_s(p_s) \equiv s$ and $u_b(p_b) \equiv b$. With the SM Feynman rules [42], the amplitude reads

$$iA = V_{ts}^* V_{tb} \frac{g_W^2}{8} Qe \int d\tilde{k}\, \frac{\bar{s}\gamma^\mu(1-\gamma^5)\,(\not{k} - \not{q} + m_t)\,\gamma^\alpha\,(\not{k} + m_t)\,\gamma_\mu(1-\gamma^5)b}{[(k-q)^2 - m_t^2][k^2 - m_t^2][(k-p_b)^2 - m_W^2]}. \tag{A.1}$$

Applying the Feynman parametrization, yields the result

$$\int d\tilde{k}\, \frac{1}{\underbrace{[(k-q)^2 - m_t^2]}_{xy}\underbrace{[k^2 - m_t^2]}_{1-x}\underbrace{[(k-p_b)^2 - m_W^2]}_{x(1-y)}} = 2\int_0^1 dy \int_0^1 x\, dx \int d\tilde{l}\, \frac{\text{Num.}}{[l^2 - \Delta]^3} \tag{A.2}$$

with

$$\begin{aligned} l^\mu &= k^\mu - x\left[q^\mu y + (1-y)\, p_b^\mu\right], \\ \Delta &= (1 - x + xy)\, m_t^2 + x\,(1-y)\, m_W^2. \end{aligned} \tag{A.3}$$

The numerator reads

$$\begin{aligned}\text{Num.} &= \gamma^\mu(1-\gamma^5)\,(\not{k}-\not{q}+m_t)\,\gamma^\alpha\,(\not{k}+m_t)\,\gamma_\mu(1-\gamma^5)\\ &= 2\gamma^\mu(\not{k}-\not{q})\gamma^\alpha\not{k}\gamma_\mu(1-\gamma^5)\\ &= 2\gamma^\mu\left[-(1-xy)\not{q}+x(1-y)\not{p}_b\right]\gamma^\alpha\left[xy\not{q}+x(1-y)\not{p}_b\right]\gamma_\mu(1-\gamma^5).\end{aligned}\tag{A.4}$$

It gives rise to four different structures

$$\begin{aligned}&\gamma^\mu\not{q}\,\gamma^\alpha\not{q}\gamma_\mu(1-\gamma^5) = -2q^\alpha\not{q}(1-\gamma^5),\\ &\gamma^\mu\not{q}\,\gamma^\alpha\not{p}_b\gamma_\mu(1-\gamma^5) = -4m_bq^\alpha(1+\gamma^5)+2m_b\not{q}\gamma^\alpha(1+\gamma^5)-4\not{q}p_b^\alpha(1-\gamma^5),\\ &\gamma^\mu\not{p}_b\gamma^\alpha\not{q}\gamma_\mu(1-\gamma^5) = -4m_bq^\alpha(1+\gamma^5)+2m_b\gamma^\alpha\not{q}(1+\gamma^5),\\ &\gamma^\mu\not{p}_b\gamma^\alpha\not{p}_b\gamma_\mu(1-\gamma^5) = -4m_bp_b^\alpha(1-\gamma^5),\end{aligned}\tag{A.5}$$

where we made use of the Dirac equation $\not{p}_b(1-\gamma^5)b = m_b(1+\gamma^5)b$. Since $\epsilon(q)\cdot q = 0$, where $\epsilon(q)$ denotes the photon polarization, we can discard terms proportional to q^α. Thus we obtain

$$\begin{aligned}\text{Num.} = &-4m_bx(1-y)(1-xy)\not{q}\gamma^\alpha(1+\gamma^5)+8x(1-y)(1-xy)\not{q}p_b^\alpha(1-\gamma^5)\\ &+4m_bx^2y(1-y)\gamma^\alpha\not{q}(1+\gamma^5)-8m_bx^2(1-y)^2p_b^\alpha(1+\gamma^5).\end{aligned}\tag{A.6}$$

Using the Gordon identity

$$\bar{s}\left(\frac{p_b^\mu+p_s^\mu}{2}\right)(1-\gamma^5)b = \frac{1}{2}m_b\bar{s}\gamma^\mu(1+\gamma^5)b+\frac{i}{2}\bar{s}\sigma^{\mu\nu}q_\nu(1-\gamma^5)b,\tag{A.7}$$

and the relations

$$\begin{aligned}\not{q}\gamma^\alpha &= \frac{1}{2}\left(\{\gamma^\alpha,\not{q}\}-[\gamma^\alpha,\not{q}]\right)\simeq -\frac{1}{2}\left[\gamma^\alpha,\not{q}\right],\\ \gamma^\alpha\not{q} &= \frac{1}{2}\left([\gamma^\alpha,\not{q}]+\{\gamma^\alpha,\not{q}\}\right)\simeq \frac{1}{2}\left[\gamma^\alpha,\not{q}\right],\end{aligned}\tag{A.8}$$

we simplify the numerator to

$$\text{Num.} = \left[2x^2y(1-y)+2x^2(1-y)^2\right]m_b\left[\gamma^\alpha,\not{q}\right](1+\gamma^5).\tag{A.9}$$

The amplitude reads

$$A = -iV_{ts}^*V_{tb}\frac{g_W^2}{8}Qe\,2\int_0^1 dy\int_0^1 x\,dx\int d\tilde{l}\,\frac{\text{Num.}}{[l^2-\Delta]^3}.\tag{A.10}$$

Now we can perform the integration. First the momentum integration

$$\int\frac{1}{[l^2-\Delta]^3}\,d\tilde{l} = \frac{-i}{(4\pi)^2}\frac{1}{2}\frac{1}{\Delta}.\tag{A.11}$$

The only thing left is the Feynman integration, which yields

$$A = -\frac{1}{(4\pi)^2} V_{ts}^* V_{tb} \frac{g_W^2}{8} Qe \left(\frac{4 - 9x + 5x^3 + 6x\,(1 - 2x)\ln(x)}{6m_W^2 (x-1)^4} \right) m_b \left[\gamma^\alpha, \not{q}\right] \left(1 + \gamma^5\right). \quad \text{(A.12)}$$

Using the replacement

$$\sigma^{\mu\nu} F_{\mu\nu} \to -2i\sigma^{\mu\nu} q_\nu, \qquad \sigma^{\mu\nu} = \frac{i}{2}\left[\gamma^\mu, \gamma^\nu\right], \quad \text{(A.13)}$$

the requirement eq.(5.6) reads

$$A_{\text{full}} \stackrel{!}{=} A_{\text{eff}} = \frac{G_F}{\sqrt{2}} V_{ts}^* V_{tb} C_{7\gamma} \frac{e m_b}{8\pi^2} \bar{s}\left[\gamma^\alpha, \not{q}\right](1 + \gamma^5) b. \quad \text{(A.14)}$$

With this we can easily identify the coefficient $C_{7\gamma}$

$$C_{7\gamma} = \frac{4 - 9x + 5x^3 + 6x\,(1 - 2x)\ln(x)}{18(x-1)^4}, \qquad x = \frac{m_t^2}{m_W^2}. \quad \text{(A.15)}$$

The calculation of the other diagrams is straightforward. The Wilson coefficient eq.(5.7) is the sum of the contributions of the single diagrams.

Gordon Identity

For the calculation of the Wilson coefficients the use of the Gordon Identity was necessary. Here I derive the identity for the case $b \to s\gamma$. We begin with

$$\bar{s}\sigma^{\mu\nu} q_\nu (1 - \gamma^5) b = \bar{s}\frac{i}{2}\left(\gamma^\mu\gamma^\nu - \gamma^\nu\gamma^\mu\right)(p_b - p_s)_\nu (1 - \gamma^5) b. \quad \text{(A.16)}$$

That leads to four different Dirac structures, which we compute using the Dirac equation and setting the s quark mass to zero

$$\begin{aligned}
\bar{s}\gamma^\mu \not{p}_b (1 - \gamma^5) b &= m_b \bar{s}\gamma^\mu (1 + \gamma^5) b, \\
\bar{s}\gamma^\mu \not{p}_s (1 - \gamma^5) b &= 2\bar{s} p_s^\mu (1 - \gamma^5) b, \\
\bar{s}\not{p}_b \gamma^\mu (1 - \gamma^5) b &= -m_b \bar{s}\gamma^\mu (1 + \gamma^5) b + 2\bar{s} p_b^\mu (1 - \gamma^5) b, \\
\bar{s}\not{p}_s \gamma^\mu (1 - \gamma^5) b &= m_s \bar{s}\gamma^\mu (1 - \gamma^5) b = 0.
\end{aligned} \quad \text{(A.17)}$$

Inserting this into eq.(A.16), we obtain

$$\frac{i}{2}\bar{s}\sigma^{\mu\nu} q_\nu (1 - \gamma^5) b = -\frac{1}{4}\left[2m_b \bar{s}\gamma^\mu (1 + \gamma^5) b - 2\bar{s} p_s^\mu (1 - \gamma^5) b - 2\bar{s} p_b^\mu (1 - \gamma^5) b\right] \quad \text{(A.18)}$$

which we can finally rewrite as

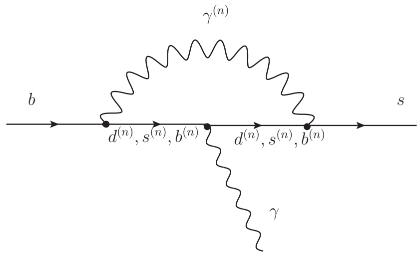
$$\bar{s}\left(\frac{p_b^\mu + p_s^\mu}{2}\right)(1-\gamma^5)b = \frac{1}{2}m_b\bar{s}\gamma^\mu(1+\gamma^5)b + \frac{i}{2}\bar{s}\sigma^{\mu\nu}q_\nu(1-\gamma^5)b. \tag{A.19}$$

Appendix B

Other Contributions

In this appendix I will give some other contributions to $C_{7\gamma}$ and C_{8g} suppressed by M_{KK}.

Photon Contributions

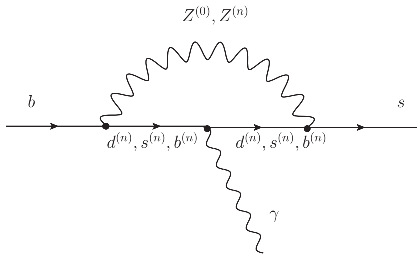


Performing an analogous calculation as in the gluon case (see section 6.1), we obtain

$$C_{7\gamma} = -\frac{1}{\kappa}\frac{e^2}{2M_n^2}Q^3 I^L_{n2m} I^R_{nm3}\left[\frac{1-x^2+2x\ln(x)}{2(x-1)^3}\right]\frac{m_m}{m_b}, \qquad x = \frac{m_m^2}{M_n^2}, \tag{B.1}$$

where m_m denotes the mass of a KK fermion and M_n the mass of a KK photon.

Z Boson Contributions



$$C_{7\gamma} = -\frac{1}{\kappa}\frac{g_W^2}{2M_n^2}\frac{Q\,(g_V - g_A)^2}{\cos^2\theta_W} I^L_{n2m} I^R_{nm3} \left[\frac{1 - x^2 + 2x\ln(x)}{2(x-1)^3}\right]\frac{m_m}{m_b}, \qquad x = \frac{m_m^2}{M_n^2} \tag{B.2}$$

where we used the following definitions

$$g_V = \frac{1}{2}T_3 - Q\sin^2\theta_W \qquad \text{and} \qquad g_A = -\frac{1}{2}T_3. \tag{B.3}$$

The contributions to C_{8g} are given by $C_{8g} = -3C_{7\gamma}$. The other diagrams shown in figure 6.1 lead to contributions suppressed by M^2_{KK}. Since these contributions turn out be very small they won't be discussed any further.

Other sizable contributions are expected to arise from the exchange of the longitudinal modes of the Z and the W bosons [37]. However since the corresponding Feynman rules are at present still not derived, their contributions were not calculated in this thesis.

Appendix C

Numerical Input Parameters

Below I give a compilation of input parameters that were used in the numerical part of this thesis.
The reference values for the quark masses are

$$\begin{array}{cc} m_u = (1.5 \pm 1.0)\ \text{MeV} & m_d = (3.0 \pm 2.0)\ \text{MeV} \\ m_c = (550 \pm 40)\ \text{MeV} & m_s = (50 \pm 15)\ \text{MeV} \\ m_t = (140 \pm 5)\ \text{GeV} & m_b = (2.2 \pm 0.1)\ \text{GeV} \end{array} \tag{C.1}$$

These values are the running quark masses in the $\overline{\text{MS}}$ scheme evaluated at the scale $M_{KK} =$ 1.5 TeV, obtained from the low-energy values [16].
Furthermore, the following input parameters were used [16]

$$\begin{aligned} M_W &= (80.398 \pm 0.025)\ \text{GeV} \\ M_Z &= (91.1876 \pm 0.0021)\ \text{GeV} \\ m_t &= (172.6 \pm 1.4)\ \text{GeV} \\ \alpha_s(M_Z) &= (0.118 \pm 0.003) \\ G_F &= 1.1664 \cdot 10^{-5}\ \text{GeV}^{-2} \end{aligned} \tag{C.2}$$

Unless noted otherwise, the reference value for the Higgs boson mass is $m_h = 150$ GeV.

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